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A sensitivity analysis of the Activated Sludge process in a single reactor configuration

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Abstract: The activated sludge process is the most widely used process for the biological treatment of domestic and industrial wastewaters. Wastewater treatment plants using the activated sludge process are in widely used in developed and developing countries. The activated sludge model number 1 (ASM #1) is an internationally accepted standard for activated sludge modelling. It describes nitrogen and chemical oxygen demand within suspended-growth treatment processes, including mechanisms for nitrification and denitrification.

We analysed the biological treatment of wastewater when a single aerated reactor was investigated. The residence time, or hydraulic retention time, was used as the bifurcation parameter and investigate how the autotrophic and heterotrophic biomass concentrations varied.

The configuration analysed consisted of a continuously stirred tank reactor (CSTR) with an attached settling tank. The settling tank concentrated the particulate matter that flowed from the reactor, before being returned back into the reactor.

The model leads to three types of behaviour

1. For low hydraulic retention times, the flow rate is too fast for either biomass to develop, and therefore, there is no reaction occurring in the reactor. We call this the Washout state, as the wastewater flows through the system without any change.
2. For slightly higher hydraulic retention times, a transcritical bifurcation occurs where the heterotrophic biomass concentration first becomes non-zero.
3. Finally, there is a higher hydraulic retention time when a second transcritical bifurcation occurs, where both the heterotrophic and autotrophic biomass concentrations become non-zero.

However, the main focus of this study is to explore how sensitive these metrics are to the twenty system parameters which describe the model.

Keywords: *Activated sludge, modelling, recycling, stirred tank, wastewater, sensitivity analysis*

1 INTRODUCTION

The activated sludge process is the most commonly used aerobic process for the biological treatment of both domestic and industrial wastewaters (Wei et al., 2003). As such, most wastewater treatment plants contain a unit employing the activated sludge process. The key to this process is the presence of highly concentrated micro-organisms, typically 2–4 g/L, present in the form of flocs, which grow through consuming organic pollutants. The mixture of flocs and particulate matter is known, for historical reasons, as "activated sludge". Without sufficient quantities of sludge, i.e. micro-organisms, the process cannot work.

The simplest reactor configuration for the activated sludge process entails the use of two steps: a series of biological reactors and a settling tank (or clarifier). In the former, the pollutants are degraded by microorganisms (the active agent that puts the 'activated' into 'activated sludge'). Inside each of the bioreactors the microorganisms flocculate to form settleable solids. These solids are removed from the effluent stream by sedimentation in a settling tank and then returned to the first reactor in a more concentrated culture. The use of a clarifier ensures the presence of a culture of highly concentrated micro-organisms and is one of the keys to the success of the activated sludge process.

Modelling has become an important tool to develop an understanding of the processes that govern the behaviour of the activated sludge process. A widely used model that describes the biological processes occurring in the activated sludge process is the activated sludge model #1 (ASM #1) (Henze et al., 1987). This consists of a biochemical model for the particulates and soluble materials and a sub-model describing nitrifying processes.

We investigate the steady-state behaviour of the ASM #1 as a function of the hydraulic retention time. (The hydraulic retention time is defined as the volume of the reactor divided by the feed flow rate. It represents the average amount of time that a soluble compound remains in the reactor.) From the steady-state values of the state variables we can calculate the corresponding steady-state values of various quantities associated with the operation of the activated sludge process, such as the concentrations of soluble ammonium, nitrate and oxygen. In particular, we will explore how sensitive these quantities are to the system parameters.

