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Analysis of sonar dipping regimes by analytical means

M van der Merwe [a]

[a] *Defence Science and Technology Group, Human and Decision Sciences Division, Edinburgh, South Australia, 5111, Australia*
Email: martijn.vandermerwe@defence.gov.au

Abstract: Within the Asia-Pacific region, and in particular within Australia's maritime domain of interest, there is growing concern of the potential threat posed by undersea vessels, including both crewed and uncrewed submarines. Many navies, including the Royal Australian Navy, use helicopters to dip sonars into the ocean to detect underwater threats.

In this paper, the cookie cutter detection model is used to determine both the optimal and robust dipping strategies. The optimal solution, in the absence of a dipping duration, is continuous dips, analogous to a sonar being towed through the water. Whereas, when the dipping duration is taken into account, the optimal dipping interval is dependent on the model parameters. Exceeding the optimal dipping interval can significantly decrease the total area covered, hence this paper discusses a method for choosing a robust dipping strategy.

Keywords: *Dipping sonar, coverage, anti submarine warfare*

1 INTRODUCTION

Australia has a significant maritime jurisdiction and is dependent heavily on shipping services for its importing and exporting of goods (Colmer and Ellem, 2021). Within the Asia-Pacific region, and in particular within Australia's maritime domain of interest, there is growing concern of the potential threat posed by undersea vessels, including both crewed and uncrewed submarines. This concern has increased in recent years from observation of increased foreign undersea activity and the evolving capability of countries of potential threat to Australian security (Cocking et al., 2016). Effective methods to detect underwater threats are important as ever.

Dipping sonars are specialized underwater acoustic systems that are designed to be lowered into the water from a helicopter on a cable, allowing for the detection, classification, and tracking of underwater targets. The Royal Australian Navy utilises MH-60R Seahawk helicopters to perform sonar dipping operations. Launched from ships, helicopters equipped with a dipping sonar are a first line of defence for detecting underwater threats. Effective dipping strategies can increase the likelihood of detecting low-acoustic signature threats.

Several simulation studies, and game theoretical approaches have investigated dipping strategies to maximise the likelihood of detection of an underwater threat. However, there is no literature that considers analytical approaches to determining the optimal search strategies for maximising coverage. This paper addresses the gap in research with regards to empirical solutions to optimal dipping strategies as identified in the literature review in Section 2. The description of the geometry of the problem and the model definition is presented in Section 3. The optimal dipping strategy for a linear flight is calculated in Section 4. Finally, the paper concludes in Section 5 with a discussion of the challenges and opportunities in the development of optimal search strategies for the deployment of dipping sonars.

2 LITERATURE REVIEW

A number of publications examining the sonar dipping problem follow a game theory approach. The earliest published work on the subject of dipping sonars for underwater threat detection was conducted by Danskin (Danskin, 1968). The authors utilised a cookie cutter detection model and modelled the problem as a two-person zero-sum game. A search is undertaken for a submarine which initial location is known, called the search datum. It is assumed that the submarine is unaware of the helicopters search pattern and that the maximum submarine velocity is known. The submarine chooses some velocity and direction, which are constant throughout the search, to attempt to evade the search. The author was able to determine the best search strategy for the helicopter using the Nash-Equilibrium to find stable strategies.

Yoash et al. (Yoash et al., 2018) extends Danskin's work. In their study, the submarine's velocity is constant and known to the searcher, and consider two cases, where the direction is unknown but uniformly distributed, and the direction is in one of three wedges with some likelihood. The focus of the study is on the trade-off between increasing dipping frequency to maximise the effectiveness of each dip and reducing the search overlap to cover more area. The authors were able to prove that disjoint dips are optimal and created the associated optimal dipping pattern.

Ristic et al. (Ristic et al., 2022) conducts a study in dipping sonar search patterns using a simulated game theory approach. The authors are able to confirm that the optimal search pattern for a threat moving at a known constant speed from a known initial location in an unknown constant direction is a spiral. A distance based detection probability is used.

Young (2022) considered an version of the submarine search problem where the submarine is closing in on a ship with the intention to initiate a torpedo attack. This constrains the submarines movement, in contrast to Yoash et al. (2018) where the submarine could travel in any direction from its initial point of being detected. Young (2022) limits the submarines direction based on the limiting lines of approach of the submarine, and then applies Yoash et al. (2018) dipping strategy to the reduced search space.

