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Passive Sonar Sensor Placement for Undersea Surveillance

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Abstract: The undersea detection of small acoustic signature threats is an important problem for navies worldwide, since such threats may be used to conduct stealthy surveillance of strategic regions as well as to initiate attacks on both sea- and land-based high value assets. Such threats will likely have sensing capabilities, and so will be alerted to the presence of active underwater sensing systems. Hence passive sonar sensors may be utilised to provide an early warning system of such threats, without revealing their presence. This can then allow the detection, tracking and localisation of threats to be established prior to the deployment of active sensor systems and countermeasures. Consequently it is useful to explore modelling frameworks for the placement of passive sonar sensors within a given surveillance region.

Towards this end, this paper is concerned with introducing a novel way in which passive sonar sensors may be placed so that a given region of interest is guaranteed to have maximal coverage against threats of certain acoustic signatures. The problem is formulated in a three-dimensional setting but the applications will be restricted to two-dimensions for simplicity. The approach is to cover the region of interest with a series of square subregions, where a omnidirectional passive sensor is placed in each centrally, and such that the subregions have maximal probability of detection against a range of acoustic signature levels. Consequently the surveillance region is constructed so that the probability of detection is maximised against threats of interest. This improves on coverage achieved via cookie-cutter methods in two ways. Firstly, there is no need for overlapping regions and secondly it will require a smaller number of sensors than required through the cookie-cutter approach.

Since one may be searching for undersea targets with unknown and varying acoustic signatures a passive broadband sensor network is considered. This means that the sensor network will detect acoustic signatures over a frequency band. This requires the construction of the probability of detection of a threat through a single passive broadband sensor, whose derivation follows by considerations of the passive sonar equation, specialised to the broadband passive sonar case. This is then extended to a network of such sensors using a simple data fusion model. However, the problem is developed from the point of view of establishing an initial detection, and so does not correlate detections over an observational window. Instead it is assuming a stationary distribution or long-run average to remove the necessity of cumulative detections. This is done to provide an initial model to illustrate the sensor placement approach.

A series of examples are then provided, where sonar characteristics are sourced from the open literature, to demonstrate the way in which coverage of a surveillance region may be undertaken, optimised to particular acoustic signatures. The numerical analysis is completed by an application of the approach to the problem of detecting a submarine approaching a sensor field. This example will demonstrate the performance of the sensor network when it is optimised to the submarine's acoustic signature, as well as the case when the sensor field is optimised to a mismatched acoustic signature.

Keywords: *Passive broadband sonar, sensor coverage, cookie-cutter models, performance prediction, undersea target detection*

1 INTRODUCTION

Undersea surveillance of regions of strategic importance is critical in modern naval settings (Soldi et al., 2023; Zhu et al., 2024). In particular it is useful to deploy a series of passive sonar sensors to provide an early warning of the presence of an intruder (Fillinger et al., 2010). Passive sonar modelling and performance prediction have thus received considerable attention in the literature (Wahlstedt et al., 1997; Johnson and Matthews, 2016; Alaie and Farsi, 2018; Wang et al., 2024). The advantage of a passive sensor is that it does not ping and hence such a network may operate without the intruder's knowledge (Biediger, 2010). This can then permit detection, localisation and tracking of the intruder to be undertaken prior to the deployment of active sensors to confirm it is a threat and then initiate suitable defensive measures. However, the problem with passive sensors is that they are dependent on sound from targets, which can be attenuated in the operational environment (Wang and Wang, 2016).

This paper is concerned with developing a method which allows the determination of the placement of passive sensors to ensure that the probability of threat detection (PD) is maximised within the surveillance region, in the presence of ambient noise. The general model development will be in three-dimensional space but the applications will be restricted to two-dimensions. It will be supposed that the surveillance region is a two-dimensional square, which is possible in ocean surveillance contexts and shipping channel situations. In cases where this assumption is invalid one may approximate the surveillance region by a series of rectangular subregions, since the analysis may be readily extended to such cases. The square region under consideration is then subdivided into a series of smaller squares where a passive sensor is located centrally in each, such that the PD is maximised within each such subregion. This results in the region having maximal PD for threats of certain signatures within an ambient noise environment.

