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INVENTORY LEVEL OF RECYCLED MATERIAL FOR DETERIORATING ITEMS UNDER LIFO POLICY

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ABSTRACT

Increasing environment concerns have resulted in efforts for companies to recycle, repair, remanufacture, or reuse their products collected from the end user. This study deals with recycling (material recovery) system where recycled material is used as raw material for an EPQ-type production system. It is assumed that the recycled material arrives at the system with a constant rate and deteriorates continuously in accordance with a general probability distribution. We develop mathematical expression of the inventory level of the recycled material under Last- In-First- Out (LIFO) issuing policy. As special cases, we study the exponential and Weibull distribution for the time to deterioration of an item. An approximate formula is derived using perturbation techniques for the case of Weibull distribution. An example problem is carried out to demonstrate the model.

1. INTRODUCTION

During several past decades many companies have been interested in product recovery due to tighter environmental regulation, lack of disposal sites, and perhaps most importantly, a chance to gain additional profits (Teunter and Vlachos, 2002). Several research articles dealing with product recovery systems appeared in the literature. Schrady(1967) considered a remanufacturing system with constant demand and return rate, and fixed lead times for external orders. It was extended by Nahmias and Rivera(1979) for finite remanufacturing rate. Koh *et al*(2002) developed a joint EOQ and EPQ model in which the stationary demand can be satisfied by remanufactured products as well as newly purchased products.

In a paper manufacturing process, virgin wood pulp has been a principal source of fiber from which paper is made. Increasing demand of paper and decrease of forests have raised price of the virgin wood pulp during past several decades. Thus the paper manufacturer tried to find an alternative to the virgin wood pulp and realized that recycled pulp made from wastepaper (paper returned from customers) can be used as raw material for a relatively low class of paper. Generally, the wastepaper is kept outdoors due to its large volume in inventory and deteriorates because of rain, sunshine, and other environmental elements. Since the recycled paper arrives continuously at the system, the recycled material has time-varying deterioration rate during its storage. The arriving wastepaper is stacked sequentially either in front of or on those previously arrived. Consequently, it becomes difficult to pull out wastepaper piled inside so that those arriving recently tend to be used first for the production of final product. This leads to Last-In-First-Out (LIFO) issuing policy.

There are a few papers dealing with item which has a variable rate of deterioration. Based on EPQ model, Hwang(1982) developed an inventory level for items deteriorating with a general distribution under LIFO policy. Later, for the same problem situation Hwang and Hwang(1982) investigated the effect of LIFO issuing policies and compared the results with those of FIFO. Wu(2001) studied an EOQ inventory model for items with Weibull deterioration. In his paper, demand rate was assumed to be a ramp type function and shortages and partial backlogging were allowed.

This paper develops inventory models for recycled items under LIFO issuing policy. It is assumed that the recycled material deteriorates continuously in accordance with a general probability distribution. We develop mathematical expression of the inventory level of the recycled material under LIFO issuing policy. As special cases, we study the exponential and Weibull distribution for the time to deterioration of an item.

2. MATHEMATICAL MODEL

Figure 1 shows a framework of the recycling system.

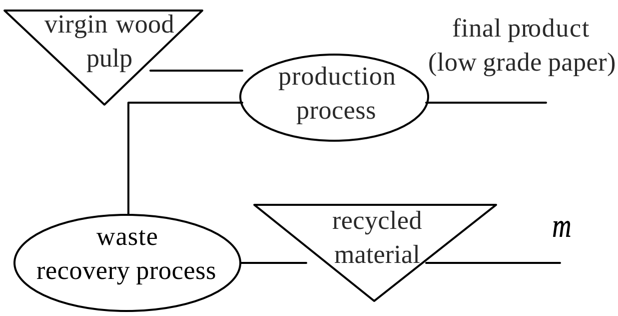

Figure 1: Framework of the Recycling System

Recycled material arrives at the system with a constant rate m. They go through recovery process to become recycled pulp (recovered raw material), a source of fiber. Together with the virgin wood pulp (procured raw material), they are used as raw material for the production of relatively low grade paper. The mixture of pulps goes through several chemical and mechanical processes to be converted into the low grade paper. In this study, we are only interested in expressing inventory level analytically for the recycled material. (This study has a focus on expressing…)

