*Proceedings of the 2005 International Conference on Simulation and Modeling*
*V. Kachitvichyanukul, U. Purintrapiban, P. Utayopas, eds.*

# AN INVENTORY POLICY FOR RECYCLING SYSTEM

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## ABSTRACT

An important new trend in supply chain management is repair, remanufacturing, recycling, or reuse of products collected from the end user after they have reached the end of their useful life. This paper deals with inventory control a recycling (material recovery) system. For the system, we assume that demand is deterministic, and a fixed fraction of demands is returned, and all demands are met directly. We propose inventory policies and present procedures for determining the optimal policy parameters.

## 1 INTRODUCTION

The roots of environmentalism can be traced to the period during World War II when severe material shortages occurred worldwide. As a result of these shortages, people were forced to reuse or recycle many different materials. Many organizations are now undertaking initiatives to restructure their supply chain processes and products due to several reasons. First, producers and consumers became more environmentally conscious, and started to realize that it is time to abandon the 'throw-away age'. Second, tighter legislation in some countries forced producers to take back products after use and either recover them or dispose of them properly. Third, and perhaps most important, some producers realized that recovery operations can lead to additional profits. They are now extending their distribution channels beyond the end customer to include the acceptance and "disassembly" of final products for reuse in new products. Moreover, some organizations are seeking to "close the loop" and eventually transform used products into new products and/or materials that can be returned to the earth without harming the environment. In short, organizations are actively working to improve their "reverse logistics" functions, to manage the flow of products and services moving backward through the supply chain (Handfield and Nichols, 1999, Teunter and Vlachos, 2002).

According to Thierry et al. (1995), there are four forms of reuse, (1) direct reuse, (2) repair, (3) recycling, and (4) remanufacturing. 'Reused directly' implies items are reused without prior repair operations (though possible after cleaning and minor maintenance). The goal of 'repair' is to restore failed products to working order, though possibly with a loss of quality. 'Recycling' denotes material recovery without conserving any production structure. 'Remanufacturing' conserves the product identity and seeks to bring the product back into an 'as new' condition by carrying out the necessary disassembly, overhaul, and replacement operations.

Many research papers appeared in the literature on inventory models for remanufacturing. Assuming constant demand and return rate and fixed lead times for external orders and remanufacturing, Schrady (1967) proposed a deterministic model. The costs considered were fixed setup costs for orders and remanufacturing process and linear holding costs for serviceable and returned item inventory. To solve the model he proposed a control policy with fixed lot sizes serving demand as far as possible from remanufactured products. Nahmias and Rivera (1979) extended the Schrady's model for finite remanufacturing rates and determined an optimal remanufacturing batch size in conjunction with optimal ordering quantities. Recently, another extension of the Schrady's model was proposed by Mabini, Pintelon and Gelders (1992). They considered stockout service level constraints and a multi-item system where items share the same remanufacturing facility. For these extended models numerical solution methods were proposed. A model similar to the Schrady's but with a different control policy was proposed by Richter (1996a and b). For the proposed policy he gave expressions for the optimal control parameter values and discussed their dependence on the return rate. Koh et al. (2002) developed a joint EOQ and EPQ model in which the stationary demand can be satisfied by remanufactured products as well as newly purchased products. Taking into account of the system parameter values and operating policies, they examined six different models. Recently, Minner and Lindner (2003) wrote an overview article about lot-sizing decisions for remanufacturing system.

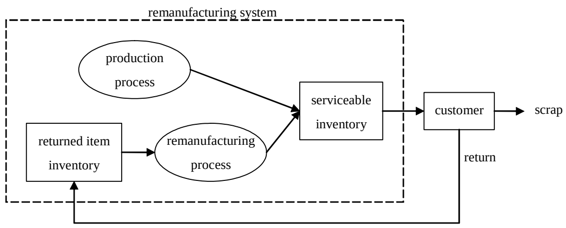


Figure 1: Framework of the remanufacturing system

The remanufacturing system in Figure 1 shows that to satisfy a given demand either returned items are remanufactured to as good as new ones (remanufacturing process) or new items need to be produced from raw material (production process).

In this study, we analyze the recycling system in Figure 2 in which returned items are used for raw material of the manufacturing process. The system is similar to the remanufacturing system in that a known proportion of stationary demand of an item returns to be used as raw material. Examples are metal recycling from scrap, glass and paper recycling, and plastic recycling. Because not all the used items return to the system, additional raw materials must be purchased from outside to satisfy a given demand rate. For a recycling system, the optimal production lot size of the item, order size of the additional raw material procurement and the number of setup/ordering times are determined such that the total cost is minimized. Note that in the system we ensure that the setup/ordering times in a cycle are integer valued.

The remainder of the paper is organized as follows. The modeling and optimal solution procedure for the recycling system is presented in section 2. Conclusions and further studies follow in section 3.