2 MATERIALS AND METHODS

We implement the Activated Sludge Model number 1 (ASM #1) (Henze et al., 1987) in a configuration with a single reactor and a settling tank (part of the exit stream from the settling tank is discharged as waste). A schematic of the reactor configuration is shown in Figure 1. See Henze et al. (1987) for a detailed description

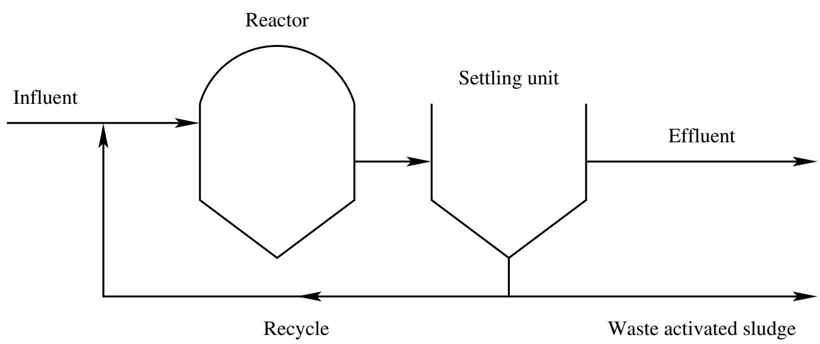

Figure 1. A schematic of the bioreactor configuration with a settling tank.

of the ASM #1 and to Nelson et al. (2019) for a full nomenclature and a complete listing of parameter values. The latter also provides the differential equations for a single reactor connected to an ideal settling tank. The ideal settling tank is characterised by three parameters. These are the recycle ratio (R), the concentration factor of particulates ($\mathcal{C}$), and the fraction of the non-effluent stream leaving the settling tank that is discharged to waste (w).

The ASM #1 incorporates differential equations for six soluble and six particulate components. The six soluble components are: inert soluble organic material, readily biodegradable soluble substrate, soluble oxygen, soluble nitrate & nitrite nitrogen (one component), soluble ammonium (NH_4^+ and NH) nitrogen, and soluble biodegradable organic nitrogen. The particulate components are: particulate inert organic matter, slowly biodegradable particulate substrate, active heterotrophic particulate biomass, active autotrophic particulate biomass, non-biodegradable particulate products arising from biomass decay, and particulate biodegradable organic nitrogen.

2.1 Formulation of reaction rates

The ASM #1 includes eight fundamental processes which describe nitrogen and chemical oxygen demand within suspended-growth treatment processes, These are: (1) aerobic growth of heterotrophic biomass, (2) anoxic growth of heterotrophic biomass, (3) aerobic growth of autotrophic biomass, (4) decay of heterotrophic biomass, (5) decay of autotrophic biomass, (6) ammonification of soluble organic nitrogen, (7) hydrolysis of entrapped particulate organic matter and (8) hydrolysis of entrapped organic nitrogen. These processes include mechanisms for nitrification and denitrification.

In writing the equations, we use the symbol S_O to denote the concentration of soluble oxygen in the reactor and the symbol S_j ($j = 1 \ldots 5$) to denote the concentration of the five other soluble components in the reactor. We denote the concentration of the six particulate components in the reactor by X_l ($l = 1 \ldots 6$). The rate functions are specified in a column vector which depends upon the corresponding concentration of the soluble and particulate components. In this formulation the variable $\mathbf{S}$ includes all six of the soluble components and $\mathbf{X}$ includes all six of the particulate components. The reaction rate expression may contain as few as one of the twelve state variables.

For each of the state variables there is a stoichiometric column vector $\alpha_{i,j}$. The second index ($j = 1, 2, \ldots, 12$) identifies the state variables being considered. The first index ($i = 1, 2, \ldots 8$) indicates the process being considered (as numbered above). If the entry $\alpha_{i,j}$ is positive, then component j is a product in reaction i. If the entry $\alpha_{i,j}$ is negative, then component j is a reactant in reaction i. If the entry $\alpha_{i,j}$ is zero, then component j is neither a product nor a reactant in reaction i.

3 MODEL EQUATIONS

The equations consist of two components - the flow component, which consists of influent flow and the recycled flow from the settling tank, where τ is the hydraulic retention time and is the ratio of the reactor volume and the inflow rate, and the reactions between the various chemical species.

