

Hybrid forecasting of crude oil prices using WT-QPSO-ANFIS

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Abstract: Accurate crude oil price forecasting is essential due to the market's high volatility and complex nonlinear dynamics. This study proposes a hybrid model integrating Wavelet Transformation (WT), Quantum-Behaved Particle Swarm Optimization (QPSO), and Adaptive Neuro-Fuzzy Inference System (ANFIS), termed WT-QPSO-ANFIS, to enhance predictive performance. The wavelet decomposition extracts multi-scale features, while QPSO optimizes ANFIS parameters to avoid local minima and improve convergence. Using daily Brent crude oil price data from 2015 to 2025, the model is benchmarked against ARIMA, ANN, PSO-ANFIS, and Sugeno-ANFIS. Results show that WT-QPSO-ANFIS achieves the lowest errors in testing datasets across all key metrics (RMSE, MAE, MAPE, SEM), demonstrating strong generalization and robustness. This research highlights the effectiveness of quantum-enhanced optimization and multiscale learning in modelling complex energy price behaviour.

Keywords: *Crude Oil Price Forecasting, Wavelet Transformation, Quantum-Behaved Particle Swarm Optimization (QPSO), Adaptive Neuro-Fuzzy Inference System (ANFIS), Hybrid Forecasting Models.*

1. INTRODUCTION

Accurate forecasting of crude oil prices is essential for energy policy, investment strategies, and risk management, given the high volatility and nonlinear behaviour of oil markets. Crude oil prices are influenced by a variety of complex factors, including geopolitical tensions, supply-demand imbalances, and financial speculation. Traditional econometric models such as Autoregressive Integrated Moving Average (ARIMA), Generalized Autoregressive Conditional Heteroskedasticity (GARCH), Cointegration Models, have been widely used for price and volatility prediction (Frey et al. 2009, de Albuquerque et al. 2018, Fan and Li 2015, Zhang et al. 2018). However, these models rely on assumptions of linearity and stationarity, which limit their ability to capture abrupt shifts, structural breaks, and nonlinear dependencies common in real-world oil price dynamics.

In response to these challenges, machine learning models, such as Artificial Neural Networks (ANN) and Long Short-Term Memory (LSTM) networks, have been adopted for their nonlinear approximation capabilities (Sehgal and Pandey 2015, Chiroma et al. 2016, Odion et al. 2024). In recent years, machine learning (ML) techniques have gained prominence in crude oil price forecasting due to their ability to model nonlinear relationships and handle high-dimensional data. Tree-based ensemble methods such as Random Forest and XGBoost have demonstrated superior accuracy and robustness compared to traditional econometric models (Guliyev and Mustafayev, 2022). Deep learning approaches, including LSTM networks, have also been employed to capture temporal dependencies in volatile markets (Chiroma et al., 2016).

Furthermore, Hybrid ML models that combine decomposition techniques such as Ensemble Empirical Mode Decomposition (EEMD), which breaks down non-linear and non-stationary time series into intrinsic mode functions, and wavelet transform with predictive models like Support Vector Machines (SVM) or Adaptive Neuro-Fuzzy Inference Systems (ANFIS) have shown improved forecasting performance by isolating patterns at different temporal frequencies. (Abdullah and Zeng 2010, Yu et al., 2008). Optimization-based enhancements, such as Particle Swarm Optimization (PSO), have been widely used to fine-tune ML model parameters, although recent studies suggest that Quantum-Behaved PSO (QPSO) offers better global convergence capabilities (He et al., 2012). These developments underline the potential of hybrid and interpretable machine learning models in addressing the complexities of oil price dynamics.

Despite their potential, these models suffer from issues such as overfitting, high sensitivity to hyperparameters, and difficulty in interpreting results. Moreover, both traditional and machine learning models often fail to account for multiscale features in time series data and rely on static optimization methods that risk convergence to local minima.

To overcome these limitations, this study apply a hybrid forecasting model that integrates Wavelet Transformation (WT) (Chimprang and Tansuchat 2022) for multiscale signal decomposition, Quantum-Behaved Particle Swarm

Optimization (QPSO) for global parameter optimization, and the Adaptive Neuro-Fuzzy Inference System (ANFIS) for nonlinear prediction. This integrated model, referred to as WT-QPSO-ANFIS, is designed to address the key forecasting challenges of nonstationarity, nonlinearity, and model adaptability.