An alternative approach to that of a game theory is to formulate the dipping problem as a coverage problem, where the goal is to maximise the area of the search space that is covered by the total sensing area. The coverage problem has been investigated independently from the sonar dipping problem. See for example the paper by Birgin et al. (Birgin et al., 2021) who provides a short introduction to covering problems in their paper. An example of a coverage problem is the circle packing problem, in which the goal is to determine the maximum radius r_n, of n non-overlapping circles that fit in the unit square. A slight variation on the circle packing problem is the disk covering problem, in which the aim is to find the smallest $r(n)$, for n circles with radius $r(n)$ covering the unit circle. These problems are useful for providing upper and lower bounds on the number of dips or sensors that are required to achieve a coverage, but do not take into account practical constraints, for example an evading enemy or a limit on the number of dips a helicopter can achieve on single flight before refuelling.

Although dipping strategies have been studied using game theoretical approach, there has been limited investigation into dipping strategies that maximise coverage of a search area. Maximising coverage may be desirable in the scenario where the aim is to achieve full coverage with limited resources or the case where there is no prior knowledge of the location of the target.

3 MODEL DEFINITION

Consider a helicopter flying on a linear path dipping a sonar at regular intervals with the aim of detecting an underwater target. The helicopter is flying at a speed of s performing dips at regular intervals of time t_i (or distance $2x$), each dip has a duration of t_d. Let n be the total number of dips and A the total area covered. The flight has a total mission duration of t_T with the first dip at time $t = 0$ and final dip at some time $t < t_T$.

The total area covered is determined using a cookie cutter approach with a maximal sensing distance of ϵ. The cookie cutter is the region that is being ensonified such that if the underwater target is within ϵ, the echo it returns will be detectable.

The total area covered by three dips is illustrated in Figure 1. The total area covered by n dips is

$$A(n) = n\pi\epsilon^2 - h(n)$$

where $h(n)$ is the area covered by the intersections of sensing areas. In the case where the intersection of the ith dip with the area covered by preceding dips is C, that is $C_i \cap C_{i-1} = (C_i \cap C_{i-1}) \cup (C_i \cap C_j)\ \forall\, j < i$, then

$$A(n) = n\pi\epsilon^2 - (n-1)C. \tag{1}$$

The intersecting area C of two dipping areas for consecutive dips on a linear flight path is given by $C = 2\epsilon^2 \arccos\left(\frac{x}{\epsilon}\right) - 2x\sqrt{\epsilon^2 - x^2}$.

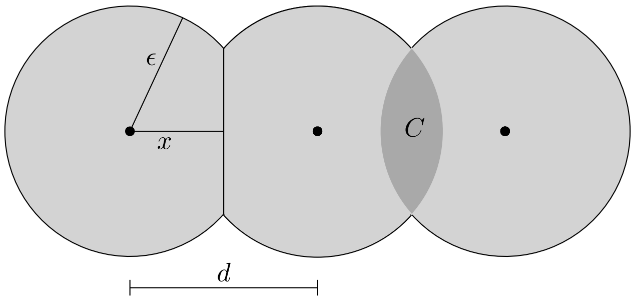


Figure 1. The area covered by a cookie cutter dipping model - three dips at regular intervals d (or $2x$) is shaded in light and dark gray. The area where the sensing area of two consecutive dips overlap, C, is shaded in dark gray.

