MODELING FINANCIAL DATA USING CONTINUOUS–TIME ECONOMETRIC METHODS (Part I)

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ABSTRACT

It is commonly accepted that some financial data may exhibit long-range dependence, while certain financial data exhibit short–range dependence. Both behaviours may be fitted to a continuous–time fractional stochastic model. The estimation procedure proposed in this paper is based on a continuous–time version of the Gauss–Whittle objective function to find the parameter estimates that minimise the discrepancy between the spectral density and the data periodogram. These estimates are asymptotically consistent. In addition, the finite sample results support the asymptotic consistency.

KEYWORDS: Continuous–time model, diffusion process, long–range dependence.

1 INTRODUCTION

Since the publication of Merton (1969), continuous–time processes have been closely associated with finance. Thus, the variation of a security price is roughly calculated as the sum of its multiple variations during the given time period. The main assumption of the continuous–time theory is that these security price variations happen over infinitesimal intervals of time. Perhaps the most popular application of this theory has been the contribution to option pricing by Black and Scholes (1973) and Merton (1973), in which the option price problem is reduced to finding the solution to a partial differential equation. In general, any contingent claim that has an unpredictable outcome in the future can be modelled in continuous–time by a Brownian motion process. Consider a stochastic differential equation (SDE) of the form,

$$dX(t) = \mu(X(t))dt + \sigma(X(t))dB(t), \qquad (1)$$

where $\mu(X(t))$ is the drift function and $\sigma(X(t))$ is the volatility function of the process. Analytical solutions to these models are not always available. This motivates the development of numerical and estimation techniques. For instance, Platen (1999) and Kloeden and Platen (1999) extend numerical methods used to find approximations of solutions of ordinary differential equations to find approximations of solutions of SDEs. At the same time, there has been an important development of estimation techniques for continuous–time models which can be grouped into: maximum likelihood methods, generalised method of moments, simulated method of moments, efficient method of moments, nonparametric approaches and methods based on empirical characteristic functions (see Sundaresan 2000).

In recent years, there have been both theoretical and applied studies for dealing with cases where data may exhibit long–range dependence (LRD) (see Ding, Granger and Engle 1993; Robinson 1994, 1999; Baillie and King 1996; Comte and Renault 1996, 1998; Ding and Granger 1996; Anh and Heyde 1999; Heyde 1999; Deo and Hurvich 2001; Gao, *et al.* 2001; Gao, Anh and Heyde 2002; Gao 2004; and others). For the case of continuous–time models, Comte and Renault (1996) prove that classical SDE models can be extended to embrace LRD models. They also show that how this extension is more suitable in a continuous–time framework than in a discrete time framework. The main characteristic of these extended models is the substitution of the classical Brownian motion by the so–called fractional Brownian motion of the form $B_\beta(t) = \int_0^t \frac{(t-s)^\beta}{\Gamma(1+\beta)} dB(s)$, where $B(t)$ is the standard Brownian motion and $\Gamma(x)$ is the usual Γ function. A Hurst index, H, with values in the interval $(\frac{1}{2}, 1)$ quantifies that the data exhibit LRD. The parameter β is related to the Hurst index through the expression $H = \beta + \frac{1}{2}$ (see Beran 1994, p.52–53), therefore β is defined as the LRD parameter when $0 < \beta < \frac{1}{2}$. For $0 < \beta < \frac{1}{2}$ (i.e., $\frac{1}{2} < H < 1$) the process is said to have LRD, for $\beta = 0$ (i.e., $H = \frac{1}{2}$) the observations are uncorrelated, and for $-\frac{1}{2} < \beta < 0$ (i.e., $0 < H < \frac{1}{2}$) the process is said to have short–range dependence (SRD). In practice, Ding, Granger and Engle (1993) suggest that financial aggregate data, such as the absolute return for Standard & Poor 500 Stock Price Index, may display LRD. That is, transformations of the autocorrelation function for large lags have non–negligible values.

