## STRATEGICALLY OPTIMUM MAINTENANCE OF MONITORING-ENABLED MULTI-COMPONENT SYSTEMS USING CONTINUOUS-TIME JUMP DETERIORATION MODELS

Aparna Gupta and Chaipat Lawsirirat

Department of Decision Science and Engineering Systems School of Engineering Rensselaer Polytechnic Institute Troy, NY 12180 USA

### ABSTRACT

Efficient usage of high-tech, costly industrial equipment requires not only a good operations schedule, but also a well-designed maintenance schedule to prevent losses of capital and lives. Condition-based maintenance (CBM) uses real time information to schedule a maintenance. With today's sensor technology, CBM is implemented in practice and supports the delivery of Long-Term Service Agreements (LTSA) by companies (such as G.E., UTC). An LTSA is a service contract, sold bundled with the products, making a manufacturer responsible to maintain their products over a specified contract period. In this paper, we address the strategic maintenance problem from the manufacturer's perspective. The goal is to find a strategically optimal maintenance action for a multicomponent system, which deteriorates under continuously with jumps. The deterioration of the system is analyzed using a continuous-time simulation and a search algorithm to find the optimal strategic maintenance actions is proposed.

#### **1** INTRODUCTION

Efficient usage of high-tech, costly industrial equipments requires not only a good operation schedule, but also a well-designed maintenance schedule. Generally a maintenance schedule requires a trade off between costs of inspections, repairs and replacements of parts and a cost incurred due to equipment failure. Too many inspections and repairs result in unneeded maintenance but too few maintenances can result in very costly failures. Today condition-based maintenance is used to better balance this trade-off. According to Rao (1996), the condition-based maintenance costs and improve profits of a plant by 20-60%. The goal of CBM is to make a real-time assessment of equipment in order to make maintenance decisions,

consequently reducing unnecessary maintenance and related costs. Our focus in this article is to strategically analyze maintenance actions for a monitoring-enabled multi-component systems based on the context of CBM by using continuous-time deterioration models with jumps.

With the pace of today's technology, many sophisticated products have sensors-enabled monitoring unit embedded in the products. The monitoring units provide real time condition information for the system (product). Thus, CBM is implemented in practice and supports the delivery of Long-Term Service Agreements (LTSA)by companies (such as G.E., UTC). An LTSA is a service contract, bundled with the products, which makes a manufacturer responsible for maintaining their products over a specified contract period. As such, managing a portfolio of LTSAs is an organization's strategy. The maintenance schedule and resources need to be well managed to meet customers' requirement while minimizing cost and maximizing profit to the organization (Bollapragada et al. (2004)). In our paper, meeting the customers' requirement implies preventing unexpected breakdowns of the products, resulting in losses of capital and lives

A failure occurs when a system, which deteriorates over time due to aging, fatigue, usage, environmental conditions or extreme events, suffers a certain level of deterioration. With the view toward CBM, information concerning a system condition, such as temperatures, vibrations, pressures, crack lengths and etc, is available to help predict the condition or the deterioration of the system. A maintenance decision, either to continue to use, repair or replace a system or its component(s), is determined from the current deterioration of the system. We assume that a function can be constructed that transforms the available condition information of the system to indicate the deterioration of the system.

$$D = f(X_1, X_2, ..., X_n),$$
(1)

where D is the deterioration level of a system or its component(s). X is a condition information vector such as crack temperature. size. vibration. etc. а In this paper, we identify trigger events corresponding to deterioration levels for a multi-component system and determine optimal maintenance actions for these triggers in order to obtain a strategically optimum maintenance policy. Components may require somewhat different interpretation in different contexts. When a system is very complex, it may have thousands of parts; therefore capturing all of them as components into our model will be prohibitively hard. As an abstract on modeling, it is critical to decide what level of components or subsystems resolution is kept in a model such that the model is not too complicated, yet good at mimicking the real system. In our context, a component means a module of a system that is essential for the system's functionality whose deterioration affects system performance criticality, thus needs to be incorporated into the model. A maintenance action is considered if the deterioration level of the system or its components fall in specified trigger zones. Failure Mode and Effect Analysis is used to derive the interrelation of deterioration level of components and their impact on the system's deterioration. Maintenance actions are analyzed to maximize the system's functionality while controlling system's deterioration level.