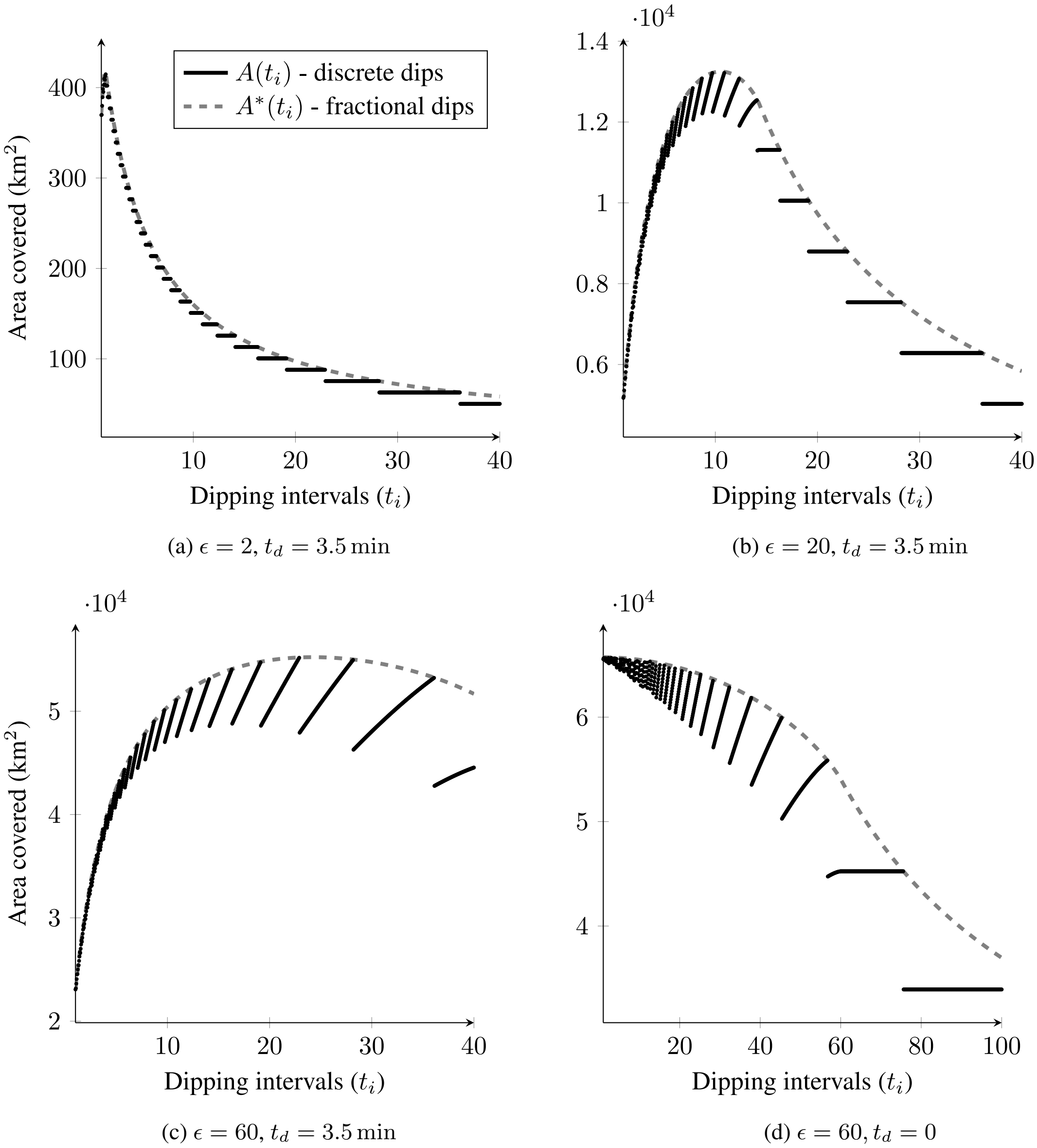
The mission duration imposes the constraint $nt_d + (n-1)t_i \leq t_T$, and since $n \in \mathbb{N}_0$ it follows that

$$n = \left\lfloor \frac{t_T + t_i}{t_d + t_i} \right\rfloor. \tag{2}$$

Considering that there is no overlap between the sensing areas when $x \geq \epsilon$ and substituting the expression for n in (1) yields

$$A(t_i) = \begin{cases} \left\lfloor \frac{t_T+t_i}{t_d+t_i} \right\rfloor \pi\epsilon^2 - \left(\left\lfloor \frac{t_T+t_i}{t_d+t_i} \right\rfloor - 1 \right) \left(2\epsilon^2 \arccos(\frac{st_i}{2\epsilon}) - st_i\sqrt{\epsilon^2 - (st_i)^2/4} \right) & x \leq \epsilon \\ \left\lfloor \frac{t_T+t_i}{t_d+t_i} \right\rfloor \pi\epsilon^2 & x > \epsilon \end{cases} \tag{3}$$