Throughout this paper, the models used are determined by the continuous–time fractional stochastic differential equation of the form

$$dX(t) = -\alpha X(t)dt + \sigma dB_\beta(t),\ X(0) = 0, \qquad (2)$$

for values of $t \in (0, \infty)$. The solutions to this diffusion equation are processes with a spectral density defined by

$$\phi(\omega) = \phi(\omega, \theta) = \frac{\sigma^2}{\Gamma^2(1+\beta)} \frac{1}{|\omega|^{2\beta}} \frac{1}{\omega^2 + \alpha^2}, \qquad (3)$$

where the parameter θ belongs to the set $\Theta = \left\{\theta = (\alpha, \beta, \sigma) : \alpha > 0, -\frac{1}{2} < \beta < \frac{1}{2}, \sigma > 0\right\}$. In this equation, α is the drift parameter, σ is the volatility parameter and $B_\beta(t)$ is as defined before. The well–known short–term interest rate model proposed by Vasicek (1977) is a special case of model (2) with $\beta = 0$. The spectral density described in equation (3) corresponds to that of an Ornstein–Ulhenbeck process of the form (2) driven by fractional Brownian motion with Hurst index $H = \beta + \frac{1}{2}$.

The solutions $X(t)$ of (2) are given by

$$X(t) = \int_0^t A(t-s)dB(s) \qquad (4)$$

with $A(x) = \frac{\sigma}{\Gamma(1+\beta)}\left(x^\beta - \alpha\int_0^x e^{-\alpha(x-u)}u^\beta du\right)$. It follows from equation (4) that $X(t)$ belongs to a family of non–stationary Gaussian processes. It is known though, that a stationary version, $Y(t)$, of $X(t)$ can be found as follows:

$$Y(t) = \int_{-\infty}^t A(t-s)dB(s).$$

Comte and Renault (1998) were among the first to study the estimation of the LRD parameter β involved in model (2). In their study, an approximation to the solution given by equation (4) is found using a path-wise fractional integration method. As an application, they use this method to estimate β as a parameter of a fractional stochastic volatility (FSV) model of the form

$$d\ln(S(t)) = v(t)dB(t), \qquad (5)$$
$$d\ln(v(t)) = -\alpha\ln(v(t))dt + \sigma dB_\beta(t), \qquad (6)$$

where the parameters and Brownian motion are defined as above.

As can be seen, models (2) and (6) are also determined by the drift parameter, α, and the volatility parameter, σ^2. More recently, Gao (2004) proposes an estimation procedure for the case where $\theta \in \Theta_1 = \left\{\theta = (\alpha, \beta, \sigma) : \alpha > 0, 0 < \beta < \frac{1}{2}, \sigma > 0\right\}$ for model (2) and establishes some asymptotic properties for the proposed estimation procedure.

To check whether the estimation procedure proposed in Gao (2004) remains applicable to some well–known financial data, such as the S&P 500 Stock Price Index, we choose several different sections of the data and then check whether all the chosen sections of the data exhibit LRD. Our empirical studies show that the estimated values of β based on some sections of the data appear to be within the interval of $\left(0, \frac{1}{2}\right)$ while the resulting estimated values of β based on other sections of the data look negative. This motivates the extension of the proposed estimation procedure to the case where $\theta \in \Theta = \left\{\theta = (\alpha, \beta, \sigma) : \alpha > 0, -\frac{1}{2} < \beta < \frac{1}{2}, \sigma > 0\right\}$.

The main contribution of the current paper thus includes that the proposed estimation procedure is applied to check whether the LRD or SRD property of two well–known financial data sets: a) the S&P 500 Stock Price Index and b) the Treasury Bill rate. Our results support those obtained by Ding, Granger and Engle (1993). This paper finishes with a summary of the results.