Accordingly, this study investigates how effective the WT-QPSO-ANFIS hybrid model is in enhancing the accuracy and robustness of crude oil price forecasts compared to traditional econometric models, machine learning methods, and standard optimization-based approaches. To achieve this, the study develops a forecasting framework that integrates wavelet-based multiscale decomposition, adaptive neuro-fuzzy inference, and quantum-inspired optimization. The proposed model is then benchmarked against ARIMA, ANN, PSO-ANFIS, and Sugeno-ANFIS. The predictive performance was evaluated using four standard metrics: Root Mean Squared Error (RMSE), measuring the square root of the average squared differences between predicted and actual values; Standard Error of the Mean (SEM), assessing the precision of the sample mean estimate; Mean Absolute Percentage Error (MAPE), expressing prediction errors as a percentage of actual values; and Mean Absolute Error (MAE), representing the average absolute difference between predictions and observations.

This study contributes to the field of energy price forecasting by proposing a novel hybrid model, WT-QPSO-ANFIS, that effectively addresses key challenges in crude oil price prediction, including nonstationarity, nonlinearity, and local optimization traps. By integrating wavelet transformation for multiscale signal decomposition, quantum-behaved particle swarm optimization for enhanced global search, and adaptive neuro-fuzzy inference for nonlinear pattern learning, the model achieves superior forecasting accuracy and generalization performance.

Through empirical validation using Brent crude oil price data from 2015 to 2025, the proposed model outperforms traditional econometric approaches (e.g., ARIMA), conventional machine learning models (e.g., ANN), and existing hybrid frameworks (e.g., PSO-ANFIS and Sugeno-ANFIS) across all key performance metrics. This research not only demonstrates the practical advantages of quantum-enhanced optimization in hybrid learning systems but also provides a scalable and interpretable forecasting framework for volatile and complex financial time series.

2. MODELS AND DATA

2.1. Wavelet Decomposition

To address the nonstationary nature of crude oil prices (oil_t), Wavelet Decomposition is employed as a signal decomposition technique. This approach enables multiscale analysis, effectively capturing both low- and high-frequency dynamics in the data (Percival and Walden, 2000; Gencay et al., 2002). oil_t can be reconstructed using wavelet decomposition as follows:

$$F(oil_t) = \sum_k S_{j,k} \phi_{j,k}(oil_t) + \sum_{j=1}^J \sum_k d_{j,k} \Psi_{j,k}(oil_t) \quad (1)$$

where $\phi_{j,k}(oil_t)$ and $\Psi_{j,k}(oil_t)$ denote the father and mother wavelet functions, respectively. These are defined as:

$$\phi_{j,k}(t) = 2^{-\frac{j}{2}} \phi\left(\frac{t - 2^j k}{2^j}\right), \quad \Psi_{j,k}(t) = 2^{-\frac{j}{2}} \Psi\left(\frac{t - 2^j k}{2^j}\right) \quad (2)$$

Here $S_{j,k}$ and $d_{j,k}$ represent the approximation and detail coefficients, respectively, at scale j and position k . The signal components at each level are given by:

$$S_j(oil_t) = \sum_k S_{j,k} \phi_{j,k}(oil_t), \quad D_j(oil_t) = \sum_k d_{j,k} \Psi_{j,k}(oil_t) \quad (3)$$

Therefore, the original signal $F(oil_t)$ can be expressed as the sum of its approximation and detail components:

$$F(oil_t) = S_J(oil_t) + D_J(oil_t) + D_{J-1}(oil_t) + \dots + D_1(oil_t) \quad (4)$$

where J denotes the wavelet decomposition level. This wavelet decomposition input is the raw Brent crude oil price time series. The decomposition resulting in the form of approximation coefficients (S_3) capturing the long-term trend and detail coefficients (D_1, D_2, D_3) capturing short-term fluctuations and noise. These coefficients serve as the input features for the ANFIS model.