As an example, consider the case where $t_d = 3.5\,\mathrm{min}$ (Forecast International, 2021), $s = 168\,\mathrm{km\,h^{-1}}$ (Eyre, 2019) and $t_T = 2.7\,\mathrm{h}$ (Naval Techonology, 2015). Three values for the maximum detection radius are considered, $\epsilon = 2\,\mathrm{km}, 20\,\mathrm{km}$, or $60\,\mathrm{km}$ - these values are chosen to explore the typical ranges of dipping sonars. For example, Young (2022) and Yoash et al. (2018) assumed a detection radius of $1\,\mathrm{Nm}$ ($1.852\,\mathrm{km}$) and $2\,\mathrm{Nm}$ ($3.704\,\mathrm{km}$), respectively, and the a typical dipping sonar can detect targets at a distance of up to $111\,\mathrm{km}$ (L3Harris, 2025). The area covered as a function of the dipping interval is shown in Figure 2.



(a) $\epsilon = 2$, $t_d = 3.5\,\mathrm{min}$

(b) $\epsilon = 20$, $t_d = 3.5\,\mathrm{min}$

(c) $\epsilon = 60$, $t_d = 3.5\,\mathrm{min}$

(d) $\epsilon = 60, t_d = 0$

Figure 2. The total area covered, $A(t_i)$, as a function of the dipping interval as defined in (3) with $s = 168\,\mathrm{km\,h^{-1}}$, $t_T = 2.7\,\mathrm{h}$ and remaining parameter values are indicated in the figure captions. The gray dashed line is the area covered if fractional dips are allowed, *i.e.* $A(x)$ without the floor function.

Note that for a given number of dips n, increasing the distance between these dips decreases the overlap of two consecutive dips, hence the total area covered by n dips increase as the dipping interval increases. Noting that increasing the distance between dips sufficiently will reduce the number of dips that can be performed within the time limit t_T. Since a reduction in the number of dips decrease the area covered, $A(t_i)$ is a noncontinuous function with discontinuities at values of t_i satisfying $\left\lfloor \frac{t_T+t_i}{t_d+t_i} \right\rfloor \in \mathbb{N}_0$ as illustrated in Figure 2.

4 MAXIMISING THE AREA COVERED

The dipping interval that maximises the area covered is calculated in this section. Two cases are considered for dipping duration, that is, $t_d = 0$ and $t_d > 0$.

4.1 Zero cost dipping

In the absence of a dipping duration, *i.e.* $t_d = 0$, the distance covered by the helicopter is independent of the dipping interval. Let the total distance travelled be donated by $l = s \cdot t_T$. Then the expression for the area covered (3) simplifies to

$$A_1(x) = \begin{cases} (\left\lfloor \frac{l}{2x} \right\rfloor + 1)\pi\epsilon^2 - \left\lfloor \frac{l}{2x} \right\rfloor \left(2\epsilon^2 \arccos(\frac{x}{\epsilon}) - 2x\sqrt{\epsilon^2 - x^2}\right) & x \leq \epsilon \\ (\left\lfloor \frac{l}{2x} \right\rfloor + 1)\pi\epsilon^2 & x > \epsilon \end{cases} \tag{4}$$

As $x \to 0$ the total area covered $A_1(x)$ approaches that of a rectangle with sides of lengths 2ϵ and l and two semicircles of radius ϵ. The maximum value of $A_1(x)$ is achieved at this limit as stated without proofs in the following two lemmas.

Lemma 1.

$$\lim_{x \to 0} A_1(x) = \pi\epsilon^2 + 2l\epsilon$$

Lemma 2. *Considering a linear flight path with zero dipping duration. Then $x = e$, with $e \to 0$ is the optimal dipping strategy where the maximum coverage is achieved.*

4.2 Non-zero dipping cost

Two methods are considered for finding the number of dips that maximises the area covered for the case where $t_d > 0$. The first method is through an exhaustive search, and the second method is by finding the local maximum point(s) of $A(t_i)$ by calculating the derivative.

The following lemma enables the exhaustive search approach.