2 ESTIMATION PROCEDURE

2.1 Continuous–time Estimation

The spectral density function $\phi(\omega, \theta)$ given in equation (3) is well–defined for all values $\omega \in \Re$. Thus for values of $\beta \in (0, \frac{1}{2})$, the spectral density behaves as a usual LRD spectral density: decreasing to zero as $|\omega| \to \infty$ and increasing to ∞ as $|\omega| \to 0$. For values of $\beta \in (-\frac{1}{2}, 0)$ the spectral density, $\phi(\omega, \theta)$, decreases to zero as $|\omega| \to \infty$ and $|\omega| \to 0$ and has the maximum at $\omega = \alpha\sqrt{\frac{-\beta}{1+\beta}}$. We say, the latter describes a SRD behaviour. The autocorrelation and spectral density function for LRD and SRD processes is illustrated in Figure 1.

Some detailed discussion about spectral analysis involving short–lange dependent stationary time series can be found in §10 of Brockwell and Davis (1991) and Priestly (1981). For the LRD case, Gao, Anh and Heyde (2002) propose a continuous–time periodogram of the form

$$I_N^Y(\omega) = \frac{1}{2\pi N}\left|\int_0^N e^{-i\omega t} Y(t)dt\right|^2,$$

where $N > 0$ is the upper bound of the interval $[0, N]$, on which each $Y(t)$ is observed.

As in Gao (2004), this paper uses an extended continuous–time version of the discrete Gauss–Whittle contrast function used by Dahlahaus (1989) of the form

$$L_N^Y(\theta) = \frac{1}{4\pi}\int_{-\infty}^{\infty}\left\{\log(\phi(\omega,\theta)) + \frac{I_N^X(\omega)}{\phi(\omega,\theta)}\right\}\frac{d\omega}{1+\omega^2}.$$

We then define the minimum contrast estimator of θ as

$$\bar{\theta}_N = \arg\min_{\theta\in\Theta_0} L_N^Y(\theta),$$

where Θ_0 is a compact subset of Θ. Similar to Gao (2004), it can be shown that $\bar{\theta}_N$ is a consistent estimator of θ_0, the true value of θ.

2.2 Discrete Estimation

In many practical circumstances, observations on $Y(t)$ are made at discrete intervals of time, even though the underlying process may be continuous. In addition, it is computationally easier to find a consistent estimate of θ based on a sequence of discrete observations on $Y(t)$. This section considers the following discrete process:

$$Z_t = Y(t), \quad t = 1, 2, \ldots, T-1 \quad \text{and } T = [N].$$

Such $\{Z_t\}$ is stationary and normally distributed with $\mathbb{E}[Z_t] = 0$ and auto-covariance function obtained as the inverse Fourier transform of its density function. As can be seen in equation (2), $\phi(\omega, \theta)$ is symmetric with respect to ω and therefore the complex terms of the transformation cancel out. Thus, the auto–covariance function is calculated as follows:

$$\gamma(\tau) = 2\int_0^{\infty}\phi(\omega,\theta)\cos(\omega\tau)d\omega.$$

Equivalently, $\phi(\omega, \theta)$ is the Fourier transform of the covariance function of the stationary process $\{Z_t\}$ (see Priestly 1981) given by

$$\phi(\omega,\theta) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\gamma(\tau)e^{-i\tau\omega}d\omega.$$

It can be seen from Bloomfield (1976, §2.5) that the corresponding spectral density of $\{Z_t\}$ is defined by

$$f_Z(\omega) = f(\omega,\theta) = \sum_{k=-\infty}^{\infty}\phi(\omega - 2k\pi, \theta).$$

Given T observations $Z_1, \ldots, Z_T$, we may estimate the spectral density function $f_Z(\omega, \theta_0)$ by

$$I_T^Z(\omega) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} e^{-i\omega t}Z_t\right|^2.$$

As a discrete approximation to $L_N^Y(\theta)$ defined in Section , we use a discrete version of the form

$$W_T(\theta) = \frac{1}{2T}\sum_{s=1}^{T-1}\left\{\log(f(\omega_s,\theta)) + \frac{I_T^Z(\omega_s)}{f(\omega_s,\theta)}\right\}$$

with $\omega_s = \frac{2\pi s}{T}$.

