MODELING FINANCIAL DATA USING CONTINUOUS–TIME ECONOMETRIC METHODS (Part II)

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ABSTRACT

In this paper, we demostrate how to implement the proposed estimation procedure in Casas and Gao (2005) to examine the compounded return for Standard & Poor 500 Stock Price Index. Our studies show that there is some kind of weak evidence that certain sections of the data may exhibit long-range dependence. We also conclude that the T-Bill rate should be treated as short–range dependent time series.

KEYWORDS: Continuous–time model, diffusion process, long–range dependence.

1 INTRODUCTION

Consider a stochastic differential equation (SDE) of the form,

$$dX(t) = \mu(X(t))dt + \sigma(X(t))dB(t), \tag{1}$$

where $\mu(X(t))$ is the drift function and $\sigma(X(t))$ is the volatility function of the process.

There is a vast list of references related to developments on the short–term interest rate as a stochastic diffusion process. Vacisek (1977) proposes a model of type (1) with the variance independent of the interest rate. Cox, Ingersoll and Ross (1985) extend this case to a model where the variance is proportional to the interest rate. Such a model is termed as the well–known CIR process. Hull and White (1987) amongst others, study the logarithm of the stochastic volatility (SV) as an Ornstein–Uhlenbeck process. Andersen and Lund (1997) extend the CIR model to associate the spot interest rate with stochastic volatility process through estimating the parameters with the efficient method of moments. Other closely related studies include Aït–Sahalia (1996, 1999), Arapis and Gao (2004), and Hong and Li (2004).

This paper considers the estimation of θ using the estimation procedure in Casas and Gao (2005) for: i) the compounded return of the S&P 500, ii) the compounded return of the T-Bill rate, and iii) the first difference of the T-Bill rate. The estimation of β is of particular interest, as this parameter hints whether the data exhibits LRD or short–range dependence (SRD). Thus, for $0 < \beta < \frac{1}{2}$ the process is said to have LRD, for $\beta = 0$ the observations are uncorrelated, and for $-\frac{1}{2} < \beta < 0$ the process is said to have SRD.

2 FINANCIAL DATA

A good estimation procedure must be able to solve some real data problems if it is to be of any practical value. To test whether the proposed estimation procedure works adequately for real data, two data sets have been studied:

i) the daily values of the S&P 500 Stock Price Index from January 1928 to December 1987 and,

ii) the monthly values of the three–month Treasury Bill rate from January 1963 to December 1998.

The first step is to prepare the data under study such that a set of stationary Gaussian data can be obtained.

In this Section, two transformations to produce stationary data are considered:

- The first difference of the original data set is defined as follows

$$V_t = Z_t - Z_{t-1} \text{ for } t = 1 \dots T. \quad (2)$$

- The compounded return of $\{Z_t\}$ is the first difference of the natural logarithm of the original data set, given by

$$W_t = \ln\left(\frac{Z_t}{Z_{t-1}}\right) \text{ for } t = 2 \dots T. \quad (3)$$

In some of the cases studied in this paper, once the stationarity was assured, the data needed to be slightly truncated to ensure Gaussianity.

2.1 S&P 500

For the first financial example, a section of the S&P 500 Stock Price Index from January 1928 to December 1987 is considered and four subsets taken: the whole set with 16,128 daily values; a set of 10,000 daily values from the 21^{st} of September 1948; a set of 2,000 daily values from the 4^{th} of February 1980; and a set of 500 daily values from the 10^{th} of January 1986. The trajectory of the S&P 500 Stock Price Index is illustrated in Figure 1.