Coverage of a surveillance region by sensors has received considerable analysis and a multitide of solutions have been proposed (Wang, 2011). In the context of antisubmarine warfare (ASW), many studies have examined the optimal placement of active sensors, examples including (Craparo et al., 2019; Craparo and Karatas, 2020) and the references contained therein. The design of multistatic sonobuoy fields for ASW has also received much attention (Ozols and Fewell, 2011). Passive sonobuoy fields have been explored in (Wahlstedt et al., 1997), while optimisation of passive sonar placement was investigated in (Johnson and Matthews, 2016) to facilitate interdiction. Improving passive sonar modelling by including more accurate environmental considerations, as well as sensor placement problems, has also been examined in (Wang et al., 2024). A study more closely aligned with the current paper is (Stolkin and Florescu, 2009) who constructs an optimal sensor placement strategy for threshold-based detection schemes. The latter study assumes that a target's range is modelled through a Gaussian process, and selects sensor placement to minimise this distance.

The approach proposed in the current paper differs from previous studies in a number of ways. Firstly cookie-cutter models for detection are replaced by one based upon the passive sonar equation. The approximation of a surveillance region by squares, with a sensor located centrally in each, is equivalent to setting up a passive sonobuoy field. By construction, within each square the PD is maximised so that for threats of interest detection is certain. Consequently there are no coverage gaps in the sensor field, and a simple approach may then be used to work out sensor placement without the need to resort to optimisation algorithms. A basic rule of thumb for the number of required sensors can then be determined quite easily.

To illustrate the method, consider Figure 1 which shows a square surveillance region. In the left subplot the region has been subdivided into four subregions, each containing a passive sonar located centrally. As a consequence of the method developed in this paper each such subregion will have a PD maximised for threats of particular source levels of interest within ambient noise. Consequently the entire surveillance region will also have maximised PD for the same threat class. The right subplot in Figure 1 illustrates the case where omnidirectional receivers are deployed with a maximal detection range given by the radius of each circle, as in the conventional cookie-cutter approach. Within each circle the sensor is located at the centre. If for convenience it is assumed that the surveillance region is a square of length 1 unit, and consequently the circles are of radius $1/4$ then the shaded region, which is the area where there is no coverage, is 21.26% of the total area. Therefore the network of sensors in the right subplot covers 78.54% of the surveillance area. This means that a threat can be missed through the lack of coverage in the right subplot. As surveillance is undertaken over consecutive timesteps this lack of coverage can result in an actual target being interpreted as a false alarm due to lack of sequential detections. The solution would be to introduce more sensors to eliminate the areas of no coverage, and if overlapping surveillance subregions are allowed, then this will rectify the problem. Therefore the solution proposed in the current paper will guarantee 100% coverage with a minimum of passive sensors,

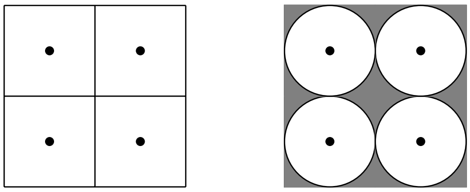

Figure 1. The left subplot shows a square surveillance region covered by subsquares with sensors located centrally. The right subplot illustrates coverage using circular surveillance regions. The shaded area in the latter is the subregion where there is no coverage.

which is a more robust solution. It is worth noting that there have been studies undertaken examining shape optimization for surveillance coverage which may provide an alternative to the approach adopted in this study Birgin and Laurain (2023).