Rate of change of soluble components (excluding oxygen) in the reactor ($j = 1 \ldots 5$)

$$\frac{\mathrm{d}S_j}{\mathrm{d}t} = \frac{1}{\tau}\left(S_{j,\text{in}} - S_j\right) + \frac{R}{\tau_1}\left(S_j - S_j\right) + \sum_{i=1}^{8} \alpha_{i,j} \cdot \mathbf{f}_i\left(\mathbf{S}, \mathbf{X}\right). \tag{1}$$

Rate of change of soluble oxygen in the reactor

$$\frac{\mathrm{d}S_\mathrm{O}}{\mathrm{d}t} = \frac{1}{\tau}\left(S_{O,\text{in}} - S_\mathrm{O}\right) + \frac{R}{\tau}\left(S_\mathrm{O} - S_\mathrm{O}\right) + K_\text{L,a}\left(S_{\mathrm{O},\max} - S_\mathrm{O}\right) + \sum_{i=1}^{8} \alpha_{i,O} \cdot \mathbf{f}_i\left(\mathbf{S}, \mathbf{X}\right). \tag{2}$$

Rate of change of particulate components in the first reactor ($l = 1 \ldots 6$)

$$\frac{\mathrm{d}X_l}{\mathrm{d}t} = \frac{1}{\tau}\left(X_{l,\text{in}} - X_l\right) + \frac{R}{\tau}\left(\mathcal{C}X_l - X_l\right) + \sum_{i=1}^{8} \alpha_{i,l} \cdot \mathbf{f}_i\left(\mathbf{S}, \mathbf{X}\right). \tag{3}$$

Note that there are zero terms in equations (1) and (2), where the soluble components feed straight through the settling tank, back into the reactor. In these equations the parameters are: $K_\text{L,a}$, the oxygen transfer coefficient; R, the recycle ratio of the settling tank; $S_{j,\text{in}}$, the concentration of soluble component j in the feed; $S_{O,\text{in}}$, the concentration of soluble oxygen in the feed; $S_{O,\max}$, the maximum concentration of soluble oxygen; $X_{l,\text{in}}$ is the concentration of the particulate components l in the feed, and τ, the hydraulic retention time in the reactor. The concentration factor for particulates in the settling tank, $\mathcal{C}$, is defined as

$$\mathcal{C} = \frac{R+1}{R+w},$$

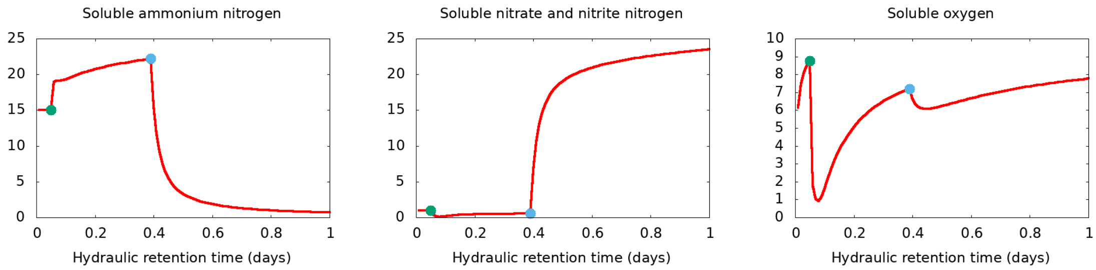

Figure 2. Variation of the soluble ammonium nitrogen, soluble nitrite and nitrate nitrogen and soluble oxygen for the baseline system parameters. The green and circle is the location of the first and second trans-critical bifurcation points where the heterotrophic and autotrophic biomass first appear, respectively.

where w is the wasted fraction of sludge. See Nelson et al. (2019) for details of the reaction functions $\mathbf{f}_i\left(\mathbf{S},\mathbf{X}\right)$ in equations (1)–(3). The reaction functions are combinations of Michaelis-Menten rate limiting growth functions (M) and inhibition functions (I), of the form

$$M(S) = \frac{S}{K+S}, \quad I(S) = \frac{K}{K+S}$$

4 RESULTS AND DISCUSSION

From a starting concentration of each of the state variables, the state variable will evolve to one of a number of solution paths. In general, there will be only one possible solution path for a given set of parameters. As the system parameters change, the solution path followed can change if the solution path becomes unstable when one of the eigenvalues of the linearised system becomes positive. For this paper, the only parameter that is being varied is the hydraulic retention time (τ). All other parameters are being kept constant. For this study, we used a Newton-Raphson iterative method. An advantage of using this method, compared to integrating the system numerically using MATLAB, for example, is that the linearised system is a by product of the method.