## 2 RECYCLING SYSTEM

### 2.1 System description

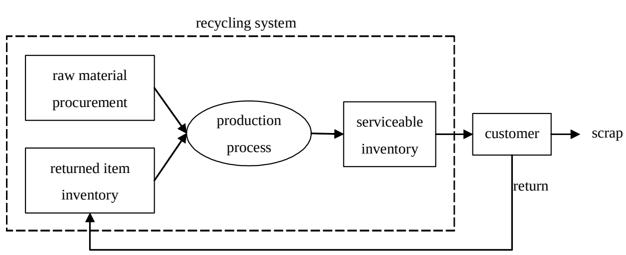


Figure 2: Framework of the recycling system

In this section, recycling system is considered whose general framework is shown in Figure 2. The supplier receives a fixed portion of recyclable material from customers. Like used cans and crashed bottles, recyclable materials become raw materials of new ones. In order to meet the demand, he also purchases additional raw material from outside. The objective of the system is to control raw material orders and production process to satisfy demand and minimize the sum of fixed and variable costs. Before formally stating the problem, we introduce the following notations and assumptions.

Notations:
Known parameters:
$d$ demand rate of the item [units]/[time]
$p$ production rate of the system ($p > d$)[units]/[time]
$f$ return fraction of the item ($0 < f < 1$)
$C_o$ ordering cost of raw material [$]/[order]
$C_p$ setup cost per production [$]/[setup]
$Ch_1$ inventory holding cost of the raw material (type 1) [$]/[unit]/[time]
$Ch_2$ inventory holding cost of the serviceable items (type 2) [$]/[unit]/[time] $\left(Ch_1 \le Ch_2\right)$

Decision variables:
$P$ number of production setups in a cycle
$O$ number of raw material orders in a cycle
$Q_p$ production lot size
$Q_o$ order size of raw material
$T$ cycle time of the model

Assumptions:
1. Demand and production rates are constant and known.
2. Recyclable materials are collected from customers at a fixed and known rate ($fd$). All the collected materials can be recycled and the inventory holding cost of recyclable materials is the same as that of raw materials.
3. The cost parameters are known constants.
4. Purchase and production lead times are assumed to be zero.
5. Shortages are not allowed.
6. Recycled materials are more economical than purchased raw materials.
7. All production lot and order size of raw material are of equal size.
8. For the simplicity of both analysis and implementation, only the following one case are considered, one order of raw material and multiple production setups in a cycle.

### 2.2 Formulation

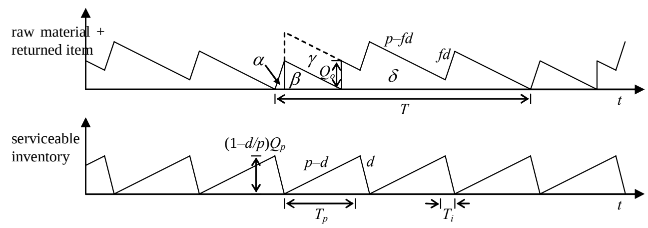


Figure 3: Inventory graph of the system when $P$ = 3

The inventory graph for this case is depicted in Figure 2. The relationships among variables are as follows.

$$T = \frac{P \cdot Q_p}{d} \tag{1}$$

$$T_i = \frac{p-d}{dp} Q_p \tag{2}$$

$$Q_o = P(1-f)Q_p \tag{3}$$

The objective function consists of four cost elements: raw material ordering cost, production setup cost, inventory holding cost for serviceable items, and inventory holding cost for raw material. The first three cost elements are:

(i) raw material ordering cost

$$\frac{C_o}{T} \tag{4}$$

(ii) production setup cost

$$\frac{PC_p}{T} \tag{5}$$

(iii) inventory holding cost for serviceable items

$$Ch_2 \frac{1}{2} dT_i \tag{6}$$

To find the inventory holding cost for raw material, let $A(s)$ denote the area of polygon $s$. The average inventory level of raw material can be found by $A(\alpha+\beta+\delta)/T$. By adding dotted lines to the inventory graph, it can be seen that $A(\alpha+\beta+\delta)$ equals $A(\beta+\gamma+\delta)+A(\alpha)-A(\gamma)$. For the area of the three different polygons, we have the following results.

$$\begin{aligned} A(\beta+\gamma+\delta) &= \frac{1}{2}Q'(T-T_i) \\ &= \frac{1}{2}(fdT_r+Q_o)(T-T_i) \end{aligned} \tag{7}$$

$$A(\alpha) = \frac{1}{2} fdT_i^2 \tag{8}$$

$$A(\gamma) = Q_o \frac{fdT_i}{p-fd} \tag{9}$$

Thus the average inventory holding cost for raw material becomes

$$\begin{aligned} &\frac{Ch_1}{T}\left\{\frac{1}{2}(fdT_i+Q_o)(T-T_i)+\frac{1}{2}fdT_i^2-Q_o\frac{fdT_i}{p-fd}\right\} \\ &= \frac{Ch_1}{T}\left\{\frac{1}{2}fdT_iT+\frac{1}{2}Q_o(T-T_i)-Q_o\frac{fdT_i}{p-fd}\right\} \\ &= Ch_1 \cdot d\left\{\frac{1}{2}fT_i+\frac{1}{2}(1-f)(T-T_i)-(1-f)\frac{fdT_i}{p-fd}\right\} \end{aligned} \tag{10}$$