The initial data set, $X(t)$, is transformed to obtain a stationary set using equation (3). Afterwards, the new data set is truncated by the 1% and 99% quantiles to assure normality. Next, the estimation procedure is applied to the transformed data set. The estimates of the parameters involved in the density function (**??**) of the S&P 500 Stock Price Index are found. These are shown in Table 1. The estimate of the spectral density is shown in Figure 2. The β estimates that correspond to the two large data sets within the interval $(0, \frac{1}{2})$ suggest that these sets may display LRD. However, the β estimates corresponding to the two smaller sections of the data set are negative therefore, the smaller sets do not display LRD. For the two larger sections of the data, moreover, our findings are consistent with those results obtained by Ding, *et al.* (1993). They analyse the autocorrelation function (ACF) of the compounded return, W_t, $|W_t|$ and W_t^2 for a large section of the S&P 500 Stock Price Index, from January 1928 to August 1991. Their analysis is repeated in this paper for different sections of the S&P 500 Stock price Index compounded return. The results are shown in Figures 3 and 4 and Table 3. In these figures, the ACF dies off for the smaller sets, but it is still important for large lags for the larger sets.

In summary, our studies show that there is some weak evidence of LRD for large sections of the S&P 500 Stock Index Price, while small sections of the data exhibit SRD.

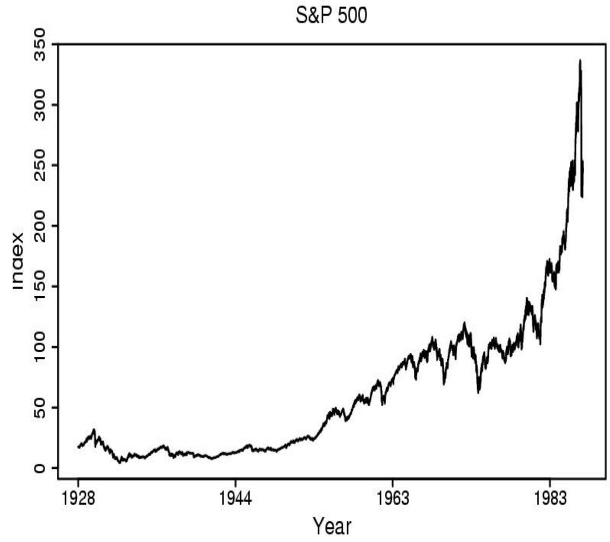

Figure 1: A Section of the S&P 500 Index from Jan. 1928 to Dec. 1987.

T	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$
500	1.810	-0.0979	0.0195
2000	2.4566	-0.0444	0.0188
10000	1.8824	0.0053	0.0131
16128	2.4165	0.0120	0.0197

Table 1: Estimation of S&P 500 Stock Index parameters.