The system parameters are those reported by Nelson et al. (2019), which in turn are taken from Yoon and Lee (2005); Fikar et al. (2005); Henze et al. (1987) and Lu et al. (2001). In addition, we have set the recycle rate at 45% (or $R = 0.45$) and the wasted fraction of sludge of 10% (or $w = 0.1$). The baseline behaviour is shown in Figure 2. In addition to showing the performance measures, the figure shows the values of hydraulic retention time where one solution branch becomes unstable and another solution branch becomes stable. This corresponds to when the heterotrophic biomass concentration is non-zero, in the first instance, and when both the autotrophic and heterotrophic biomass concentrations become non-zero, in the second instance.

4.1 Varying one parameter at a time

For the first sensitivity analysis, we will vary one of the twenty system parameters between 20% above and and 20% below the baseline value while keep the other nineteen values the same. Two of the parameters that lead to significant performance variation are the maximum specific growth rate of the heterotrophs and autotrophs. Increasing or decreasing these parameters increases or decreases the hydraulic retention time when each biomass first appears. This is shown in Figure 3 for the maximum specific growth rate of autotrophs. There is a similar change for heterotrophs, but is not as pronounced. Another parameter that affects the performance is the heterotrophic yield coefficient. This leads to decrease in the nitrate and nitrate and ammonium nitrogen concentrations and an increase in the soluble oxygen concentration as the parameter is increased, and the opposite when it is decreased. Comparing that to the autotrophic yield coefficient, where the difference is negligible. The parameters that change the performance are summarised below (where the information in the brackets denotes whether the change is positive (+), negative (-) or negligible (0) as the parameter is increased for the concentrations of soluble ammonium nitrogen, soluble nitrate and nitrite nitrogen and soluble oxygen, respectively)

1. shifted onset of heterotrophs
    - maximum specific growth rate for heterotrophs (- - -)

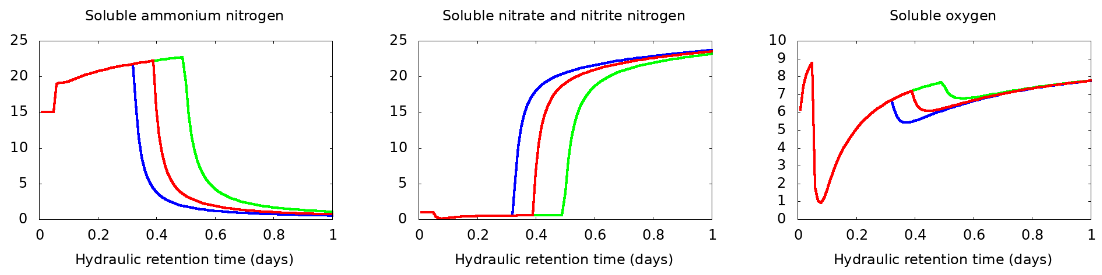

Figure 3. Variation of the soluble ammonium nitrogen, soluble nitrite and nitrate nitrogen and soluble oxygen with the variation of the maximum specific growth rate of autotrophs. The baseline value is the red line, the value 20% above the baseline value is the blue line and the value 20% below the baseline value is the green line.

