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Hydrological modelling under uncertainty: Can two-state residuals improve error representation?

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Abstract: This study introduces a novel two-state residual modelling framework designed to improve streamflow prediction by decomposing hydrological residuals into their constituent components. Rainfall-runoff models always produce residuals, the difference between observed and modelled catchment response, that exhibit an autocorrelative nature, suggesting they contain a predictable signal. Our framework separates these residuals into state-dependent components representing modelling errors and observation errors. This decomposition offers more accurate estimates of hydrological errors, even in the absence of observed flow data. The framework holds significant promise for improving streamflow estimation in un-instrumented catchments and enhancing the correction of peak flow events. These practical applications are beneficial for both hydrological research and operational water resources management.

Keywords: *Runoff correction, residual modelling, modelling error, observation error*

1. INTRODUCTION

Residuals provide a practical basis to quantify uncertainty in hydrological modelling. The temporal dependencies within the hydrological residuals make it a predictable signal (Kuczera, 1983). Moreover, Li et al. (2021) proposed that residuals produced by a hydrological model can be predicted in absence of observed flow. Although Li et al. (2021)'s residual model, referred as to baseline model hereafter, parsimoniously estimates total residuals using modelled flow only, but a critical question. We argue that uncertainty in hydrological modelling is intrinsically state-dependent and can differ during different precipitation and flow regimes. This disparity hints towards the multimodal behaviour of residuals as well. Therefore, the assumptions underlying existing residual modelling frameworks, including Li et al. (2021), that justify unimodal treatment of residuals across all hydrological states, can be questioned.

When separated according to absence or presence of precipitation, or based on increasing or decreasing flows, residuals exhibit significantly different predictability, characterized by lag-1 autocorrelation (ρ_1), and variance (σ^2). The difference in residual behaviour in contrasting precipitation and flow states necessitates the need of a targeted modelling strategy to effectively capture the full range of residuals in hydrologically distinct processes. Therefore, we propose a two-stage residual model that acknowledges the heteroscedastic nature of residual and treats them through multimodal framework for enhanced flow corrected and peak floe estimation.

2. METHODS AND RESULTS

The proposed two-stage residual model is grounded on physical information from hydrological model and predictive abilities of Long Short-term Memory (LSTM) network. First we use the hydrological model under three different calibration settings; 1) when modelling parameters are fully calibrated, 2) when paramters are partially (or poorly) calibrated, and 3) without any paramter calibration, to mimic all possible real world scenarios. We will discuss second case further in details here. Time series of residuals is then disaggregated into hydrlogical state-depended components; dry period errors (when rainfall is almost zero, $P \approx 0$) and wet period errors (for non-zero rainfall, $P \neq 0$).

$$e_t = e^{P\approx 0} + e^{P\neq 0} \tag{1}$$

As the dry period errors occur in absence of rainfall, it is assumed that modelling error is the only constituent of the total error during the dry period. Thus stage-I residual model, built on LSTM (L_1),is trained and tested to estimate the dry period error using only the modelled streamflow as input varaible. L_1, further expanded to wet period error, gives reliable estimates of modelling error in presence of rainfall. The difference between total wet period error and the estimated modelling error is then used to train the stage-II residual model (L_2) that estimates the remainder observation error during the wet period error, given the rainfall instances as input variable. This two-stage modelling setup gives estimates of modelling error during both; the dry period and wet period, as well as observation error during the wet period. Thus accumulation all estimated constituents gives a better estimation of the total residuals which are further used to correct the modelled streamflow (as well as peak flows), or estimate the observed flow in ungauged catchments as in equation 2.

$$\hat{Q}_o = \hat{Q}_m + \hat{e}_t \tag{2}$$

The proposed framework is applied to Warren River Catchment located south of Perth in Western Australia. In parallel to the proposed model, Li et al. (2021)'s residual model is also applied as a baseline approach. While this baseline model exhibits its ability to esitmate residuals and correct modelled flows, the performance significantly drops at high magnitude flows. The baseline model reduced mean squared error (MSE) of hydrological model from 2.41 x 10^{-4} mm/d to 5.30 x 10^{-3} mm/d, whereas the proposed model reduced it to 2.70 x 10^{-3} mm/d. Also, at high (q > 95^{th} percentile) and extreme (q > 99th percentile) flows the performance of baseline model dropped significantly. Meanwhile, the proposed two-stage residual model shown superior performance in estimating residual and correcting overall flows with Kling-Gupta Efficiency (KGE) of 0.84 and compared to 0.76 of the baseline model and 0.24 of the partially calibrated hydrological model alone (without any residual model). The proposed model also demonstrates better suitability to estimate high and extreme magnitude flows with KGE of 0.78 and 0.45 respectively, against 0.66 and 0.03 respective KGEs of baseline model. Table 1 provides the performance summary of a stand-alone partially calibrated hydrological model, baseline residual model and proposed two-stage residual model. Similar model performance trends were found for fully calibrated and un-calibrated settings of the hydrological model.

Table 1 Performance summary of residual models

Model Settings	MSE (x 10^{-3}) [mm/d]	NSE	KGE
Stand-alone hydrological model	24.10	0.38	0.24
Baseline residual model	5.30	0.86	0.76
Proposed residual model	**2.70**	**0.93**	**0.84**
Performance at high flows (q > 95^{th} percentile)			
Stand-alone hydrological model	388.30	(4.56)*	(0.14)
Baseline residual model	47.2	0.32	0.76
Proposed residual model	**22.3**	**0.69**	**0.84**
Performance at extreme flows (q > 99^{th} percentile)			
Stand-alone hydrological model	802.70	(41.71)	(0.01)
Baseline residual model	47.2	(1.51)	0.03
Proposed residual model	**27.3**	**0.14**	**0.45**

All error values are in mm/day. * Values of NSE range from -∞ to 1, where NSE < 0 means that the model performs poor and using simple mean of observed data can produce better results instead.

3. CONCLUSIONS

We introduced a two-stage residual model that treats hydrological residuals as a combination of state-dependent constituents; modelling and observation errors and estimates them separately. This approach produces estimation of total residuals of partially-, fully- calibrated, and standalone hydrological model at low and high flows, as compared to baseline residual model. Thus, the proposed model offers better and reliable correction of hydrological model responses across all flow regimes and calibration settings. We propose this model as a reliable alternative to estimate peak flows, specially in catchments with poor or no observed flow records and recalibrated hydrological models. Also, in un-instrumented catchments this method can be viable for flow estimation and eventually improved water resources management.

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