Notations and assumptions used for developing the model are introduced as follows:

Notations:

p : Production rate for the final product

m : Arriving rate of the recycled material at the producer

Assumptions:

1. Recycled material arrives at the producer with a fixed and known rate m.
2. The recycled material is used as raw material to produce final product. The production follows an EPQ-type model.
3. The production rate p is constant and known. It is equal to or larger than m.
4. The life-time of collected materials is regarded as random variable with probability density function $f(t)$ and cumulative distribution function $F(t) = 1 - R(t)$.

LIFO issuing policy is adopted for the use of the recycled material

2.1 Development of the Model

Figure 2 shows inventory graph for the recycled material.

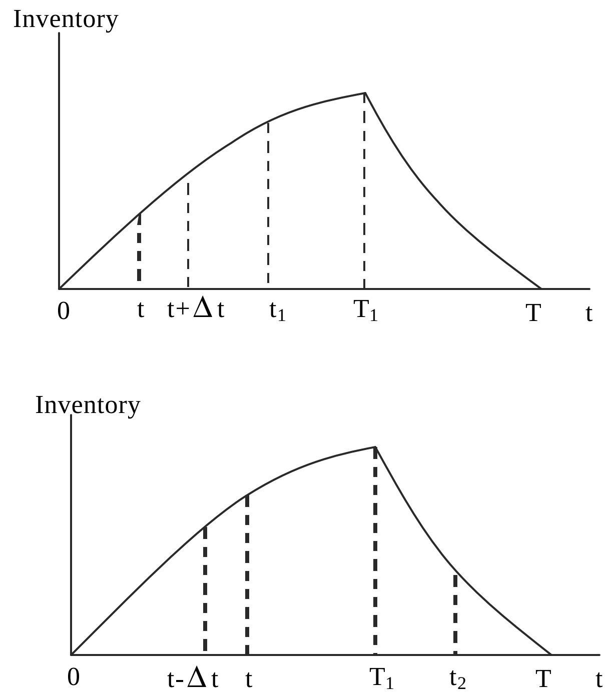

Figure 2: Inventory Level of the Recycled Material during T

During time interval (0, T_1) , there is no production for the final product and inventory of the recycled material increases with constant rate m while decreases due to deterioration. At time t_1 where $0 \le t_1 \le T_1$, the quantity $m\Delta t$ which entered the inventory during (t, $t+\Delta t$) reduced to $m\,\Delta t \cdot R(t_1 - t)$ due to deterioration.

Inventory level at time t_1, I_{t_1} is expressed as

$$I_{t_1} = \int_0^{t_1} m \cdot R(t_1 - t)\, dt \tag{1}$$

When time t reaches T_1, production for the final product gets started. During time interval (T_1, T), the recycled material is needed to produce the final products with the constant rate p. Due to LIFO policy, the recycled material newly arrived is used first. Therefore, stacked recycled material leaves the inventory with constant rate $p-m$. Demand for the returned products during (t_2 , $t_2 + \Delta t_2$) with $T_1 \le t_2 \le T$ is $(p-m)\Delta t_2$ and this is satisfied with the recycled material accumulated during ($t(t_2) - \Delta t,\ t(t_2)$). Inventory at time t_2 does not contain products returned during $(t(t_2), t_2)$ because of LIFO policy.

The above argument gives

$$(p-m)\Delta t_2 = m \cdot R(t_2 - t) \cdot (-\Delta t). \quad (2)$$

Similar to the case that $0 \le t \le T_1$, inventory level at time t_2 with $T_1 \le t_2 \le T$ is obtained as

$$I_{t_2} = \int_0^{t(t_2)} m \cdot R(t_2 - t)\, dt. \quad (3)$$

2.2 Special cases

As special cases of the general inventory level developed in the previous section, exponential distribution and Weibull distribution for the deterioration time are considered.