Thus, the (discrete) minimum contrast estimator of θ can be defined by

$$\hat{\theta}_T = \arg\min_{\theta\in\Theta_0} W(\theta),$$

which approximates $\bar{\theta}_N$ for large enough N, i.e. it can be shown that

$$\lim_{N\to\infty} P(|\hat{\theta}_T - \bar{\theta}_N| \geq \epsilon) = 0$$

for any given small $\epsilon > 0$. Thus, we may approximate $\bar{\theta}_N$ by $\hat{\theta}_T$ in practice.

2.3 Estimation of θ

Samples for different parameters, θ_0, and different lengths, T, were generated. The discrete estimation procedure explained in Section 2.2 was applied to these samples to obtain estimators of θ_0. The aim was to show that the estimation procedure was reliable for any sample path that follows a model of the form (2).

The results in Tables 1 and 2 show the empirical means, the empirical standard deviations and the empirical mean square errors. When the number of points generated increases the empirical mean gets closer to the value of the real parameter and its standard error reduces. This shows that there is an asymptotic convergence of the estimates to the real parameters. The estimates obtained with 100 and 1000 simulations do not differ strongly from each other. This may show that the method is also robust for small numbers of simulations. The tables show that the parameter β may be estimated quite accurately. In addition, the simulations have been carried out for values of $\beta \in (-\frac{1}{2}, \frac{1}{2})$ including the case with $\beta = 0$. The results confirm that this procedure can be used to estimate the parameters of financial data with possible LRD or SRD.

The parameter β is restricted to the interval $(-\frac{1}{2}, \frac{1}{2})$, whereas α and σ can take any positive value. Large values of α and σ need larger data sets for some good estimation. For instance in the simulation for $\theta = (1, -0.2, 0.05)$, there were 15 outliers out of 1000 estimates. This is less than 2% of the number of estimates but these have a large effect on the empirical mean. As the size of the data increases, the occurrence of these outliers decreases.

In summary, the proposed estimation procedure works well numerically.

3 DISCUSSION

Recently, several methods and models have been proposed to model data with LRD property. This paper has extended one of the models proposed in Gao (2004) to accommodate cases where some sections of the data may exhibit LRD while other sections may not exhibit LRD. The simulation results corroborate asymptotic convergence of the estimates.

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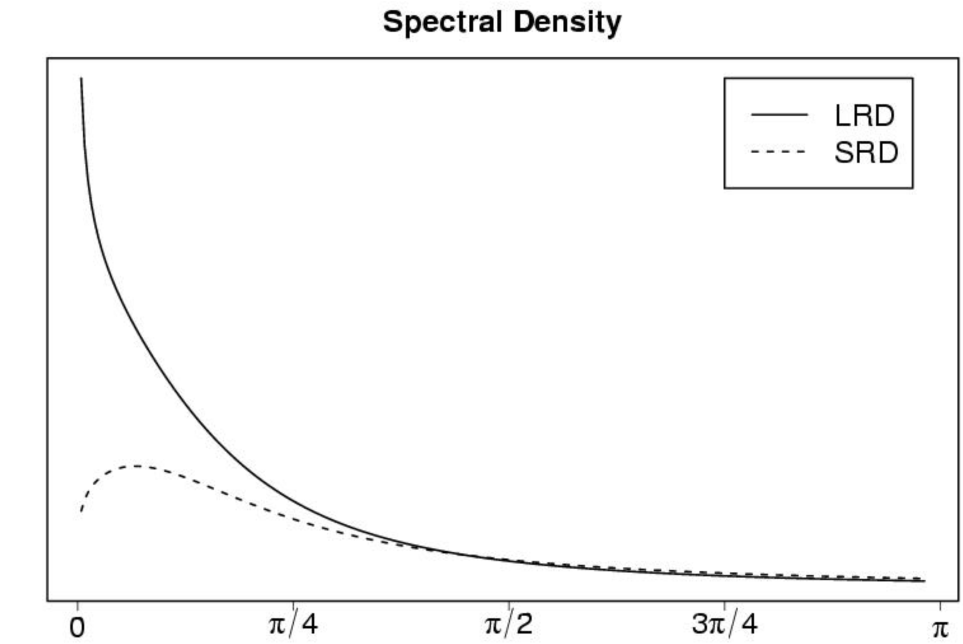
a)