2.2. Adaptive-Network-Based Fuzzy Inference System (ANFIS) Model

ANFIS is a hybrid intelligent system that combines the learning capability of neural networks with the reasoning approach of fuzzy logic. In this study, ANFIS is employed as a neuro-fuzzy model based on the Takagi–Sugeno

fuzzy inference system (Takagi and Sugeno 1985), which is well-suited for modeling nonlinear dynamic systems. The ANFIS architecture and layer function typically comprises five layers:

Layer 1: Fuzzification: Each adaptive node computes the degree of membership for input variables using predefined fuzzy sets (e.g., low, medium, high). Bell-shaped membership functions are applied to wavelet-based inputs: approximation coefficient S_3 , and detail coefficients D_1 , D_2 and D_3 .

$$O_{1,i} = \mu_{N_j}(x) = \frac{1}{1 + \left| \frac{x - c_i}{a_i} \right|^{2b_i}}, \quad x \in \{S_3, D_1, D_2, D_3\} \quad (5)$$

Layer 2: Rule Firing: This layer calculates the firing strength of each fuzzy rule by multiplying the corresponding membership values of all inputs.

$$O_{2,i} = w_i = \prod_k \mu_{N_k}(x_k) \quad (6)$$

Layer 3: Normalization: Each node normalizes the firing strengths by dividing each rule's firing strength by the sum of all rule activations.

$$O_{3,i} = \bar{w}_i = \frac{w_i}{\sum w_i} \quad (7)$$

Layer 4: Rule Output: Each rule's output is computed as a weighted first-order linear function of the input variables, using the normalized firing strength as the weight.

$$O_{4,i} = \bar{w}_i f_i = \bar{w}_i (p_i S_3 + q_i D_1 + m_i D_2 + n_i D_3 + r_i) \quad (8)$$

Layer 5: Final Output: The final output is obtained by summing all weighted rule outputs to produce a crisp prediction

$$O_5 = \sum_i \bar{w}_i f_i \quad (9)$$

The ANFIS model receives as input the wavelet-derived coefficients S_3 , D_1 , D_2 , and D_3 representing the long-term trend and multi-scale fluctuations of the crude oil price series. The model outputs the forecasted Brent crude oil price for the specified prediction horizon.

2.3. Quantum-Behaved Particle Swarm Optimization (QPSO)

QPSO is an advanced variant of the traditional PSO algorithm, inspired by quantum mechanics, in which each particle is modeled as a quantum entity with a probabilistic position governed by a potential well rather than a deterministic trajectory (Sun et al. 2004a, Sun et al. 2004b). In QPSO, the particle's local attractor is computed as:

$$P_{id}^* = \varphi \cdot P_{id} + (1 - \varphi) \cdot P_{gd}, \quad \varphi \sim U(0,1) \quad (10)$$

where P_{id}^* represents the local attractor, calculated as an interpolation between the personal best P_{id} of particle i and the global best P_{gd} on dimension d , where φ is a uniformly distributed random variable in the interval $[0,1]$. The new position of particle i on dimension d is updated by:

$$X_{id,t} = P_{id}^* \pm \alpha \cdot |mbest_d - X_{id,t-1}| \cdot \ln(1/u), \quad u \sim U(0,1) \quad (11)$$

where $X_{id,t}$ denotes the current position of particle i on dimension d at time t , while $mbest_d$ represents the mean best position across all particles in that dimension d . The parameter α is the contraction–expansion coefficient controlling the exploration range, and u is a uniformly distributed random number in the interval $(0,1)$.

$$mbest = \frac{1}{M} \sum_{i=1}^M P_i = \left(\frac{1}{M} \sum_{i=1}^M P_{i1}, \frac{1}{M} \sum_{i=1}^M P_{i2}, \dots, \frac{1}{M} \sum_{i=1}^M P_{in} \right) \quad (12)$$

QPSO improves global search ability, reduces parameter dependency, and is particularly effective in high-dimensional and nonlinear optimization problems. The algorithm takes as input the initial ANFIS model parameters (both premise and consequent) along with the training data, and iteratively updates these parameters to minimize forecasting error. The output is an optimized ANFIS model with improved generalization and predictive performance.