The paper is organized as follows. The related works in the field of CBM and Failure Mode and Effect Analysis (FMEA) are discussed in section 2. Section 3 describes a single component system, followed by a generalization to a multiple component system in section 4. Section 5 discusses a simulation procedure to find the deterioration of the system and the optimal maintenance search procedure. Numerical examples are discussed in section 6, followed by conclusions.

# 2 LITERATURE REVIEW

# 2.1 Condition Based Maintenance Models

Several models for CBM in the literature assume that the condition of a system is found by a periodic inspection. After the inspection, they assume to have perfect information of the condition of a system. Many models use a Markov model to find a control limit of a maintenance action (repair or replacement), Chen et al. (2003), Grall et al. (1998), Yeh (1997)). Honzalez et al. (1996) and Wijnmalen and Honzalez (1997) considered a problem where an inspection did not give perfect information about the condition of a system. Barbera et al. (1999) studied a two-unit series model using a dynamic program to find the optimal maintenance action, i.e. repair only one unit or both units. Under an assumption of

economy of scale and a cost set up, they showed that a repair of both units was the preferred maintenance action. Castanier et al. (2003) assumed that the condition of the system is known continuously, instead of periodically from the inspections. Barata et al. (2002) used Monte Carlo simulation to find a maintenance schedule. They assumed that failure occurs if the deterioration of components exceeds their maximum deterioration level. If a failure occurs, the component is replaced with an associated replacement cost. However, it is possible to repair a component before it fails with an associated repair cost. They proposed a search over the set of possible deteriorations of each component to find a threshold value for repair such that the expected long run cost was optimal. In a shock model, a shock causes a system to suffer large deteriorations over time. One method to find a control limit was to find the stopping time when failure was modeled as a counting process (Aven and Bergman (1986), Aven (1996)). Chiang and Yuan (2000) modeled both a shock and an aging deterioration by using Markov Chain. However, their system could go to failure stage only by a shock. The aging deterioration contributed only to worsen the condition of the system. They showed that it was optimal to replace a system upon failures or a system reaching a certain age.

### 2.2 Failure Mode and Effect Analysis

FMEA is a powerful tool assisting engineers or design engineers to improve the design of an equipment or a process. The process of FMEA starts with analyzing failures at the components level and proceeds to a system level analysis. Once we know failures at the components level, we can then evaluate their effects on the failure characterization of the system. A key outcome of FMEA is a rank order of criticality of components. The criticality is ranked by using a risk priority number (RPN) which is a function of severity, occurrence and detectability of failure (Franceschini and Galeto (2001), Pillay and Wang (2000), Stamatis (2003)).

The most serious drawback of FMEA method is its incapability to address a tradeoff between cost of failure and performance of a system. Enhancement to address the trade off issue such as a behavior model, an Advance FMEA, and a Qualitative Simulation model can be found in Eubank et al. (1996,1997), Kmenta and Ishii (1998), Snooke (1999).

### 3 A SINGLE COMPONENT DETERIORATION MODEL

We begin this section by discussing a deterioration model for a single-component system, degrading continuously over time. Next, we will incorporate jumps to the deterioration model.

t.

## 3.1 A Single Component Deterioration Model Subjected to a Continuous Deterioration

Consider a system which has only one component, the system degrades randomly over time. The main objective of the model is to indicate the deterioration level of the system at some time, t. The degradation of the system is between  $D^0$  and  $D^{\max}$ , where  $D^0$  is the initial deterioration of a component at time  $t_0$ . and  $D^{\max}$  is the maximum deterioration of the system. The system fails if its deterioration exceeds the maximum deterioration,  $D^{\max}$ .