b)

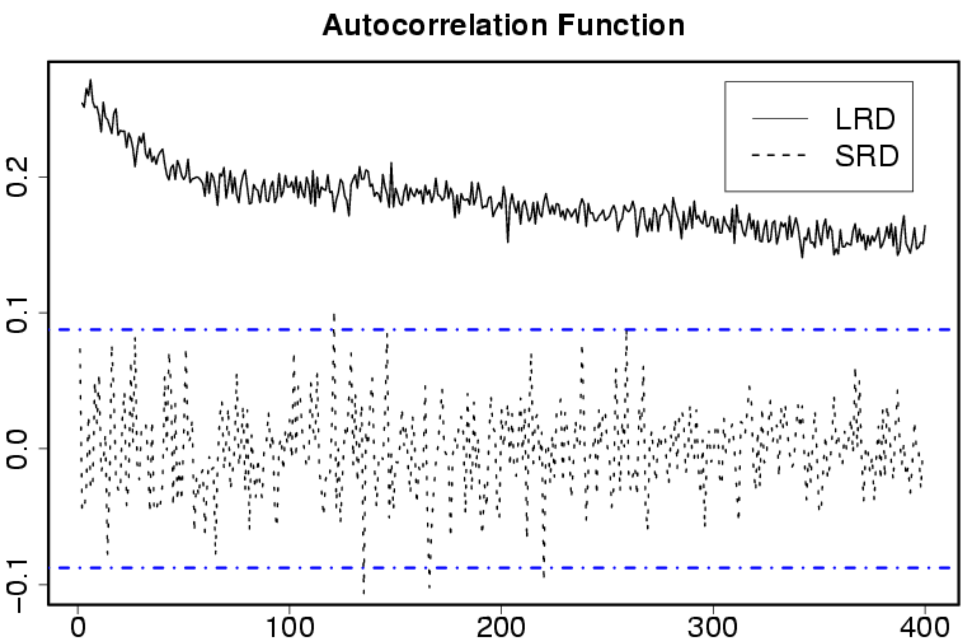


Figure 1: a) Spectral density function for LRD and SRD processes, b) Autocorrelation function for LRD and SRD processes.

