26th International Congress on Modelling and Simulation, Adelaide, SA, Australia, 30 Nov–4 Dec 2025
mssanz.org.au/modsim2025

A stochastic nonlinear model of influence in synchronisation dynamics

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Abstract: We propose a mathematical model that represents the influence of an external actor (influencer) on a network of actors (influenced). The model is an adaptation of control systems within the family of formulations inspired by the Kuramoto Model of synchronisation. Capturing influence as a capacity to effect the character or behaviour of another, we study an external node's ability to control the synchronisation of the Kuramoto system while its global behaviour is pulled to the external control's frequency and away from its natural mean frequency.

Keywords: *Synchronisation, control, stochastic, networks, influence*

1. INTRODUCTION

The modern phenomenon of social media and its impact on societal behaviours suggests there is value in research on the mechanisms of 'influence' within networked systems of interacting agents. We propose a mathematical model of such a system, where synchronisation is the underlying dynamic occurring on a network. Influence can be defined as the capacity (of an agent or set of agents) to have an effect on the thoughts, character, development or behaviour of one or more individuals, and is a concept that commonly arises in work on social networks and collective behaviour. We consider the synchronisation of entities in a complex system which is pivotal in many scientific areas including social systems. To propose a mathematical model that represents the influence of an external node on another, we adapt a model of control systems within synchronisation. The control problem, such as that solved by the Kalman filter, is whether a system can be controlled such that any final state is reachable by any initial state. A network version of this has been developed in Liu. For network synchronisation this becomes whether synchrony can be achieved to a desired frequency, say Ω, from any initial oscillator state. Previous work in Brede and Kalloniatis, has shown that for deterministic dynamics, adaptive mechanisms can be more efficient than traditional 'pinning control' approaches, such as Skardal. We extend this concept of controlling a system to influence, whereby the influencing agent seeks to shape or direct the system away from its natural behaviour: either speed up or slow down the collective behaviour of the influenced system away from its natural preferred collective frequency $\overline{\omega}$. Our model utilises stochastically generated dynamical weights assigned to the links between the external agent and the system, describing a temporal network whereby control links from one system to the other draw upon non-Gaussian distributions. We use one-sided heavy tail noise specifically here, generated by the Gamma distribution. This now generalises approaches such as in Brede and Kalloniatis but is comparable to stochastic pinning control formulations seen in recent work such as Wu and Zhu, allowing us to study the influence that the external node has on the system's collective synchronisation.

2. METHODOLOGY AND RESULTS

We consider a network A_{ij} of N oscillators, φ_i, with an external control node θ, connected to nodes via B_j. The dynamics are given by

$$\dot{\theta} = \Omega$$

$$\dot{\varphi_i} = \omega_i - \sigma \sum_{j=1}^{N} A_{ij} sin(\varphi_i - \varphi_j) - \tau \sum_{j}^{N} B_j sin(\varphi_j - \theta)$$

where ω_i are the natural frequencies of the network, Ω is the natural frequency of the control and σ, τ are coupling coefficients. We consider the influence links as a random variable, such that at time t

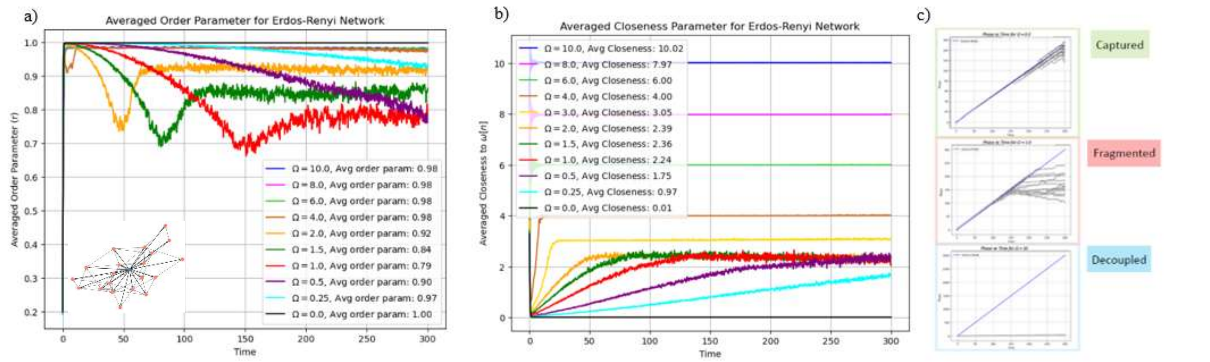
$$B_j \sim \Gamma\left(1, \frac{1}{(\varphi_i - \theta)^2 + 1}\right)$$

We perform numerical experiments to examine transition points in the ability of the influencer to alter the behaviour of the influenced, while retaining synchronisation but driving to collective frequencies outside the bounds of what is achievable through internal synchronisation and strictly deterministic control models. Specifically we consider this through the order parameter r and the closeness Δ defined as

$$r = \frac{1}{N}\left|\sum_{j=1}^{N} e^{i\varphi_j}\right| \qquad\qquad \Delta = \frac{1}{N}\sum_{j=1}^{N}|\dot{\varphi}_j - \Omega|$$

Figure 1 highlights the change in global synchronisation with changing control frequency *Ω*. We see transition behaviours in system under different parameter choices as demonstrated by the change in synchronisation and average order-parameter of the oscillators, shown in a) and c). Combined with the average closeness to the control, shown in b), we see that there are regions where the system is captured by the external node, where it fragments and where it completely decouples from the control node. Analytical calculations based on a clustering approximation reveal Lyapunov stability of the capture regime in this system.

Figure 1 Average order parameter for Erdos-Renyi Networks



a) The average order parameter over an ensemble of 100 Erdos-Renyi graphs (inset an example graph (red) with single control node (blue)) demonstrating synchronisation, for different values of control parameter Ω. b) The average closeness over an ensemble of 100 Erdos-Renyi graphs, demonstrating capture by the control node for different values of control parameter Ω. c) Example phase plot for $\Omega = 0.5$, $\Omega = 1$ and $\Omega = 10$ demonstrating capture, fragmentation and decoupling of the network nodes by the control node.

3. CONCLUSIONS

We present a mathematical model for the influence of one actor on a network of actors, adapting control systems within the Kuramoto Model with stochastic coupling coefficients drawn from a time-dependent Gamma distribution for the influence links. Numerical experiments show a region where the external node captures the network to its frequency, while the network maintains synchronisation seen in the order parameter, r, and the closeness, Δ. We demonstrate the dependence of these on the frequency of the external node, Ω, where for high driving frequency control the system decouples while retaining a synchronised state, and smaller values show synchronisation at a frequency shifted from the mean natural frequency. When comparing to deterministic models, the region of capture shows higher order parameter for the stochastic system: noisy influencers do better.

ACKNOWLEDGEMENTS

This research forms part of a Work-Based PhD at Defence Science and Technology Group through the Australian National University.

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