2 Models of Passive Sonar

In the first instance what is required is the PD for a single passive sonar sensor. Since the sensor network will be designed to monitor for threats which may emit sound waves over a frequency band it will be assumed that the sensors are broadband. This means that they are designed to monitor for sounds over a bandwidth around a centre frequency. The passive broadband sonar equation is therefore used to construct the desired PD. This equation is given by

$$SE = SSL - TL - NSL + DI - DT, \tag{1}$$

where SE is the signal excess, SSL is the source spectrum level, or equivalently the dB strength of a signal received at the sensor, TL is the transmission loss, which is a measure of the attenuation of the signal in the maritime environment. NSL is the noise spectrum level which captures acoustic interference from the environment, either due to natural phenomena or naval vessels. DI is the directivity index, which provides a measure of spatial directivity of the receiving hydrophone, also providing a quantification of the amount of noise reduction the sensor provides. Finally, DT is the detection threshold, which provides a measure of the required signal to noise ratio (SNR) to achieve a PD for prescribed probability of false alarm (PFA). All the terms in (1) are measured in dB. In the following it is assumed that each passive sonar uses an amplitude spectrum for detection decisions for a small number of returns. Hence one is justified in assuming that the noise distribution is Rayleigh (Dawe, 1997).

Since the sensors will be omnidirectional the $DI = 0$. Based upon the discussion in Dawe (1997) on passive broadband sonar detection thresholds, one may apply the expression

$$DT = 10\log_{10}\left(0.273d + 1.045\sqrt{d}\right) \text{ with } d = \left(\frac{\pi}{4-\pi}\right)\left(\sqrt{\frac{\log_e \mathrm{PFA}}{\log_e \mathrm{PD}}} - 1\right)^2 \tag{2}$$

for the detection threshold (which is relative to a bandwidth B), within a detection timestep for an integration factor of unity, for a desired PD and PFA. Throughout the following the PD was set to 0.5 with a PFA of 10^{-4}, for application in (2). This results in $DT = 10.8917$. The SSL and NSL are obtained by averaging mean estimates over the sensor bandwidth. The TL is more involved and merits further discussion at this point. This version of it is adopted from (Ainslie, 2010) and is based upon a spherical spreading model. This is also averaged over frequency, and under the assumption of linear variation of attenuation with frequency a simplified expression for broadband TL can be sourced in Ainslie (2010). In particular when the absorption coefficient is given by

$$\alpha(f) = \alpha_m + \alpha_f(f - f_m), \tag{3}$$

where α_m and α_f are constants, in units of dB per kiloyard, the broadband TL is given by

$$TL(r) = (40\alpha_m \log_{10} e)r + 30\log_{10} r + 10\log_{10}(2\alpha_f B) - 10\log_{10}\left[e^{2\alpha_f Br} - e^{-2\alpha_f Br}\right], \tag{4}$$

for the case where r, which is the distance between the sensor and the target, is measured in nautical miles (nmi). In order to estimate the frequency-dependent absorption coefficient in (3) the expression for it in Tyler (1992) will be used together with a linear interpolation between the limits of the broadband frequency range.

The PD for a single passive broadband sensor is the probability that the $SE > 0$. Using the Rayleigh assumption, one may show that

$$\mathbf{P}(\text{Detection Made}) = 1 - e^{-10^{0.2[SSL-TL-DT+DI-\gamma]}}, \tag{5}$$

where γ is the average NSL over the frequency bandwidth, expressed in dB.

Next an expression for the PD is derived for a passive broadband sensor network. However, this will be based upon the simplification of applying (5) without integration over time of detection decisions. Hence it is being asssumed that at given time steps a detection decision is made from the sensor network, to provide an initial indication of the presence of a potential threat. In an operational setting it would be necessary to correate detections over time steps to then make an accurate detection decision. Here it is being supposed that modelling the detection process as a stationary distribution is a reasonable approximation in the long run.

Assume that the surveillance region is a cube of length R, defined through the Cartesian product $[0, R] \times [0, R] \times [0, R]$, for some known $R > 0$. Denote by $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N$ the coordinates within this region of each passive sensor, assumed to be fixed. Define $\mathrm{P_D}(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N)(t)$ to be the probability of detection of a threat, at a given time instant t, by the sensor network. Hence this is a not a cumulative probability of detection but is that for a single time step t.