The deterioration of the system evolves randomly over time. Using a continuous stochastic process, we can define an increment in the deterioration as follows.

$$dD_t = \alpha(D_t, t)dt + \beta(D_t, t)dX_t, \qquad (2)$$

where  $dD_t$  is the infinitesimal of the deterioration in time,  $\Delta t \cdot \alpha(D_t, t)$  is the drift term and  $\beta(D_t, t)$  is the diffusion term.  $X_t$  can be any stochastic processes such as a Wiener process or an Ornstein-Uhlenbeck process. (Ciampolli (1998,1999), Le Breton and Soler (1999), Boreta (1999)).

Using the Euler scheme, a solution for Eq. 2 can be numerically obtained as follows,

$$D_t = D_{t-1} + \Delta D_t, \qquad (3)$$

$$D_t = D_{t-1} + \alpha (D_{t-1}, t-1)\Delta t + \beta (D_{t-1}, t-1)dX_{t-1}, \quad (4)$$

where  $D_t$  is the deterioration of the system at time, t.

However, by the model of Eq. 2 the deterioration of the system  $(D_t)$  can decrease with time because it is possible for  $dD_t$  to take negative values. We thus modify Eq. 2 so that the deterioration level of the system is a non-decreasing function of time.

$$dZ_t = \alpha(Z_t, t)\Delta t + \beta(Z_t, t)dX_t, \tag{5}$$

$$Z_t = Z_{t-1} + dZ_t \tag{6}$$

$$D_t = D_{t-1} + f(Z_t),$$
 (7)

where  $Z_t$  represents an infinitesimal continuous stochastic process. A function,  $f(\bullet)$ , transforms  $Z_t$  to a

positive value. The function, f, can be any positive functions, such as, an exponential function, an absolute value, a squared function. This transformed model is referred to as a two-stage model (Spencer Jr. (1993), Spencer Jr. and Tang (1989)).

### 3.2 A Single Component Deterioration Model Subjected to Jumps in Deterioration

A jump in deterioration can be categorized into two categories, 1. an extreme event and 2. a non-extreme jump event. An extreme event is a rarely occurring event that yields a severe damage to a system. A non-extreme event is an event that occurs more frequently but does not yield as much damage as an extreme event. We can model a jump in deterioration as a counting process such as a Poisson process (Ciampoli (1998, 1999)).

$$J_t = U_t I_t, \tag{8}$$

where  $J_t$  is a jump in deterioration at period t.  $U_t$  is the intensity of the jump and  $I_t$  is the indicator function of the arrival of the jump.

## 3.3 A Single Component Continuous-Time with Jumps Deterioration Model

In this subsection, we combine both the continuous–time deterioration with jumps in deterioration to obtain the overall deterioration level of the system.

We use an additional model for this combination as follows.

$$D_{sys,t} = D_t + J_t, \tag{9}$$

where  $D_{sys,t}$  is the deterioration of the system at time

Assume a value of  $Z_0$ , we can find the underline  $Z_t$ process and transform to the continuous deterioration,  $D_t$ . For the discrete deterioration process, we first find when a jump occurs and then find its intensity by generating from the underlined distribution. After finding  $D_t$  and  $J_t$ , we can find the system's deterioration by adding those terms together.

## 4 A MULTI-COMPONENT DETERIORATION MODEL

In this section, we generalize the single-component system to a multi-component system, where each component interacts with other components as each component degrades randomly over time. As such, Eq. 9 in section 3.3 is used to find a deterioration level of each component. The assumptions of a multi-component system model are made as follows.

- There are *N* components in a system.
- Each component, *i*, has its deterioration process, which starts at  $D_{i,0}$  and has its maximum

possible deterioration  $D_i^{\text{max}}$  where i=1 to N.

- A component fails if it reaches its maximum deterioration level.
- The interaction coefficients of each component are known (through a prior FMEA analysis).
- The deterioration level of a system is a function of the deterioration level of its components and the components' interaction coefficients.
- The system fails if it exceeds its maximum deterioration level,  $D_{sys}^{max}$ .
- It is possible that the system fails due to a part's failure.
- A repair time and a replacement time are negligible.