2.4. Performance Evaluation

To assess and compare the predictive performance of the proposed WT-QPSO-ANFIS model against established benchmarks, namely ARIMA(p,d,q), Artificial Neural Networks (ANN), Sugeno-ANFIS, and PSO-ANFIS, four widely used evaluation metrics were employed: Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Standard Error of the Mean (SEM). These metrics collectively capture both the accuracy and consistency of each model's forecasts. The evaluation was conducted using both training and independent testing datasets to assess in-sample learning and out-of-sample generalization. All models were trained under identical input structures and experimental settings, and the reported results represent the average of multiple runs to ensure statistical robustness.

2.5. Data

The empirical data used in this study consist of daily closing prices of Brent crude oil, obtained from Yahoo Finance. The dataset spans from February 22, 2015 to May 18, 2025, covering a total of 3,742 trading days. This comprehensive time series captures both short-term volatility and long-term trends in the global oil market, offering a robust foundation for evaluating the predictive performance of the proposed model.

Prior to modeling, the original price series was decomposed using the Daubechies 4 (db4) wavelet with three levels of decomposition. This technique enables multi-resolution analysis by extracting the approximation component (S_3) and detail components (D_1 , D_2 , and D_3) from the original signal. These four wavelet coefficients were then employed as input features for the forecasting model, while the actual Brent oil price was designated as the prediction target. The input features derived from the wavelet decomposition are illustrated in Figure 1. For model development, the dataset was partitioned such that the first 80% of the data was used for training, and the remaining 20% was used for testing. This partitioning scheme allows for the evaluation of both in-sample learning and out-of-sample generalization capability of the WT-QPSO-ANFIS model.

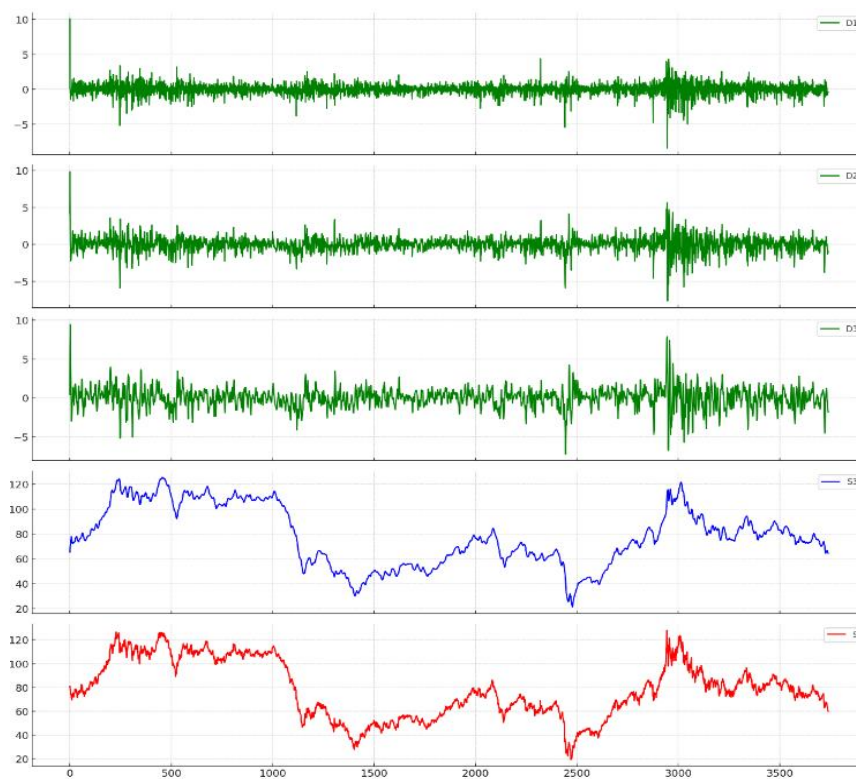


Figure 1 The complete wavelet decomposition of the Brent crude oil price series

The red line (S) represents the original Brent price signal. The blue line (S_3) denotes the level-3 approximation coefficients, capturing the dominant low-frequency movements that represent the long-term trend in the data.

Because most of the variance in Brent prices occurs at low frequencies, S_3 closely resembles the original series while filtering out short-term fluctuations. The green lines (D_1 , D_2 , and D_3) represent the detail coefficients at levels 1, 2, and 3, respectively, which capture higher-frequency variations and short-term noise in the series.