	$T = 150$			$T = 400$			$T = 1000$			$T = 2500$		
$\theta_0 = (0.1, 0.2, 0.01)$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$
Empirical mean	0.1593	0.1938	0.0076	0.1145	0.1958	0.0070	0.1042	0.1967	0.0071	0.1031	0.1986	0.0072
Empirical std.dev.	0.2501	0.1434	0.0032	0.0574	0.0603	0.0027	0.0298	0.0414	0.0015	0.0151	0.0244	2.0e-04
Empirical MSE	0.0661	0.0206	1.6e-05	0.0035	0.0036	1.6e-05	0.0009	0.0017	1.1e-05	0.0002	0.0006	7.9e-06
$\theta_0 = (0.1, -0.2, 0.01)$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$
Empirical mean	0.1509	-0.1902	0.0071	0.1255	-0.1903	0.0072	0.1052	-0.1998	0.0071	0.0996	-0.2053	0.0071
Empirical std.dev.	0.1366	0.1033	0.0014	0.1137	0.0624	4.0e-04	0.0250	0.0269	2.0e-04	0.0133	0.0169	1.0e-04
Empirical MSE	0.0212	0.0108	1.0e-05	0.0136	0.0040	8.0e-06	0.0006	0.0007	8.4e-06	0.0002	0.0003	8.4e-06
$\theta_0 = (0.1, 0.3, 1)$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$
Empirical mean	0.1685	0.2777	0.8723	0.1017	0.2720	0.7647	0.1016	0.2862	0.7319	0.1012	0.2946	0.7191
Empirical std.dev.	0.2631	0.1721	0.2224	0.0440	0.0676	0.0664	0.0253	0.0415	0.0371	0.0169	0.0228	0.0182
Empirical MSE	0.0739	0.0301	0.0658	0.0019	0.0053	0.0598	0.0006	0.0019	0.0732	0.0003	0.0005	0.0792
$\theta_0 = (0.1, -0.3, 1)$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$
Empirical mean	0.1859	-0.2832	0.7377	0.1118	-0.3045	0.7309	0.1055	-0.3074	0.7262	0.1004	-0.3105	0.7292
Empirical std.dev.	0.2538	0.1290	0.0907	0.0537	0.0536	0.0272	0.0282	0.0272	0.0171	0.0171	0.0180	0.0104
Empirical MSE	0.0718	0.0169	0.0770	0.0030	0.0029	0.0731	0.0008	0.0008	0.0752	0.0003	0.0004	0.0734
$\theta_0 = (0.1, 0, 0.4)$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$
Empirical mean	0.1564	0.0248	0.2937	0.1124	0.0110	0.2844	0.1028	0.0026	0.2833	0.1007	5.7e-07	0.2837
Empirical std.dev.	0.1615	0.1076	0.0355	0.0384	0.0522	0.0114	0.0254	0.0310	0.0071	0.0147	0.0190	0.0043
Empirical MSE	0.0293	0.0122	0.0125	0.0016	0.0028	0.0135	0.0006	0.0010	0.0137	0.0002	0.0004	0.0135
$\theta_0 = (1, 0.2, 0.05)$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$
Empirical mean	1.2415	0.1760	0.0351	1.0403	0.1872	0.0348	0.9700	0.1822	0.0350	1.0299	0.2034	0.0357
Empirical std.dev.	1.0033	0.1730	0.0237	0.3969	0.1171	0.0120	0.2279	0.0715	0.0031	0.1485	0.0431	0.0021
Empirical MSE	1.0649	0.0305	0.0009	0.1591	0.0139	0.0004	0.0528	0.0054	0.0002	0.0229	0.0019	0.0002
$\theta_0 = (1, -0.2, 0.05)$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$
Empirical mean	8.1053	-0.2016	0.0911	1.0292	-0.2253	0.0295	1.0048	-0.2114	0.0349	1.0342	-0.1991	0.0361
Empirical std.dev.	50.4346	0.1858	0.4104	0.4708	0.1388	0.0215	0.3293	0.0972	0.0078	0.2282	0.0651	0.0027
Empirical MSE	2594.134	0.0345	0.1701	0.2225	0.0199	0.0009	0.1085	0.0096	0.0003	0.0532	0.0042	0.0002

Table 1: Estimates of $\theta = (\alpha, \beta, \sigma)$ for different simulations. 100 samples generated.