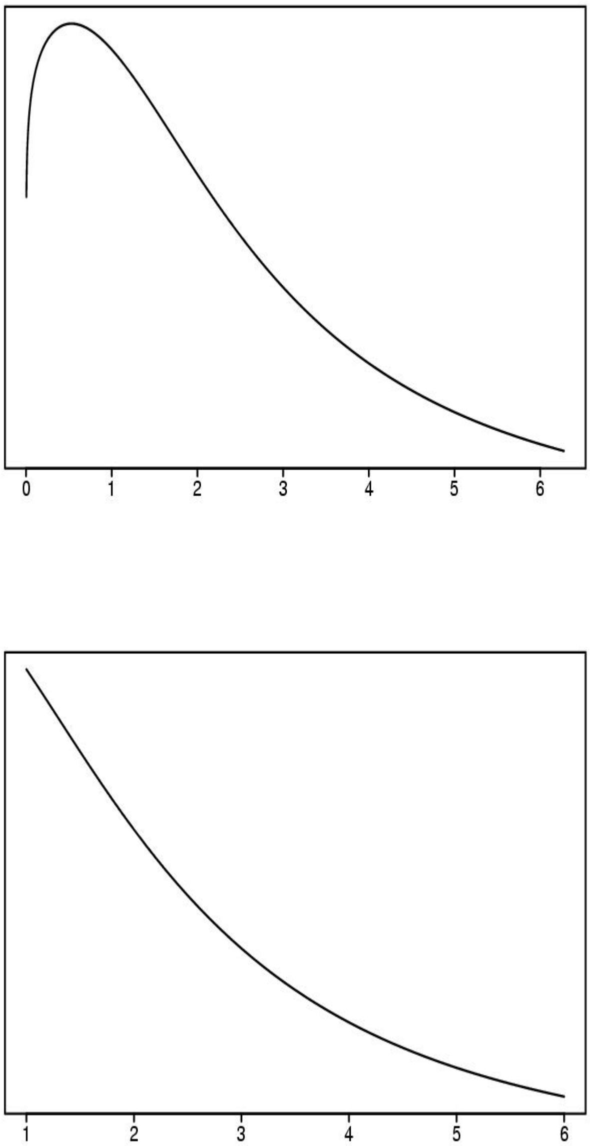

Figure 2: Estimate of the spectral density function referring to the truncated compounded return of sections of the S&P 500 Index: top) $(\hat{\alpha}, \hat{\beta}, \hat{\sigma})$ = (2.4566, -0.0444, 0.0188), bottom) $(\hat{\alpha}, \hat{\beta}, \hat{\sigma})$ = (2.4165, 0.0120, 0.0197).

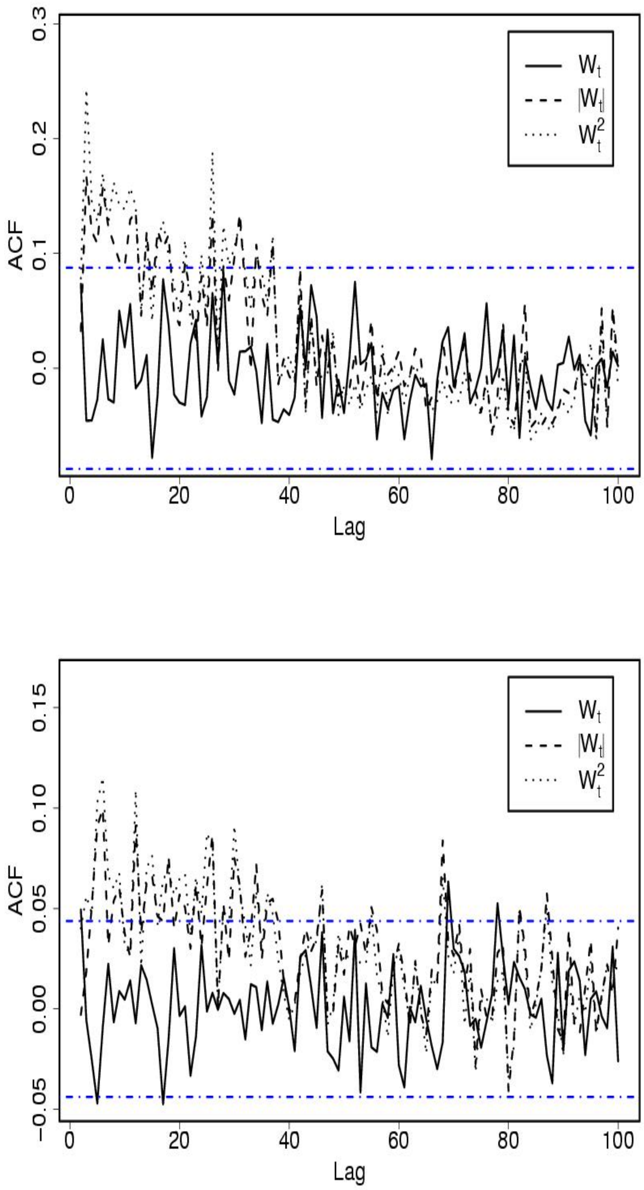

Figure 3: ACF for W_t, $|W_t|$ and W_t^2 of the: top) S&P 500 referring to Jan. 1986 to Dec. 1987, bottom) S&P 500 referring to Feb. 1980 to Dec. 1987.

2.2 T–Bill rate

The three–month T–Bill rate data, shown in Figure 5, are quarterly observations over the period from January 1963 to December 1998. An initial look at the data suggests that this set does not exhibit stationarity. This can be achieved with the appropriate transformation.