3. RESULTS

The parameters of the membership functions in the ANFIS structure are optimized using the QPSO algorithm. The primary goal of the optimization process is to minimize the prediction error measured by RMSE. The proposed WT-QPSO-ANFIS model is trained on a dataset of 2,993 daily Brent crude oil prices, corresponding to 80% of the full sample collected from February 22, 2015 to May 18, 2025. The remaining 749 daily observations (i.e., the latest 20%) are used as the test set to validate the model's generalization capability.

To ensure robust convergence of the optimization process, each training run was configured with more than 10,000 iterations per execution. Additionally, the training process was repeated 10 times independently, and the model corresponding to the lowest RMSE on the training set was selected as the final configuration. This repeated training strategy mitigates the risk of suboptimal solutions due to the stochastic nature of QPSO and enhances the reproducibility and reliability of the results.

Representative examples of the forecasting results for both the training and testing datasets are illustrated in Figure 2. The model's performance is further evaluated using standard metrics including RMSE, MAE, MAPE, and SEM. The results are summarized in Table 1, highlighting the effectiveness of the proposed hybrid forecasting approach.

The upper panel shows the model performance on the training dataset, while the lower panel presents the prediction results on the testing dataset. The close alignment between the predicted outputs and actual prices in both panels indicates strong learning capability and generalization of the proposed model.

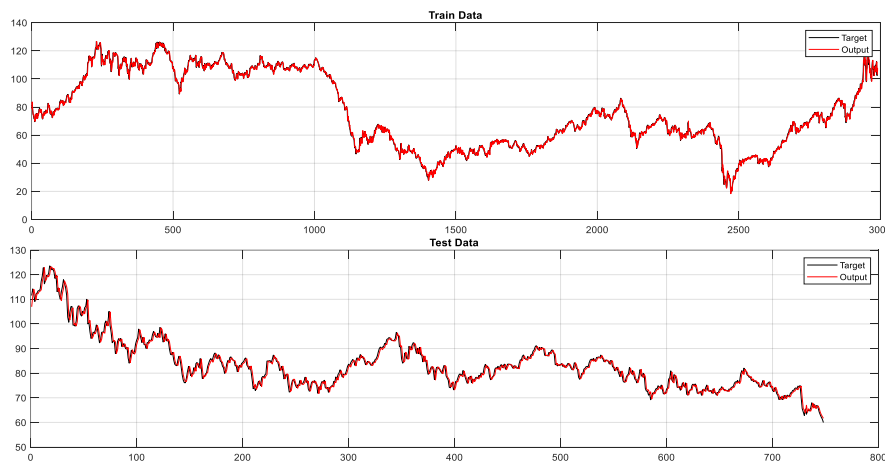


Figure 2 Predicted versus actual Brent crude oil prices using the WT-QPSO-ANFIS model

Table 1 Evaluating the WT-QPSO-ANFIS model and comparison with other methods

Model	Training dataset				Testing dataset			
	RMSE	MAE	MAPE	SEM	RMSE	MAE	MAPE	SEM
WT-QPSO-ANFIS	1.571	1.094	1.494	0.042	1.764	1.304	2.154	0.051
PSO-ANFIS	1.861	1.317	2.244	0.054	2.262	1.703	2.190	0.072
Sugeno-ANFIS	5.342	4.261	5.283	0.276	8.368	7.113	10.272	0.481
ARIMA(2,1,3)	1.525	1.044	1.516	0.028	10.942	9.456	11.866	0.262
ANN	1.704	1.202	1.734	0.031	2.012	1.506	2.281	0.074

Note: Bold values indicate the lowest (i.e., best) values among the compared models for each performance metric.

The comparative results presented in Table 1 demonstrate that the proposed WT-QPSO-ANFIS model delivers highly competitive and balanced performance across all key forecasting metrics. Although it does not attain the lowest values for every metric in the training dataset, it achieves the best performance in terms of MAPE (1.494). Its RMSE (1.571), MAE (1.094), and SEM (0.042) remain close to those of the best-performing model in training, particularly ARIMA(2,1,3), which yields RMSE of 1.525 and MAE of 1.044. Although ARIMA demonstrates slightly better accuracy during the training phase, its performance declines sharply on the testing dataset, with RMSE and MAPE rising to 10.942 and 11.866, respectively. This significant deterioration reflects overfitting and a lack of robustness in forecasting unseen data. The sharp increase in error underscores a key limitation of traditional econometric models: while effective in capturing linear patterns, they struggle to adapt to the nonlinear dynamics, structural breaks, and regime shifts often observed in crude oil markets (Frey et al., 2009; de Albuquerque et al., 2018; Deng et al. 2019).