	$T=150$			$T=400$			$T=1000$			$T=2500$		
$\theta_0=(0.1,0.2,0.01)$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$
Empirical mean	0.1503	0.1955	0.0073	0.1120	0.1911	0.0069	0.1034	0.1966	0.0071	0.1028	0.1998	0.0071
Empirical std.dev.	0.1907	0.1230	0.0037	0.0765	0.0664	0.0029	0.0254	0.0360	0.0014	0.0154	0.0215	7.0e-04
Empirical MSE	0.0389	0.0151	2.1e-05	0.0060	0.0045	1.8e-05	0.0006	0.0013	1.0e-05	0.0002	0.0005	8.9e-06
$\theta_0=(0.1,-0.2,0.01)$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$
Empirical mean	0.1702	-0.1745	0.007	0.1139	-0.1970	0.0071	0.1046	-0.2006	0.0071	0.1023	-0.2010	0.0071
Empirical std.dev.	0.2324	0.1242	0.0023	0.0798	0.0547	0.0010	0.0262	0.0290	2.0e-04	0.0144	0.0178	1.0e-04
Empirical MSE	0.0589	0.0161	1.4e-05	0.0066	0.0030	9.4e-06	0.0007	0.0008	8.4e-06	0.0002	0.0003	8.4e-06
$\theta_0=(0.1,0.3,1)$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$
Empirical mean	0.1333	0.2572	0.8540	0.1014	0.2671	0.7706	0.0977	0.2811	0.7333	0.0993	0.2927	0.7178
Empirical std.dev.	0.1966	0.1328	0.1862	0.0728	0.0724	0.0903	0.0263	0.0425	0.0372	0.0163	0.0255	0.0187
Empirical MSE	0.0398	0.0194	0.0560	0.0053	0.0063	0.0608	0.0007	0.0022	0.0725	0.0003	0.0007	0.0800
$\theta_0=(0.1,-0.3,1)$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$
Empirical mean	0.1646	-0.2857	0.7381	0.1095	-0.3067	0.7283	0.0995	-0.3122	0.7268	0.1009	-0.3018	0.7110
Empirical std.dev.	0.2222	0.1164	0.0796	0.0693	0.0512	0.0333	0.0266	0.0294	0.0176	0.0156	0.0157	0.0121
Empirical MSE	0.0535	0.0137	0.0749	0.0049	0.0027	0.0749	0.0008	0.0010	0.0749	0.0002	0.00023	0.0837
$\theta_0=(0.1,0,0.4)$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$
Empirical mean	0.1690	0.0286	0.3012	0.1156	0.0045	0.2878	0.1047	-2.0e-04	0.2848	0.1021	1.0e-04	0.2834
Empirical std.dev.	0.2037	0.1238	0.0409	0.0666	0.0584	0.0146	0.0256	0.0334	0.0072	0.0155	0.0201	0.0044
Empirical MSE	0.0462	0.0161	0.0114	0.0047	0.0034	0.0128	0.0007	0.0011	0.0133	0.0002	0.0004	0.0136
$\theta_0=(1,0.2,0.05)$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$
Empirical mean	1.2980	0.1744	0.0375	1.0053	0.1737	0.0339	1.0152	0.1932	0.0355	1.0126	0.1999	0.0356
Empirical std.dev.	2.1385	0.1739	0.0354	0.4250	0.1241	0.0128	0.2497	0.0723	0.0051	0.1412	0.0405	0.002
Empirical MSE	4.6620	0.0309	0.0014	0.1806	0.0161	0.0004	0.0626	0.0053	0.0002	0.0201	0.0016	0.0002
$\theta_0=(1,-0.2,0.05)$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$	$\hat\alpha$	$\hat\beta$	$\hat\sigma$
Empirical mean	1.4687	-0.2042	0.0323	1.1304	-0.2002	0.0311	1.0389	-0.2009	0.0339	1.0327	-0.1942	0.0356
Empirical std.dev.	3.5703	0.1714	0.0419	0.7469	0.1258	0.0209	0.3123	0.0803	0.0118	0.1973	0.0566	0.0043
Empirical MSE	12.9667	0.0294	0.0021	0.5749	0.0158	0.0008	0.0990	0.0064	0.0004	0.0400	0.0032	0.0002

Table 2: Estimates of $\theta=(\alpha,\beta,\sigma)$ for different simulations. 1000 samples generated.

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