Using Eq. 9 and the interaction coefficients,  $\rho_{i,j}$ , we

can find the deterioration level of the system as follows.

$$D_{sys,t} = \sum_{i=1}^{N} \sum_{j \ge i} \rho_{i,j} D_{i,t} D_{j,t} + J_{sys,t}, \quad (10)$$

where  $D_{i,t}$  is a deterioration level of component *i* at

time t. This model can be the outcome of several techniques such as Failure Modes and Effect Analysis, Analytical Hierarchy Process, using operational data estimation, expert opinions, etc.  $J_{SYS,t}$  represents the relationship between the failure of components and the system at period t, given by,

$$J_{sys,t} = \sum_{i=1}^{N} F_{i,t} I_{\{t, D_{i,t} \ge D_{i}^{\max}\}}, \qquad (11)$$

where  $F_{i,t}$  represents the damage of the system if one component fails.  $I_{\{t,D_{i,t} \ge D_i^{\max}\}}$  is an indicator function for a component hitting its failure state.

### 4.1 Repair or Replacement Models

A failure of the system results in a disruption of production, reducing products' quality or severe losses such as loss of lives, capital, etc. Maintenance is performed to retain the system in a functional state to minimize the losses and the unwanted incidents. In our analysis, maintenance can be a general repair activity or a replacement of component(s). Repair means restoring to a better state while replacement means restoring to its original state.

In Eq. 10, the deterioration of the system is a linear combination of its components' deterioration level. As a result, reducing a component's deterioration level also decreases the deterioration level of the system. Thus, in our analysis we use the maintenance model to find the new deterioration level of a component and apply it to find the new deterioration level of the system.

$$D_{i,t^{+}} = D_{i,t} - R_{i,t}I(R_{i,t}), \qquad (12)$$

$$R_{i,t} = uD_{i,t} \tag{13}$$

$$u \sim U(0, 0.3)$$
 (14)

$$D_{i,t^+} = (1-u)D_{i,t}, \tag{15}$$

where  $R_{i,t}$  is a repair value for component *i* at period *t*.

 $I(R_{i,t})$  is an indication function for repair at period, t.

In our study, a decision maker can choose to perform a repair or a replacement. If a replacement is chosen, we assume that a component is restored to its original zero deterioration state. As a result, the new deterioration after maintenance is as follows.

$$D_{i,t^+} = D_{i,0}E_{i,t} \tag{16}$$

where  $E_{i,t}$  in an indicator function for replacement at period, t.

A repair or a replacement are performed in response to the following trigger events (Table 1). A threshold value can be found using a search algorithm such as in Barata et al. (2002).

Table 1: Trigger Events developped for maintaining a system.

Trigger Events		
$D_{sys,t} \ge D_{sys}^{\max}$		
$D_{sys,t} \ge D_{sys}^{th} \& D_{sys,t} < D_{sys}^{max}$		

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$D_{i,t} \ge D_i^{\max}$
$D_{i,t} \ge D_i^{th} \& D_{i,t} < D_i^{\max}$

### 5 COMPUTATIONAL APPROACH

In this section, we discuss a continuous-time simulation to solve the model in section 4.

### 5.1 Simulation of the Multi-Component System

The simulation procedure starts from constructing a dependency of each component based on an FMEA analysis. After that, we simulate a continuous deterioration and a jump deterioration of a system to find the deterioration of each component (Eq. 5-8) and the deterioration level of the system (Eq. 10). The deterioration of each component and the system is compared with a trigger event. A maintenance action is performed if the deterioration of the system or a component activate a trigger event (Eq. 11). The flow chart of the algorithm is shown in Figure 1.

#### 5.2 A Simulation-Based Maintenance Optimization

We continue our computational analysis in this subsection by discussing the objective function of the problem, followed by an optimization algorithm that utilizes the simulation models of section 5.1 to obtain optimal maintenance strategy.