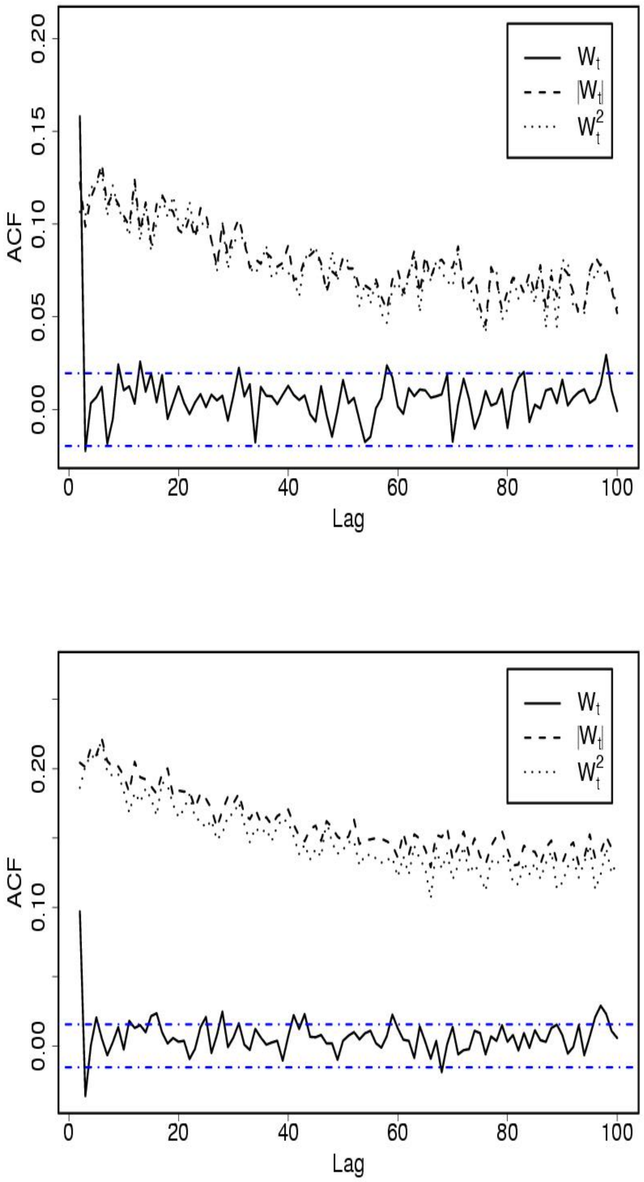

Figure 4: ACF for W_t, $|W_t|$ and W_t^2 of the: top) S&P 500 referring to Sep. 1948 to Dec. 1987, bottom) S&P 500 referring to Jan. 1928 to Dec. 1987.

The two transformations described by equations (2) and (3) were applied to this data. In some cases truncations were needed to ensure Gaussianity.

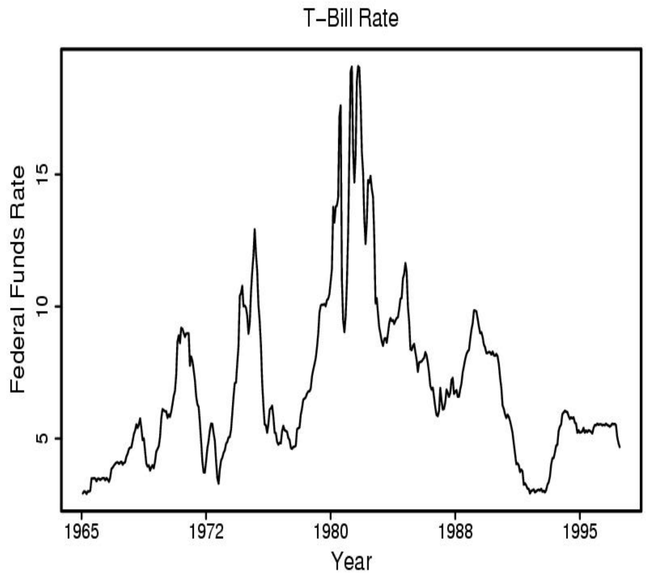

Figure 5: Original T-Bill rate.

