

Modelling heteroscedasticity and detecting shifts in rating curves for reliable streamflow records

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Abstract: River discharge is usually estimated using stage-discharge rating curves, which are often assumed to be stable over their period of application. However, river morphology is dynamic, with channel-altering events like scour and deposition causing shifts in the stage-discharge relationship. These shifts can lead to significant inaccuracies in discharge records if the validity of the rating curve is not regularly assessed via at-site gaugings. To address this challenge, this study introduces a robust, data-driven framework to address the effect of non-stationarity in rating curves on streamflow estimations.

The residual in streamflow estimation, defined as the difference between the gauged discharge and the rating curve estimate at a given stage, provides important insight into prediction errors. The variance of these residuals often increases with discharge and is heteroscedastic, meaning that it changes over time and with flow conditions. This heteroscedasticity reflects multiple sources of uncertainty, with both measurement errors and channel changes contributing to high uncertainty at low and high flows (McMillan and Westerberg 2015). While the shifts in the stage-discharge relationship can be identified using manual data examination or various change point detection tests, explicitly modelling the error structure offers a more systematic and robust way to detect these changes. This allows for more precise segmentation of rating curves into periods of hydraulic stability, ultimately improving uncertainty quantification and leading to more reliable streamflow estimates.

This study leverages an exceptionally rich dataset comprising almost daily at-site gaugings—data that are not generally available at most gauging stations. The primary objectives are: (a) to automatically detect change points in the stage-discharge relationship, (b) to develop a segmented rating curve model for distinct periods of hydraulic stability, and (c) to quantify its predictive accuracy. Using daily velocity-area measurements, the methodology was applied to selected gauging stations distributed across the Mahanadi and Godavari basins in Peninsular India, providing a testbed for evaluating the proposed framework.

To establish a benchmark, a stationary baseline was constructed using an adaptive Bayesian model that describes the entire record. In this model, error heteroscedasticity was explicitly modelled as a power function of predicted discharge, allowing for realistic representation of uncertainty in streamflow estimates. The proposed non-stationary framework builds upon this by employing a Sequential Bayesian Change Point Algorithm (SBCPA) to identify periods of hydraulic stability. For each detected segment, a deterministic rating curve was fitted, and its predictive uncertainty was characterized by modelling error heteroscedasticity with a Locally Estimated Scatterplot Smoothing (LOESS) approach.

The findings revealed that explicitly accounting for shifts is critical, with the non-stationary model yielding an improvement of up to 60% in probabilistic estimation accuracy over the baseline stationary model. By systematically detecting dynamic shifts and quantifying uncertainty, the new framework addresses a major source of error in streamflow records and provides a path toward more robust hydrological monitoring.

This approach enables hydrologists to identify likely times of change and the intervals over which rating curves can be considered stable, supporting more accurate streamflow estimation and uncertainty assessment. The methodology has broad applicability for river monitoring networks worldwide, especially as more frequent gauging data become available. Emphasizing the need for non-stationary analysis, this work advances the state of practice in hydrometric data analysis and highlights the critical role of uncertainty modelling in water resources management.

REFERENCES

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