Our objective is to minimize the expected long run cost of maintenance and failures. Each maintenance action,  $A_k$ , will have an associated cost,  $C(A_k)$ . The failure cost,  $C_F(D_{i,\nu}D_{sys,l})$ , includes a safety cost, an opportunity cost for a recovery from failure, and a penalty cost for not meeting customer satisfaction. The cost of failure in a general form of,  $C_F$ , is a function of the components' deterioration and the system's deterioration. The total cost (TC) is as follows:



Figure 1: The flow chart represents the simulation algorithm.

$$TC = \int_{t} C(A_{k,t}) + C_F(D_{i,t}, D_{sys,t}) dt \quad (17)$$

Next, we present a search algorithm to obtain an optimal maintenance strategy.

Let  $T_i$  be a trigger event *i*.

 $A_k$  is a maintenance action k.

 $A = \{A_1, A_2, \dots, A_n\}$  are ranked in a decreasing order based on their recovery capability.

 $A_{T_i}$  is the set of all possible actions for trigger  $T_i$ .

 $B_{T_i}$  is the optimal search set which is a subset of  $A_{T_i}$ .

- 1. Initialization
  - a. Select a combination of  $A_k$  for each  $B_T$ .
  - b. Search on  $B_{T_i}$  for an initial solution,  $A_{T_i}^0$ .

### 2. Optimal Search

a. Select a new set of  $B_{T_i}$  based on the

solution,  $A_{T_i}^n$ .

- i. If the position of  $A_{T_i}^n$  is on the right hand side of  $B_{T_i}$  span a set  $B_{T_i}$  to include maintenance actions on the right hand side (less recovery value).
- ii. If the position of  $A_{T_i}^n$  is on the left hand side, span a set  $B_{T_i}$  to include maintenance actions on the left hand side (higher recovery value).
- iii. If  $A_{T_i}^n$  is in the middle of the set  $B_{T_i}$  or on the far left or right of set **A**,  $A_{T_i}^n$  is the optimal solution,  $A_{T_i}^*$ .
- b. Set  $n \leftarrow n+1$
- c. Search on a new  $B_{T_i}$  to find  $A_{T_i}^n$ 
  - *i.* if  $A_{T_i}^n$  satisfies condition 3a, stop  $A_{T_i}^n$  is the optimal solution,  $A_{T_i}^*$ .
  - ii. If not, go to step a.

## 6 NUMERICAL RESULTS

In this section, simulation results are presented to demonstrate the simulation procedure. We set up a specific system configuration of five components, as shown in Fig. 2.



Figure 2: The figure represents the configuration of the system.

The parameters used for this example are given in Tables 2.1-2.7

Table 2.1: The interaction coefficient,  $\rho_{i,i}$ .

$ ho_{_{i,j}}$	1	2	3	4	5
1	1	0.5	0	0	0
2	0	1	0.5	0	0
3	0	0	1	0.5	0
4	0	0	0	1	0.5
5	0	0	0	0	1

Table 2.2: The parameters used to calculate continuous deterioration and the initial deterioration value.

Component(i)	D(i,0)	$\alpha_i$	$\beta_i$
1-5	100	1E-04	1

Table 2.3: The parameters used to calculate discrete deterioration

Component(i)	Jump Rate ( $\lambda_i$ )	Jump intensity ( $U_i$ )
1-5	2	Abs(N(50,10))

Table 2.4: The damage relationship between the
component and the system.

Component	F(i,t)	Condition
	50	if <b>a</b> >=0 and <b>a</b> <0.25
Critical Component	100	if <b>a</b> >=25 and <b>a</b> <0.75
p	150	if <b>a&gt;=</b> 0.75
	250	if <b>a</b> >=0 and <b>a</b> <0.25
Non-Critical Component	300	if <b>a</b> >=25 and <b>a</b> <0.75
component	350	if <b>a</b> >=0.75

Where **a** is a generated random number follows U(0,1).