The parameters α, β and σ are then estimated using the same estimation procedure as before. The resulting estimates are shown in Table 2. The density function estimate is shown in Figure 6. For each of the transformations, the estimate of β is negative and therefore does not suggest that the data set may display LRD.

Transformation	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$
V_t (trunc. 98%)	0.5322	-0.1440	0.4846
V_t (trunc. 96%)	0.2336	-0.3199	0.3737
W_t	0.7019	-0.0359	0.0774
W_t (trunc. 98%)	0.7091	-0.0342	0.0648

Table 2: Estimation of the T-Bill rate Parameters.

The ACF of V_t and W_t, as well as their absolute values and the square values of the transformed data are examined. The functions are displayed in Figure 7 for the first difference and in Figure 8 for the compounded return. As can be seen from Table 4, the autocorrelation values die off for long lags, i.e. the data does not display LRD as was acknowledged by the results obtained with the estimation procedure discussed above.

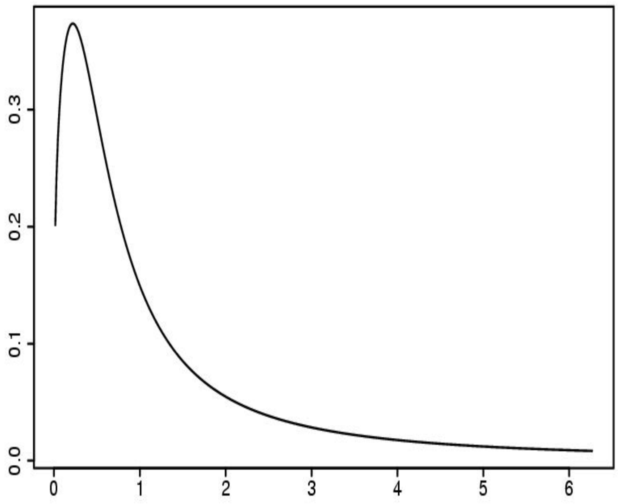

Figure 6: Estimate of the spectral density function with $(\hat{\alpha}, \hat{\beta}, \hat{\sigma}) = (0.5322, -0.1440, 0.4846)$.

3 DISCUSSION

Recently, several methods and models have been proposed to model data with LRD property. This paper has extended one of the models proposed in Gao (2004) to accommodate cases where some sections of the data may exhibit LRD while other sections may not exhibit LRD. Such an extension has then been applied to examine both the S&P 500 index and the T-Bill rate. For the the S&P 500 index, our studies have indicated that there is some kind of weak evidence of LRD for the data values recorded before 1950. In addition, our research provides a kind of answer to the question of whether or not the T–Bill rate should be treated as long–range dependent time series. We conclude that the T-Bill rate does not exhibit LRD.