By contrast, the WT-QPSO-ANFIS model demonstrates strong generalization capability and consistent predictive accuracy on unseen data. It achieves the lowest error values across all four evaluation metrics during the testing

phase, RMSE = 1.764, MAE = 1.304, MAPE = 2.154%, and SEM = 0.051, underscoring its robustness and reliability. Compared to PSO-ANFIS, the integration of quantum-inspired dynamics into the swarm optimization framework results in a consistent reduction in both training and testing errors, confirming the superior global search ability of the quantum-enhanced algorithm in avoiding local minima (Sun et al., 2004a). The quantum behavior enables particles to explore the solution space more effectively through probabilistic positioning rather than deterministic trajectories (Sun et al., 2004b). These advantages are supported by recent financial forecasting applications, where QPSO-tuned ANFIS models have demonstrated significantly improved accuracy relative to PSO-based variants. For example, Ahmad et al. (2014) and Faridi et al. (2023) reported notable reductions in prediction error when using QPSO-optimized ANFIS models for stock market and foreign exchange forecasting, further validating the enhanced convergence and generalization benefits of quantum-behaved optimization techniques.

The Sugeno-ANFIS model, by comparison, records the highest error levels during both training and testing phases (training RMSE: 5.342, testing RMSE: 8.368), underscoring its limited capacity to model complex, nonlinear temporal dependencies inherent in crude oil price movements. This underperformance may stem from its reliance on fixed rule-based structures and lack of adaptive multiscale representation, which restricts its flexibility in capturing dynamic patterns (Takagi and Sugeno, 1985). The substantial deterioration from training to testing performance (56.6% increase in RMSE) suggests that traditional Sugeno-type fuzzy inference systems, while effective for simpler control applications, may be inadequate for highly volatile financial time series without advanced enhancements such as signal decomposition or metaheuristic optimization (Abdullah and Zeng, 2010; Abdollahi, 2020).

4. CONCLUSION

This study proposed a novel hybrid forecasting framework, WT-QPSO-ANFIS, that integrates Wavelet Transformation, Quantum-Behaved Particle Swarm Optimization (QPSO), and Adaptive Neuro-Fuzzy Inference Systems (ANFIS) to improve the accuracy and robustness of crude oil price prediction. By decomposing the original price series into multiple frequency components using wavelet transformation, the model effectively isolates short-term fluctuations and long-term trends. QPSO is then employed to optimize ANFIS parameters, enhancing the learning capacity and convergence of the neuro-fuzzy system.

Empirical results based on Brent crude oil price data demonstrate that the WT-QPSO-ANFIS model outperforms benchmark models, including PSO-ANFIS, Sugeno-ANFIS, ARIMA, and ANN, across multiple error metrics (RMSE, MAE, MAPE, SEM). Notably, the proposed model achieves the best generalization performance in the testing dataset, confirming its robustness and predictive reliability. While ARIMA shows strong in-sample accuracy, its substantial error inflation in the testing phase reveals its tendency to overfit. In contrast, WT-QPSO-ANFIS maintains consistently low error levels, indicating effective model generalization. The strength of this model lies in its dual mechanism: (1) multiscale feature extraction via wavelet decomposition, and (2) enhanced global search capabilities from quantum-inspired optimization. These features allow the model to effectively capture nonlinear, nonstationary dynamics typical of crude oil markets, while avoiding local minima and overfitting.

Despite its strong predictive performance, the WT-QPSO-ANFIS model has several limitations, including high computational demands, sensitivity to parameter tuning, and a reliance on historical price data that excludes key exogenous factors such as geopolitical events and macroeconomic indicators. The fixed wavelet decomposition structure may also limit adaptability across varying market regimes. To address these challenges, future research could explore model generalization across multiple asset classes, integration of fundamental market variables, development of adaptive QPSO algorithms, ensemble model construction with diverse wavelet configurations, and applications to high-frequency data for algorithmic trading. These extensions aim to enhance the model's robustness, interpretability, and responsiveness to dynamic financial environments.

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