2.2.1 Exponential distribution

Let us consider a situation that lifetime of the returned material follows exponential distribution with probability density function

$$f(t) = a\exp(-a\, t) \quad \text{where } t \ge 0,\ a \ge 0.$$

Then $R(t)$ becomes $\exp(-a\, t)$. Substituting this result into equation (1) and evaluating the integration, we get

$$I_{t_1} = \frac{m}{a}(1-\exp(-a\, t_1)) \qquad 0 \le t_1 \le T_1 \quad (4)$$

In a similar way, the following results are derived from equations (2) and (3).

$$(p-m)\frac{dt_2}{dt} = -m\exp(-a\,(t_2 - t)) \quad (5)$$

and

$$I_{t_2} = \frac{m}{a}\exp(-a\, t_2)(\exp(at) - 1). \quad (6)$$

Solving equation (5) with the boundary condition of $t = T_1$ at $t_2 = T_1$ gives the following solution

$$m \cdot \exp(at) = p\exp(aT_1) - (p-m)\exp(at_2). \quad (7)$$

With this solution, equation (6) is converted into

$$I_{t_2} = \frac{1}{a}[p\exp(a\,(T_1 - t_2)) - (p-m) - m \cdot \exp(-at_2)] \quad (8)$$

2.2.2 Weibull distribution

In case that lifetime of the recycled material follows Weibull distribution,

$$f(t) = a\,b\,t^{b-1}\exp(-a\,t^b) \quad \text{when } t \ge 0, a > 0, b > 0$$
$$= 0 \quad \text{otherwise,}$$

$R(t)$ becomes $\exp(-a\,t^b)$ where a and b are some constants determined by the deterioration process. Substituting $R(t)$ into equations (1), (2) and (3) yield

$$I_{t_1} = \int_0^{t_1} m \cdot \exp\left(-a\left(t_1 - t\right)^b\right)dt, \quad (9)$$

$$I_{t_2} = \int_0^{t(t_2)} m \cdot \exp\left\{-a\left(t_2 - t\right)^b\right\}dt, \quad (10)$$

and

$$(p-m)\Delta t_2 = m \cdot \exp(-a\,(t_2 - t)^b) \cdot (-\Delta t). \quad (11)$$

Notice that it is essential to find $t(t_2)$ in evaluating I_{t_2}. However, solving equation (11) with respect to $t(t_2)$ is extremely difficult because finding $t(t_2)$ in a closed form with given t_2 seems impossible. Therefore, we examine two approaches in order to estimate $t(t_2)$. One is a numerical method. A computer program is developed where the Runge-Kutta method is adopted to solve the differential equation (11).

The other is an approximation under the assumption that $a << 1$. Ghare and Schrader(1973) elaborated the validity and physical meaning of the assumption.

Let $u = t_2 - t$ and $l = p - m$. Then equation (11) becomes

$$l\left(\frac{du}{dt} + 1\right) = -m \cdot \exp(-a \cdot u^b). \quad (12)$$

With the boundary condition that $t = T_1$ at $u = 0$, equation (12) is transformed to

$$\int_0^u \frac{-l}{l + m \cdot \exp(-ax^b)}dx = \int_{T_1}^t dy = t - T_1. \quad (13)$$

Using the Taylor series generated by exponential function ignoring terms with higher than third order powers, equation (13) becomes

$$\frac{-lu}{l+m}\left[\begin{matrix} 1 + \dfrac{m}{l+m}\dfrac{a\,u^b}{b+1} + \\ \dfrac{m(m-l)a^2u^{2b}}{2(l+m)^2(2b+1)} + O(a^3) \end{matrix}\right] = t - T_1. \quad (14)$$

Letting $t = g_0 + g_1(t_2)a + g_2(t_2)a^2 + O(a^3)$, then we get

$$u = t_2 - g_0 - g_1 a - g_2 a^2 - O(a^3). \quad (15)$$

Applying approximation formula, $(1-x)^b \approx 1 - bx$ after substituting equation (15) into equation (14), we obtain

$$\frac{-lu}{l+m}(t_2-g_0-g_1a-g_2a^2)\cdot\left[\begin{array}{l}1+\frac{m}{l+m}\frac{a}{b+1}(t_2-g_0)^b\,(1-\frac{a\,b\,g_1}{t_2-g_0})\\ +\frac{m(m-l)a^2}{2(l+m)^2(2b+1)}(t_2-g_0)^{2b}+O(a^3)\end{array}\right] \tag{16}$$
$$= g_0+g_1a+g_2a^2-T_1+O(a^3)$$