Within the surveillance region there is a target which is at an unknown location, potentially travelling at an unknown bearing and speed. At each time step it will be assumed to be at some randomly distributed location (x_t, y_t, z_t) within the surveillance region. Hence its location will be modelled through a spatial uniform distribution, so that its positional density is given by

$$f_{(x_t,y_t,z_t)}(x, y, z) = \frac{1}{R^3}, \tag{6}$$

provided $0 < x, y, z < R$ and is zero otherwise. This model for target position is independent of time and so is supposing that at each time step the target's position is completely unknown. If more information is available concerning this target then the model (6) can be updated or expanded to include this situational awareness, as well as time dependence. As an example, one may assess the performance of a passive sonar field by assuming that the threat trajectory is known and using deterministic values for x_t, y_t and z_t.

Denote by $\mathcal{P}_j(x, y, z, t)$ the probability that sensor j detects the threat given the threat is located at coordinates (x, y, z) at time t. This expression is given by (5), noting that the TL is dependent on the sensor and target position at time t. It is assumed that the sensors operate independently. From a sensor fusion perspective, there are a number of ways in which threat detection may be determined. Since passive sensors will be range-limited it is suitable to utilise a sensor fusion model which is based upon 1 out of N detection, or declaring a target present if at least one sensor detects a threat. Under such a target detection protocol, it can be shown that

$$\mathrm{P_D}(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N)(t) \quad = \quad 1 - \frac{1}{R^3}\int_0^R\int_0^R\int_0^R \prod_{j=1}^{N} [1 - \mathcal{P}_j(x, y, z, t)]\, dxdydz, \tag{7}$$

where independent detections has been utilised to introduce the product in the above expression. Consequently, for a given configuration of the N sensors within the surveillance region, this expression provides a quantitiative measure of the likelihood of detection of the threat whose position is unknown.

Once an algorithm is introduced for the placement of passive sensors it is then possible to assess the operational effectiveness of it by assuming particular target characteristics, as indicated previously. Hence it may be assumed that a threat is travelling towards the sensor network, with a known speed and initial position. Consequently, its position at each time step is known and one may utilise

$$\mathrm{P_D}(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N)(t) = 1 - \prod_{j=1}^{N} [1 - \mathcal{P}_j(x_t, y_t, z_t, t)], \tag{8}$$

to determine the sensor network's performance over time.

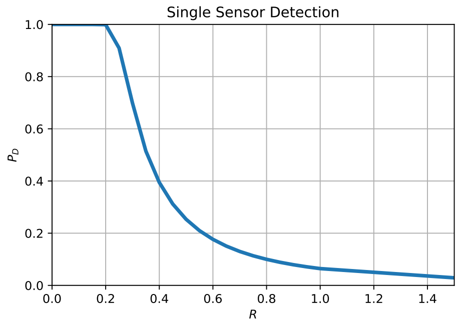

Figure 2. Single omnidirectional sensor performance as a function of the surveillance boundary parameter R.

The next section will demonstrate how one may utilise (5) to determine a sensor placement strategy to cover a given surveillance region. Based upon this an analysis of (7) and (8) will then be undertaken. However, this will be restricted to the two-dimensional setting, so that the z-coordinate in each relevant expression will be ignored and instead of a cubic surveillance volume a square region will be utilised.