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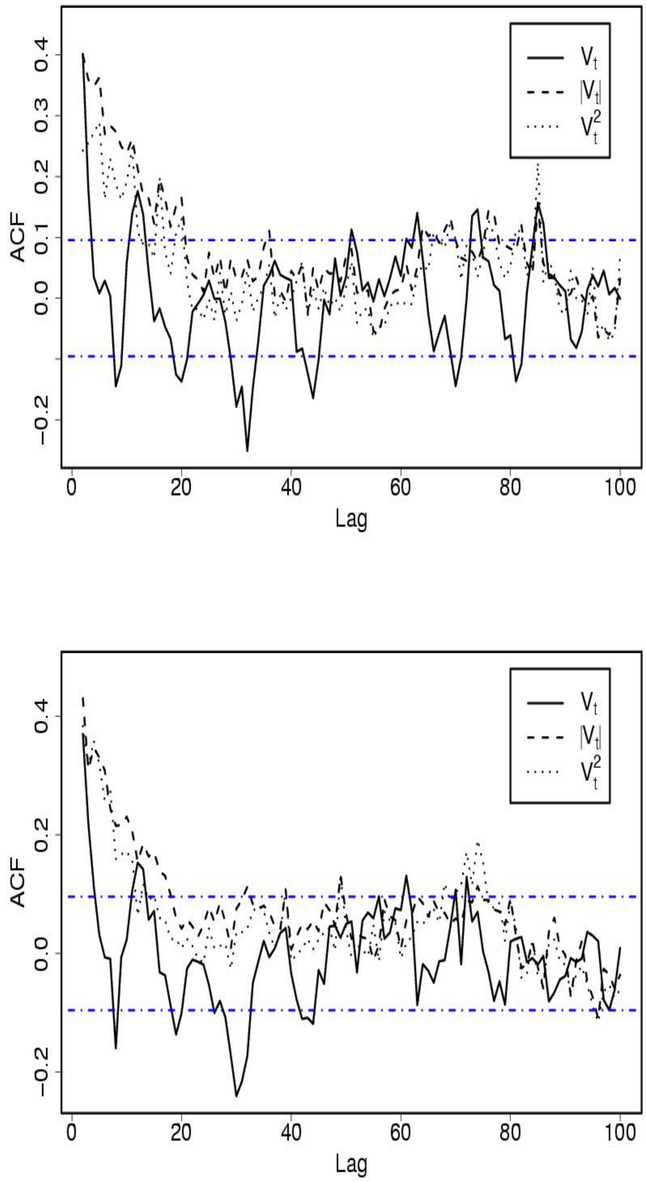

Figure 7: ACF for V_t, $|V_t|$ and V_t^2 of the first difference of the T-Bill rate: top) truncated by the 1% and 99% quantiles, bottom) truncated by the 2% and 98% quantiles.

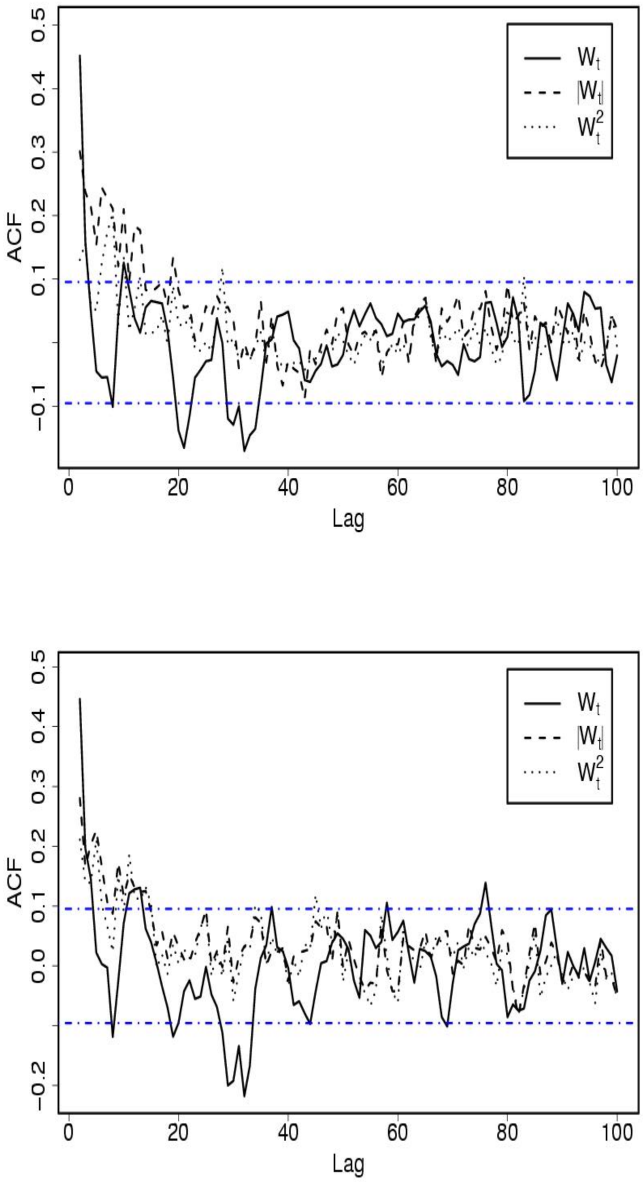

Figure 8: ACF for W_t, $|W_t|$ and W_t^2 of the compounded return of the T-Bill rate: top) without truncation, bottom) truncated by the 1% and 99% quantiles.