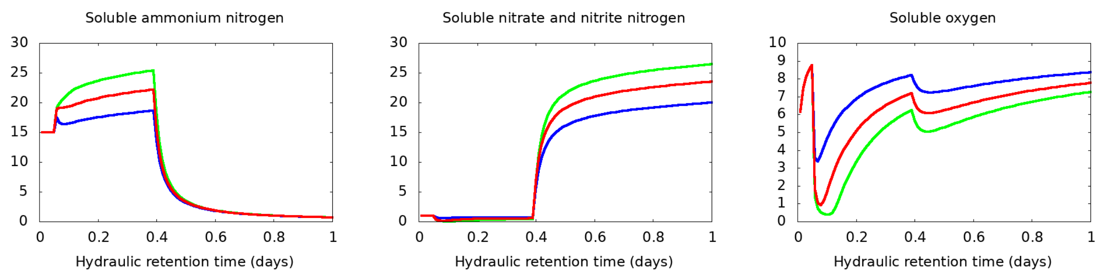

Figure 4. Variation of the soluble ammonium nitrogen, soluble nitrite and nitrate nitrogen and soluble oxygen with the variation of the heterotrophic yield coefficient. The baseline value is the red line, the value 20% above the baseline value is the blue line and the value 20% below the baseline value is the green line.

2. shifted onset of autotrophs
   - maximum specific growth rate for autotrophs (- - -)
3. shifted the concentrations up or down
   - heterotrophic yield coefficient (- - +)
   - nitrogen content in biomass (- - +)
   - maximum concentration of soluble oxygen (- - +)
4. slightly shifted the concentrations up or down
   - ammonia half-saturation coefficient for autotrophic biomass (+ + +)
   - ammonification coefficient (+ + 0)
   - hydrolysis coefficient (+ 0 0)
   - heterotrophic decay coefficient (+ + +)
   - autotrophic decay coefficient (+ + -)

4.2 Varying all parameters at once

To extend the previous analysis, we consider what happens if all of the parameters are varied about the baseline values. As before, we vary the parameters between 20% above and 20% below the baseline value and use Latin Hypercube Sampling (LHS) to hopefully cover the entire parameter space. We consider forty-one scenarios, accounting for one percentage point per scenario. The results are shown in Figure 5. We can see that, in general, using the LHS, varying all variables at once gives a wider variation in each of the concentrations.

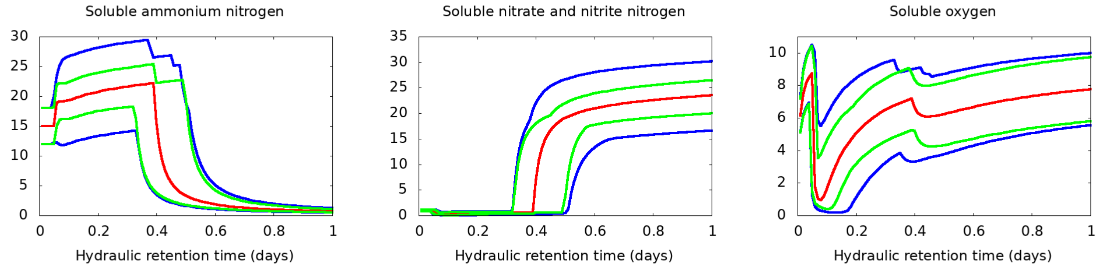

Figure 5. Variation of the soluble ammonium nitrogen, soluble nitrite and nitrate nitrogen and soluble oxygen with the variation of the heterotrophic yield coefficient using the baseline parameter values (red line), varying one parameter at a time (green line) and using a Latin Hypercube Sampling variation of the parameters (blue line).

This is not surprising, as varying one parameter at a time would be a subset of varying all parameters at once. However, there is a small part of parameter space which was possibly not captured by the LHS where the single parameter variation leads to a wider variability of performance.

5 CONCLUSIONS

We investigated the steady-state behaviour of the ASM #1 as a function of the hydraulic retention time, a measure of the time taken for the wastewater to pass through the bioreactor and settling tank. Using the system parameters gathered from the literature, a baseline performance was found. By varying these system parameters, one at a time, we found that of the twenty parameters, half had an effect on the performance of the system, to a greater or lesser extent. By varying all of the parameters, a greater variation was found. Further analysis could be carried out to explore which combination of parameter changes led to a greater change in the performance.

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