3 APPLICATIONS

3.1 Study Parameters

Throughout the following all ranges are measured in nmi and the speed of sound is taken to be constant $c = 1500$ metres per second. In order to provide tangible examples of performance it is necessary to adopt some model parameterisations. Throughout the broadband centre frequency is taken to be $f_m = 100$ Hz with a $B = 30$ Hz bandwidth. With this choice one can obtain estimates of SSL and NSL from the literature. For the NSL, Table 6.5 in (Bjørnø, 2017) allows one to obtain estimates of the mean NSL across the adopted broadband frequency band. A linear interpolation was applied to estimate the mean noise level resulting in the choice of $\overline{NSL} = 68.5$ dB. Both (Urick, 1975) and (Coppens, 1976) provide estimates of the SSL for conventional submarines on electric drive operating at periscope depth as a function of frequency and speed. A linear interpolation was applied for the case of a speed of 4 knots (kts) and the resulting estimate of the SSL is 133 dB. To provide estimates of the parameters α_m and α_f in the absorption coefficient given by (3) a linear interpolation has also been applied over the frequency band. By applying the expression for the absorption coefficient in (Tyler, 1992), given by equation (9) in the latter but adjusted to dB per nmi, one extracts $\alpha_f = 0.20347$ and $\alpha_m = 31.2218$.

3.2 Determining Number of Sensors

To illustrate the proposed approach it is necessary to first examine the performance of a single passive sensor, located at the centre of a square region of length R. Figure 2 plots (5) as a function of R under the parameters outlined above. What this figure shows is that detecton is certain for targets located within 0.2 nmi. Outside this range the PD reduces. For example, at 0.3 nmi the PD reduces to 0.8. Hence this implies that the sensor is able to provide guaranteed coverage of a square region of length 0.2 nmi. Consequently, in order to provide coverage of a square region of length R, one would require a total of $R^2/0.2^2$ sensors, each located at the centre of a square subregion of length 0.2 nmi. As examples, in the case where $R = 0.4$ nmi the number of required sensors would be 4, while when R is increased to 1 nmi, a total of 25 sensors would be required. One may reduce the number of required sensors by a corresponding reduction in the desired PD. It is important to note that this design feature has been based upon the assumed SSL and NSL. If a target's acoustic signature exceeds this SSL then it will also be detected within the constructed region. However, for reduced SSL it is reasonable to expect the PD will decrease. Therefore a suitable strategy to use is to design a standard square region, based upon a figure such as that in Figure 1, tailored to ensure detection of threats with a SSL and NSL of interest, and apply the procedure illustrated here to determine the required number of sensors to ensure coverage. One may then work out the positioning of the sensors through simple analysis of embedding a set of squares within the larger surveillance region, and if necessary, adjusting the number of squares to ensure a complete coverage of the region.

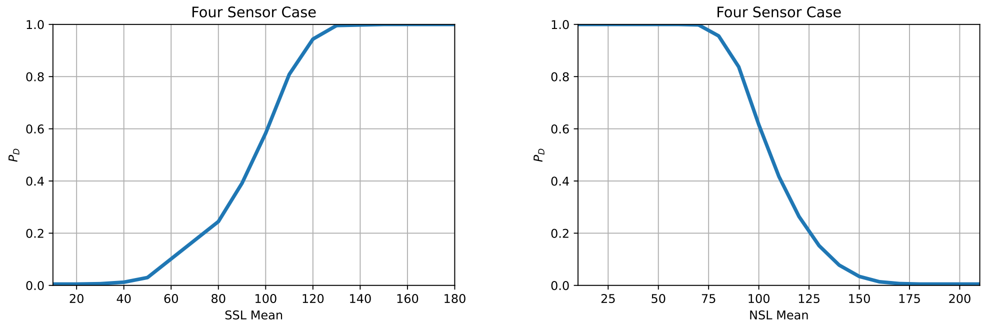

Figure 3. Left subplot: PD as a function of the average SSL, for four sensors in a 0.4 nmi square, with average noise level set to 68.5 dB. Right subplot: PD is a function of the average NSL, when the SSL is 133 dB.

