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Estimating Sparse Variance-covariance using Shrinkage Methods: Applications to Error Components Models

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Abstract: This paper aims to investigate if shrinkage estimation of sparse variance-covariance matrix can be helpful in producing more robust feasible generalised least squares estimator (FGLS) in the regression context. Consider

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{1}$$

where $\mathbf{X}$ and $\mathbf{y}$ are the $N \times K$ data matrix containing the data from K explanatory variables and $N \times 1$ vector containing the data of the dependent variable, respectively, while $\boldsymbol{\beta}$ is the $K \times 1$ vector of unknown parameter vector. The FGLS estimator is defined as

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{X}'\hat{\boldsymbol{\Omega}}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\hat{\boldsymbol{\Omega}}^{-1}\mathbf{y} \tag{2}$$

where $\hat{\boldsymbol{\Omega}}$ denotes the estimated variance-covariance matrix of $\boldsymbol{\varepsilon}$. The computation of $\hat{\boldsymbol{\Omega}}$ in practice is usually based on the context of the model and/or assumptions underlying the idiosyncratic shocks, $\boldsymbol{\varepsilon}$. The validity of these assumptions is often difficult to verify in practice. The objective of this paper is to investigate to what extend can shrinkage estimation of $\boldsymbol{\Omega}$ be helpful in reducing the assumptions required to estimate $\boldsymbol{\Omega}$.

This paper will first consider the benchmark model in the form of a three-dimensional panel data model:

$$y_{ijt} = \mathbf{x}'_{ijt}\boldsymbol{\beta} + \underbrace{\alpha_i + \gamma_j + \lambda_t + u_{ijt}}_{\varepsilon_{ijt}}, \qquad i = 1,\ldots,N_1,\ j = 1,\ldots,N_2,\ t = 1,\ldots,T, \tag{3}$$

where α_i, γ_j, λ_t and u_{ijt} are pair-wise independent. Under the assumptions that each of these random variables are independently and identically distributed, the variance-covariance matrix of ε_{ijt} is

$$\boldsymbol{\Omega} = \sigma^2_\alpha \mathbf{S}_\alpha \mathbf{S}'_\alpha + \sigma^2_\gamma \mathbf{S}_\gamma \mathbf{S}'_\gamma + \sigma^2_\lambda \mathbf{S}_\lambda \mathbf{S}'_\lambda + \sigma^2_u \mathbf{I}_N \tag{4}$$

where $N = N_1N_2T$ with σ^2_α, σ^2_γ, σ^2_λ and σ^2_u denote the variance of α_i, γ_j, λ_t and u_{ijt}, respectively and $\mathbf{S}_\alpha$, $\mathbf{S}_\gamma$ and $\mathbf{S}_\lambda$ denote $\mathbf{I}_{N_1} \otimes \mathbf{i}_{N_2} \otimes \mathbf{i}_T$, $\mathbf{i}_{N_1} \otimes \mathbf{I}_{N_2} \otimes \mathbf{i}_T$ and $\mathbf{i}_{N_1} \otimes \mathbf{i}_{N_2} \otimes \mathbf{I}_T$, respectively.

The variance-covariance matrix $\boldsymbol{\Omega}$ is not a diagonal matrix in this case even though the variance-covariance matrix of each of the error components is a diagonal matrix. It is, however, a sparse matrix. The structure of the matrix depends on the specification of the error components as well as their variance-covariance structure, which is not known in practice. This paper will investigate the effectiveness of shrinkage estimation of $\boldsymbol{\Omega}$ in the context of FGLS across different assumptions on the error components.

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