By adding up (4), (5), (6), and (10), total cost per unit time is obtained and

$$\begin{aligned} TC &= \frac{C_o}{T}+\frac{PC_p}{T} \\ &+ Ch_1 \cdot d\left\{\frac{1}{2}fT_i+\frac{1}{2}(1-f)(T-T_i)-(1-f)\frac{fdT_i}{p-fd}\right\}. \\ &+ Ch_2\frac{1}{2}dT_i \end{aligned} \tag{11}$$

Utilizing the relations in (1), (2) and (3), (11) can be rewritten as:

$$TC_1(P,Q_p) = d\left(\frac{C_o}{P}+C_p\right)\frac{1}{Q_p}+\left(Ch_1\frac{1-f}{2}P+A\right)Q_p, \tag{12}$$

where

$$A = (Ch_2-Ch_1)\frac{p-d}{2p}+Ch_1\frac{p-d}{p}\left(f-(1-f)\frac{fd}{p-fd}\right).$$

The minimum value of $TC_1(P, Q_p)$ will be found by utilizing the well known inequality between the arithmetic mean and geometric mean, i.e. $a+b \geq 2\sqrt{a+b}$ holds for two positive real numbers, $a$ and $b$ and equality holds if and only if $a = b$. Note that since $Ch_2 \geq Ch_1$ and $p > d$, $A$ is positive.

Applying the inequality to (12), we have

$$\begin{aligned} TC_1(P,Q_p) &\geq 2\sqrt{d\left(\frac{C_o}{P}+C_p\right)\left(Ch_1\frac{1-f}{2}P+A\right)} \\ &= 2\sqrt{d\left(C_pCh_1\frac{1-f}{2}P+C_oA\frac{1}{P}+C_oCh_1\frac{1-f}{2}+C_pA\right)} \\ &= TC_2(P) \end{aligned} \tag{13}$$

Note that $TC_1(P,Q_p) = TC_2(P)$ holds if and only if

$$d\left(\frac{C_o}{P}+C_p\right)\frac{1}{Q_p} = \left(Ch_1\frac{1-f}{2}P+A\right)Q_p. \tag{14}$$

Rearranging (14), the optimum value of $Q_p$ for a given $P$ is found and

$$Q_p^* = \sqrt{\frac{d\left(\frac{C_o}{P} + C_p\right)}{Ch_1 \frac{1-f}{2} P + A}} \tag{15}$$

Under the condition of (14), the derivative of $TC_2(P)$ is given by

$$TC_2'(P) = \frac{d\left(C_p Ch_1 \frac{1-f}{2} - C_o A \frac{1}{P^2}\right)}{\sqrt{d\left(C_p Ch_1 \frac{1-f}{2} P + C_o A \frac{1}{P} + C_o Ch_1 \frac{1-f}{2} + C_p A\right)}} \tag{16}$$

Let

$$P^o = \sqrt{\frac{C_o A}{C_p Ch_1 \frac{1-f}{2}}}. \tag{17}$$

It can be verified that $TC_2'(P) = 0$ with $P = P^o$, $TC_2'(P) \le 0$ with $P \le P^o$, and $TC_2'(P) \ge 0$ with $P \ge P^o$, which implies that $TC_2'(P)$ is convex in $P$ and minimized at $P = P^o$. Thus we only need to examine two neighboring integers of $P^O$ to find an integer optimal value of $P$, $P^*$, and

$$P^* = \arg\min_i \left( TC_2(i) \mid i \in \left\{ \left\lfloor \sqrt{P^O} \right\rfloor, \left\lceil \sqrt{P^O} \right\rceil \right\} \right). \tag{18}$$

In the case that $P^O$ is an integer, then $P^* = P^O$

## 3 CONCLUSIONS

We proposed a model associated with reverse logistics, recycling system. A deterministic inventory model for recycling system was developed and analyzed, in which returned items are served as raw materials. No previous studies in the literature exist about recycling system. We presented optimal solution algorithms of the model developed.

The results in this paper could be extended to the following cases. One is a system with both recycling and remanufacturing process. Secondly, the collection rate and/or demand rate could be treated as random variables. A final suggestion is the extension for multi-item case, in other words, the system producing two or more kinds of items.

## ACKNOWLEDGMENTS

This research was partly supported by Engineering Foundation and by Kitakyushu FAIS project (2004).

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