data	lag1	2	5	10	20	40	70	100
$T = 500$								
W_t	0.0734	-0.0458	0.0250	0.0559	-0.0320	-0.0255	0.0047	0.0215
$\|W_t\|^{1/2}$	-0.0004	0.1165	0.1307	0.0844	0.0605	-0.0128	0.0430	-0.0052
$\|W_t\|$	0.0325	0.1671	0.1575	0.1293	0.092	-0.0141	0.0061	-0.0004
$\|W_t\|^2$	0.0784	0.2433	0.1699	0.1573	0.1117	-0.0094	-0.0283	0.0225
$T = 2000$								
W_t	0.0494	-0.0057	-0.0090	0.0142	0.0012	-0.0209	0.0263	0.0177
$\|W_t\|^{1/2}$	-0.0214	-0.0072	0.0826	0.0222	0.0280	-0.0040	0.0359	0.0001
$\|W_t\|$	-0.0029	0.0187	0.0997	0.0258	0.0505	0.0036	0.0422	-0.0020
$\|W_t\|^2$	0.0401	0.0562	0.1153	0.0275	0.0668	0.0018	0.0376	-0.0045
$T = 10000$								
W_t	0.1580	-0.0224	0.0122	0.0125	0.0036	0.0079	0.0028	0.0071
$\|W_t\|^{1/2}$	0.1161	0.0813	0.1196	0.0867	0.0789	0.0601	0.0775	0.0550
$\|W_t\|$	0.1223	0.0986	0.1326	0.0989	0.0944	0.0702	0.0879	0.0622
$\|W_t\|^2$	0.1065	0.1044	0.1281	0.0937	0.0988	0.0698	0.0847	0.0559
$T = 16127$								
W_t	0.0971	-0.0362	0.0054	0.0180	0.0036	0.0222	-0.0061	0.0041
$\|W_t\|^{1/2}$	0.1783	0.1674	0.1879	0.1581	0.1567	0.1371	0.1252	0.1293
$\|W_t\|$	0.2044	0.2012	0.2215	0.1831	0.1835	0.1596	0.1439	0.1464
$\|W_t\|^2$	0.1864	0.2018	0.2220	0.1684	0.1709	0.1510	0.1303	0.1321

Table 3: Autocorrelation of W_t, $|W|^d$ for $d = 1/2, 1, 2$ for the S&P 500.

data	lag1	2	5	10	20	40	70	100
V_t	0.4014	0.1789	0.0285	0.1389	0.1759	0.1377	0.0419	0.0193
$\|V_t\|^{1/2}$	0.4244	0.3385	0.3012	0.2279	0.2377	0.2010	0.1882	-0.0626
$\|V_t\|$	0.3999	0.3593	0.2698	0.2622	0.2125	0.1696	0.1636	-0.0691
$\|V_t\|^2$	0.2423	0.2551	0.1626	0.2476	0.1089	0.0890	0.0899	-0.0570
W_t	0.446	0.1961	0.003	0.1212	0.128	0.1306	0.062	-0.0598
$\|W_t\|^{1/2}$	0.2893	0.1589	0.1781	0.1028	0.1381	0.1143	0.1174	0.0165
$\|W_t\|$	0.2803	0.1654	0.1654	0.1525	0.1306	0.1235	0.1250	0.0278
$\|W_t\|^2$	0.2108	0.1395	0.1020	0.1861	0.1146	0.1102	0.1314	0.0722

Table 4: Autocorrelation of V_t, $|V_t|^d$, W_t and $|W_t|^d$ for $d = 1/1, 1, 2$ for the T–Bill rate.

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