Table 2.5: Maintenance act	tions and its descript	ion.
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Action	Description
$A_1$	Replace every component regardless of its deterioration
A <sub>2</sub>	Replace a component if D(i,t)>=D(i,max) and Replace its neighbor (s) regardless of its deterioration
A <sub>3</sub>	Repair every component regardless of its deterioration
$A_4$	Replace a component if D(i,t)>=D(i,max) and Replace its neighbor (s) if its deterioration D(i,max)>D(i,t)>=D(i,th)
$A_5$	Replace a component if D(i,t)>=D(i,max) and Repair its neighbor (s) regardless of its deterioration
$A_6$	Repair a component if D(i,t)>=D(i,max) and Repair its neighbor (s) regardless of its deterioration
$A_7$	Replace a critical component if $D(i,t) \ge D(i,max)$ and Repair its neighbor (s) if its deterioration $D(i,max) \ge D(i,t) \ge D(i,t)$
A <sub>8</sub>	Repair a component if D(i,t)>=D(i,max) and Repair its neighbor (s) if its deterioration D(i,max)>D(i,t)>=D(i,th)
A <sub>9</sub>	Replace a component if D(i,t)>=D(i,max)
A <sub>10</sub>	Replace a component if D(i,max)>D(i,t)>=D(i,th)
A <sub>11</sub>	Repair a component if D(i,t)>=D(i,max) and Repair its neighbor (s) if its deterioration D(i,max)>D(i,t)>=D(i,th)
A <sub>12</sub>	Repair a component if D(i,t)>=D(i,max)
A <sub>13</sub>	Repair a component if D(i,max)>D(i,t)>=D(i,th)

A<sub>14</sub> Do nothing

Table 2.6: Repair cost and replacement cost.

]	Repair Cost	400/component
R	Replace Cost	1000/component

Table 2.7: The maximum and the threshold deterioration value.

	Maximum Value	Threshold Value
System	530	400
Component	200	150

### 6.1 Optimal Maintenance Strategy

In our example, component 3 is a critical component while the other components are less critical components. The cost of failure due to the system's failure and due to the component's failure are 5000 and 2000, respectively. First, we select the following bands of the actions,  $B_{T_i}$ , as shown in Table 3.

Table 3: The initial bands of actions used in step 1 in the search algorithm.

Trigger Events	Initial search maintenance actions.
$D_{sys,t} \ge D_{sys}^{\max}$	$A_1, A_2$ and $A_3$
$D_{sys,t} \ge D_{sys}^{th} \& D_{sys,t} < D_{sys}^{max}$	$A_5$ , $A_6$ and $A_7$
$D_{i,t} \ge D_i^{\max}$	$A_7$ , $A_9$ and $A_{11}$
$D_{i,t} \ge D_i^{th} \& D_{i,t} < D_i^{\max}$	$A_{11}, A_{13}$ and $A_{14}$

The initial search obtained the following initial solution (Table 4).

Table 4: The initial solutio	n obtained in step 1.

Trigger Events	Initial solution
$D_{sys,t} \ge D_{sys}^{\max}$	A <sub>1</sub>
$D_{sys,t} \ge D_{sys}^{th} \& D_{sys,t} < D_{sys}^{max}$	A <sub>5</sub>
$D_{i,t} \geq D_i^{\max}$	A <sub>9</sub>
$D_{i,t} \ge D_i^{th} \& D_{i,t} < D_i^{\max}$	A <sub>13</sub>
Total cost	21856

Since  $A_1$  is on the far left of A,  $A_1$  is the optimal response to the trigger event 1,  $(D_{sys,t} \ge D_{sys}^{\max})$ .  $A_9$  and  $A_{13}$  are in the middle of the set  $B_{T_3}$  and  $B_{T_4}$ , respectively, as a result  $A_9$  and  $A_{13}$  are the optimal solutions for trigger events 3 and 4  $(D_{i,t} \ge D_i^{\max} \text{ and } D_{i,t} \ge D_i^{th} \& D_{i,t} < D_i^{\max})$ , respectively. For the trigger event 2  $(D_{sys,t} \ge D_{sys}^{th} \& D_{sys,t} < D_{sys}^{\max})$ ,  $A_5$  is on the far left of the set  $B_{T_2}$ , thus we expand the search to include actions  $A_1$ - $A_4$ . Therefore, the new set  $B_{T_2}$  is as shown in Table 5.