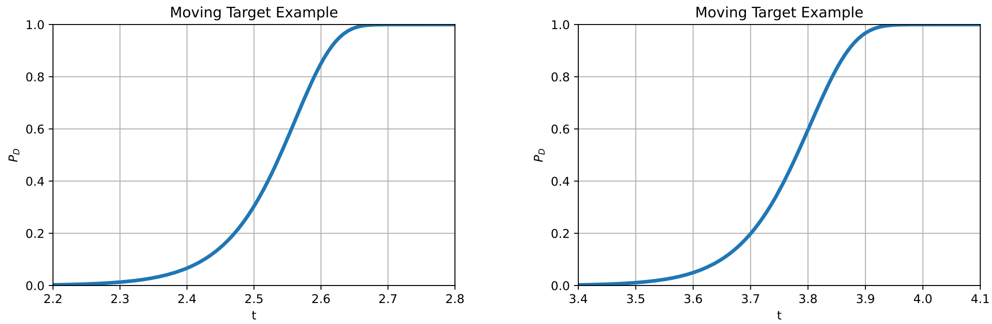

Figure 4. Left subplot is detection of a submarine travelling at 4 kts with a SSL of 133 dB, which will take 3 minutes to reach the sensor field's boundary. Right subplot is when the SSL is reduced to 90 dB. This threat will not be detected until it is within the sensor field.

3.3 An Example where $R = 0.4$ nmi

Suppose that the surveillance region is a square of length $R = 0.4$ nmi, and so from the previous discussions one requires four omnidirectional sensors to maximise coverage. Figure 1 (left subplot) is an illustration of this scenario. Suppose that the sensors are deployed at coordinates $(0.1, 0.1)$, $(0.3, 0.1)$, $(0.1, 0.3)$ and $(0.3, 0.3)$, which are the centres of four squares embedded in the region. Figure 3 (left subplot) plots the PD as a function of SSL mean but for the fixed average noise level stipulated above. At the mean SSL level of 133 dB, and above this number, the PD is almost unity, which is to be expected. Below this average SSL the PD is significantly smaller. To provide another perspective on this situation, the right subplot in Figure 3 plots the PD as a function of the average NSL, where the SSL is set to 133 dB. The subplot implies that provided the average NSL is smaller than 75 dB target detection is certain, which is to be expected in view of the sensor field design features.

As a final example, suppose that a submarine is travelling toward the sensor network analysed in this subsection, with the SSL and average NSL stipulated above, and that it is travelling at 4 kts along a linear path whose coordinates are $x_t = 0.6 - 4t$ and $y_t = 0.25$ respectively, where t is measured in hours and $0 \leq t \leq 0.15$. Hence it will take 3 minutes for the submarine to reach the boundary of the sensor field from its original position at (0.6, 0.25), whose units are nmi. Figure 4 (left subplot) plots the PD, given by (8), as a function of time, where the latter is measures in minutes in the plot. This example shows that the sensor field will detect the submarine before it reaches the sensor field boundary. At time 2.59 minutes the submarine is at a horizontal distance of 0.4273 nmi (with a PD of approximately 0.8) while at time 2.65 minutes this distance has reduced to 0.4233 with a PD of unity.

The right subplot in Figure 4 illustrates the same scenario as for the left subplot except the SSL has been reduced to 90 dB, a level for which the sensor field is not optimised. Although the plots looks identical the time scale is completely different. In this case the submarine is not detected until it is well within the sensor field. At time 3.84 minutes its probability of detection is approximately 0.8, at a horizontal distance of 0.344 nmi. From time 3.91 minutes it is detected with a probability of unity, when it is at a distance of 0.3393 nmi horizontally. Despite the reduction in SSL by just over 32% the sensor field is still able to detect this threat.

4 CONCLUSIONS

The objective of this paper has been to illustrate a simple way in which a square surveillance region may be partitioned into subsquares where each contains a sensor optimised for detection of threats of interest. An example was used to illustrate the methodology and to develop rules of thumb on the number of required sensors and their positioning. Performance of the sensor network was then assessed when a submarine is approaching it, in the cases when the sensor network is optimised for its detection, and when it is optimised for a mismatched acoustic signature level. A modelling simplification adopted was to focus on initial threat detections without integration of decisions over time. Hence it was assumed that a long-run or stationary distribution is appropriate for the univariate detection process. Further work will aim to relax this constraint and extend the analysis to three-dimensions with non-square surveillance subregions.

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