Lemma 3. *The value of t_i that maximises A satisfies $t_i = \frac{t_d n - t_T}{1-n}$ for some $n \in \mathbb{N}$.*

It follows from Lemma 3 that the optimal dipping strategy will occur at one of the jump discontinuities where $n_c \in \mathbb{N}_0$. There is at least one dipping interval and therefore least number of dips is $n = 2$. The maximum value of n is $\lfloor t_T/t_d \rfloor$, in this case $\max n(t_i) = n(0) = 46$. It follow $n \in \{2, 3, 4, ..., 46\}$. From Lemma 3 and substituting values of $n = 2, 3, ...$ in the expression for t_i, optimal strategy to dip is every 10m55s for a total of 12 dips. This provides a maximum coverage of 13 242 km^2.

Let A^* denote the total area covered when fractional dipping is allowed, that is, allow n to take on any real value $n = \frac{t_T+t_i}{t_d+t_i}$. The maximum value is achieved when the derivative is zero, that is $\frac{dA^*}{dt_i} = 0$.

It follows from the definition that $A \leq A^*$. Recall that ϵ, s, t_T, t_d are model parameters and t_i (time between dips) is the decision variable. Let

$$f(t_i) = \frac{t_T + t_i}{t_d + t_i}\pi\epsilon^2 - \left(\frac{t_T + t_i}{t_d + t_i} - 1\right)\left(2\epsilon^2 \arccos(\frac{st_i}{2\epsilon}) - st_i\sqrt{\epsilon^2 - (st_i)^2/4}\right).$$

The following notation is introduced to aid calculations:

$$h(t_i) = 2\epsilon^2 \arccos(\frac{st_i}{2\epsilon}) - \epsilon st_i\sqrt{1 - (\frac{s}{2\epsilon}t_i)^2}$$

Note that $\frac{d}{dx} arcos(x) = \frac{-1}{\sqrt{1-x^2}}$, and so

$$\begin{aligned} h'(t_i) &= 2\epsilon^2 \frac{-1}{\sqrt{1 - (\frac{s}{2\epsilon}t_i)^2}} \frac{s}{2\epsilon} - \epsilon s\sqrt{1 - (\frac{s}{2\epsilon}t_i)^2} - \epsilon st_i \frac{1}{2}(1 - (\frac{s}{2\epsilon}t_i)^2)^{-1/2}(-(\frac{s}{2\epsilon})^2 2t_i) \\ &= \frac{s^3t_i^2 - 4\epsilon^2 s}{2\epsilon\sqrt{1 - (\frac{s}{2\epsilon}t_i)^2}} \end{aligned}$$

Consequently, it follows that

$$f(t_i) = \epsilon^2 \pi \frac{t_T + t_i}{t_d + t_i} + \frac{t_d - t_T}{t_d + t_i} h(t_i)$$

and the derivative

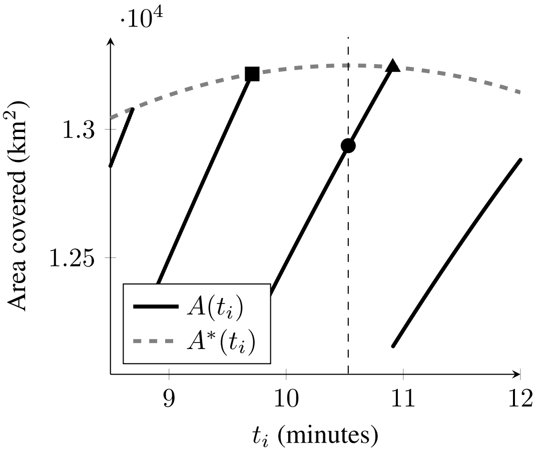
$$f'(t_i) = \epsilon^2 \pi \frac{t_d - t_T}{(t_d + t_i)^2} - \frac{t_d - t_T}{(t_d + t_i)^2} h(t_i) + \frac{t_d - t_T}{t_d + t_i} h'(t_i)$$

$$f'(t_i) = 0 \qquad \implies \epsilon^2 \pi - h(t_i) + (t_d + t_i) h'(t_i) = 0$$