By comparing parameters of the same power of a, the following results are obtained.

$$\begin{aligned} g_0 &= \frac{p}{m}T_1-\frac{p-m}{m}t_2 \\ g_1 &= -\frac{p-m}{p}\cdot\frac{(t_2-g_0)^{b+1}}{b+1} \\ g_2 &= \frac{p-m}{p}\cdot(t_2-g_0)^b\cdot \\ &\quad \left[g_1-\frac{(2m-p)(t_2-g_0)^{b+1}}{2p(2b+1)}\right] \end{aligned} \tag{17}$$

Using the above procedure with higher powers of a, it is possible to find an approximate value of t to any desired accuracy.

3. Numerical example

An example problem is carried out to demonstrate the model and the accuracy of the approximation formula. Parameters used in the problem are :

$p=30$ $\quad m=10$

$T_1=5$ $\quad d=20$

Table 1 shows values of $t(t_2)$ obtained by using the approximation formula, $t(t_2)=g_0+g_1a+g_2a^2$. The approximate value of $t(t_2)$ is compared to those by the Runge-Kutta method.

Table 1: Calculated values of $t(t_2)$

t_2	$a=0.01$ $b=1$ R	$a=0.01$ $b=1$ P	$a=0.1$ $b=1$ R	$a=0.1$ $b=1$ P
5.000	5.000	5.000	5.000	5.000
5.500	3.9924	3.9925	3.9181	3.9188
6.000	2.9695	2.9696	2.6385	2.6502
6.500	1.9308	1.9310	1.0893	1.1566
7.000	0.8758	0.8763	-	-
T	7.4088	7.4088	7.0978	7.1078

R : result by the Runge-Kutta method

P: result by the perturbation method

Similar results were obtained from both methods. Note that the approximation formula is derived based on the Taylor series at $t=T_1$. Therefore, differences between the two results increase as t_2 deviates from T_1.

Inventory level

Once $t(t_2)$ is obtained, we can calculate the inventory level from equation (10). To illustrate how it changes with time, numerical problems were additionally solved by using a computer program where the Simpson's Rule was adopted to evaluate the integration of equation (10). With the same parameters as those of the previous example, Figure 3 shows the inventory levels for $a=0.1$ with three different values of b, $b=1, 3,$ and 5 when $T_1=5$.

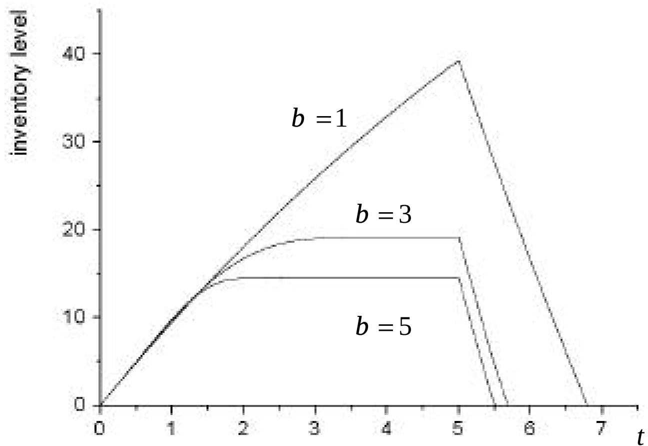

Figure 3: Inventory level

4. Conclusion

In this study we developed mathematical model of inventory level for recycled materials which deteriorate with a general distribution under LIFO issuing policy. Weibull and exponential distribution were considered as special cases of a general distribution. In case of the Weibull distribution, the inventory level is very complicated due to the fact that the amount deteriorated during a given time interval depends on the age of each item. Thus two approaches were adopted; one is an approximation using perturbation technique and the other a numerical analysis. We hope that the developed models would play an important role in developing a mathematical model for the recycling system where the manager needs to deal with three kinds of inventories, new and recycled raw material and final product, and production schedule of the system.

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