Table 5: The initial solution for the syste	em.
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Trigger Events	Initial solution
$D_{sys,t} \ge D_{sys}^{th} \& D_{sys,t} < D_{sys}^{max}$	$A_1$ , $A_2$ , $A_3$ , $A_4$ and $A_5$

Table 6: The optimal solution for the system.

Trigger Events	Initial solution
$D_{sys,t} \ge D_{sys}^{\max}$	$A_1$
$D_{sys,t} \ge D_{sys}^{th} \& D_{sys,t} < D_{sys}^{max}$	$A_1$
$D_{i,t} \ge D_i^{\max}$	A <sub>9</sub>
$D_{i,t} \ge D_i^{th} \& D_{i,t} < D_i^{\max}$	A <sub>13</sub>
Total cost	19132

From the optimal solution in Table 6, we do an opportunistic maintenance at the system level. Every part will be replaced if the system's deterioration exceeds its threshold value. At the component level, the component is replaced and repaired if its deterioration level exceeds its maximum and threshold value, respectively.

Due to the cost of the system's failure and the jump size, it is more beneficial to replace every component in the systems' warning zone  $(D_{sys,t} \ge D_{sys}^{th} \& D_{sys,t} < D_{sys}^{max})$  to prolong the system's life as much as possible. Replacing only a failed component and its neighbor will result in more system's failures as a result it incurs more cost in a long run. The maintenance strategy at the component level is the aftermath of the system's level. Since we perform the most risk averse action in the system level, it is suffices to focus our maintenance action only for a failed component or a components falling in to the component's warning zone  $(D_{i,t} \ge D_i^{th} \& D_{i,t} < D_i^{max})$  so as to minimize the expected long run cost.

Our optimal solution also matches the character of manufacturers who sells LTSAs. In the general, the manufacturer are very risk averse so as to prevent the devastating failures and losses and to meet their customer satisfaction.

## 7 CONCLUSION

The goal of our paper is to find an optimal maintenance action for a multi-component system, which deteriorates under continuously with jumps. We present a two-stage model for a deterioration process and analyze using a continuous simulation with the Euler scheme. The optimization procedure spans a search space depending on the location of the initial solution, discussed in section 4.2. The result shows that it is more beneficial to perform an opportunistic replacement in the system level.

Our result seems promising as the cost of the system's failure is much higher than the cost of the component's failure. Much research work is needed in this area. For instance, our set of maintenance actions are risk averse since we do not distinguish between a critical component and a non-critical component. More actions and trigger events distinguishing between the critical component and the non-critical component should be added into the model.

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# **AUTHOR BIOGRAPHIES**

**APARNA GUPTA,** Ph.D., is an Assistant Professor in the Decision Sciences and Engineering Systems department at Rensselaer Polytechnic Institute. Her research interests are in mathematical modeling of stochastic systems and optimization, with applications in decision-making and risk management in the service industry. Her e-mail address is guptaa@rpi.edu.

**CHAIPAT LAWSIRIRAT** is a Ph,D. candidate in Department of Decision Sciences and Engineering Systems, Rensselaer Polytechnic Institute, Troy, NY USA. He received a B.Eng in Construction Engineering from King Mongkut's Institute of Technology Ladkrabang and his M.S. in Industrial and Management Engineering from Rensselaer Polytechnic Institute. His current research has involved with Long-term Service Agreements, Part-Unit Replacement problems, Maintenance Scheduling problems and simulation modeling. His email address is lawsic@rpi.edu.