Hence the value of t_i that maximises $A^*(t_i)$ can be found by solving

$$\epsilon^2 \pi - 2\epsilon^2 \arccos(\frac{st_i}{2\epsilon}) + \epsilon s t_i \sqrt{1 - (\frac{s}{2\epsilon} t_i)^2} + (t_d + t_i) \frac{s^3 t_i^2 - 4\epsilon^2 s}{2\epsilon \sqrt{1 - (\frac{s}{2\epsilon} t_i)^2}} = 0 \tag{5}$$

Using the parameter values defined in Section 3 and solving (5) using a numeric solver results in $t_i^{\max} \approx 10.53$ minutes and $A^*(10.53) = 13\,248.76\text{km}^2$ which is achieved with 12.29 dips, as illustrated in Figure 3. Substituting the values of t_i corresponding with 12 and 13 dips, respectively, confirms that the $t_i = 10$minutes 55 seconds yields the maximum coverage of $13\,232\,\text{km}^2$. Choosing a good dipping strategy is discussed in the next section.



(a)

t_i (m:s)	A_2 (km^2)	% optimal
9:42	13 216	0.2
9:44	12 234	7.6
10:32	12 936	2.3
10:55	13 242	0.0
10:56	12 164	8.1

(b)

Figure 3. The area covered as a function of the dipping interval near the maximum possible area covered. The ▲ and ■ symbols indicate optimal dipping intervals for 12 and 13 dips respectively. The vertical dashed line indicates the optimal value of A^*, that is $t_i = $ 10m32s.

4.3 Dipping strategies

In this section a practical choice of t_i is investigated with the aim of choosing a value that provides a robust solution. That is, variation in t_i will not lead to a significant reduction in area covered. A helicopter attempting to perform a search may not be able to perform dips at exact intervals and it may be beneficial to choose a dipping interval that is is sub optimal to ensure a good mission outcome.

The maximum number of dips are made when $t_i = 0$ and $n = \lfloor t_T / t_d \rfloor$. In which case all dips are performed at the same location and the total search area is given by the area of the circle with radius ϵ, that is $A(0) = \pi\epsilon^2$. As the dipping interval increases, on average, the total area covered increase until it reaches the inflection point, $\frac{d}{dt_i} A^*(t_i) = 0$, at which point the area covered decreases, see Figure 3.

Note that for larger t_i and fewer dips, each individual dip proportionally contributes more to the total area covered, this is due to the decrease in overlap between successive dipping areas. Thus for fewer dips, the risk increase associated with not being able to execute the final dip. This may occur due to exceeding the helicopter's maximum flight time.

For example, the area covered near two jump discontinuity in $A(t_i)$ is shown in Table 3b. If the dipping interval increases by 1 second from 10m55 to 10m56, the area covered reduces from $13\,242\,\mathrm{km}^2$ to $12\,164\,\mathrm{km}^2$, which is a 8.1% decrease compared to the optimal coverage. It may be desirable to include a buffer in the dipping interval to allow for unexpected delays to ensure that there is enough time to perform the final dip. For example, performing 12 dips with a 5 minute buffer in the total time requires a dipping interval $t_i = 10$ minutes 27 seconds which results in $12\,870\,\mathrm{km}^2$ being covered, which is within 3% of the optimum value.

5 CONCLUSION

A cookie cutter model was presented in this paper with the aim of exploring the optimal dipping strategy of a helicopter searching on a linear flight path.

It was shown that in the absence of a dipping duration, the optimal solution is to perform continuous dips, analogous to a sonar being towed through the water. When the dipping duration is accounted for then there is some optimal dipping interval which is dependent on the model parameters. An example using realistic model parameters was used to demonstrate what an optimal dipping strategy may look like.

This approach uses an idealised cookie cutter model of threat detection. Further, it was assumed that an increase in coverage is directly related to an increase in the likelihood of detecting a threat. Maneuvering of a target to avoid detection is not considered in this modelling approach. However, the method investigated may be used to determine the upper bound of the area that can be covered by a helicopter deploying a dipping sonar with the aim of detecting an underwater target.

Extension of the work presented here could consider different flight paths of the helicopter, for example, a helicopter performing dips while flying on a circular flight path. Other extensions that could be considered include modelling the acceleration and deceleration of the helicopter, a probability of detection based on the distance between the sonar and the target, maneuvering of the target, and accounting for the time prosecuting a target once a contact has been made.

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