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Non-linear reservoir modelling without tears using the QuaSoARe approach

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Abstract: This abstract provides an example of the application of the "Quadratic Solution of the Approximate Reservoir Equation" (QuaSoARe) algorithm. This fast numerical algorithm solves the instantaneous reservoir equation, which is used in most hydrological models. The algorithm is applied to the GR4J daily rainfall-runoff model. In addition, a modification of the routing store is introduced to illustrate how QuaSoARe can be used to alter an existing model structure. Original and modified models are applied to an example catchment where the modified simulation is shown to improve the match between observed and simulated streamflows.

Keywords: *Differential Equation, Rainfall-runoff model, GR4J, Numerical scheme*

1. INTRODUCTION

The dynamics of most conceptual models used in hydrology can be represented by an autonomous differential equation relating the change in stored water S with fluxes affecting the system as follows:

$$\frac{dS}{dt} = \sum_{i=1}^{n} f_i(S) \qquad \text{[Equation 1]}$$

Fluxes can be inflow and outflow for a routing model, or actual ET and effective rainfall for a lumped rainfall-runoff model. Despite its widespread use, Equation 1 can be solved analytically for very simple flux functions only. In most cases, it must be solved numerically. Yet, using inappropriate numerical schemes can have disastrous consequences, leading to large errors, poor simulations, and fragile calibration, as highlighted by Kavetski and Clark (2011). This abstract presents an example of solutions obtained by the approach introduced by Lerat (2025) to solve a generic reservoir equation and compute all associated fluxes. The algorithm referred to as "Quadratic Solution of the Approximate Reservoir Equation" (QuaSoARe) is based on the approximation of flux functions f_i by piecewise quadratic polynomials. Lerat (2025) compared this algorithm against a high-accuracy implicit Runge-Kutta scheme applied to a set of highly non-linear hydrological models, and concluded that the QuaSoARe algorithm can speed up integration by a factor of 10 to 50 while maintaining its accuracy. In this work, the QuaSoARe algorithm is applied to the daily rainfall-runoff model GR4J. We briefly demonstrate how the use of QuaSoARe enables the modification of the model equation, ultimately leading to performance improvements.

2. METHOD AND RESULTS

The following equation describes the routing store of the GR4J model (Santos et al., 2018):

$$\frac{dr}{dt} = 0.9\, p_{eff} - X_2\, r^{3.5} - \frac{r^5}{4} \qquad \text{[Equation 2]}$$

where $r = R/X_3$ is the normalised store level with capacity X_3. X_2 is the inter-catchment exchange coefficient. $p_{eff} = P_{eff}/X_3$ is the normalised effective rainfall generated by the production store. This equation is used as a basis to apply QuaSoARe and obtain a complete solution of Equation 2. A similar approach is applied to the production store of GR4J (not shown), which leads to a full implementation of GR4J in QuaSoARe. In addition, a modified GR4J is run where the last term in the right-hand side of Equation 2 is changed as follows:

$$\frac{dr}{dt} = 0.9\, p_{eff} - X_2\, r^{3.5} - 2r^4 \qquad \text{[Equation 3]}$$

This modification is based on the work by Lerat et al. (2024) who suggested rebalancing the different flux terms in the GR2M routing store, which is conceptually close to GR4J. Introducing such changes impacts the solution of the store equation, but this remains a simple operation in QuaSoARe.

To demonstrate this process, the two QuaSoARe models are applied to the Wilsons River at Eltham (station ID 203014) located in North-East NSW. Additionally, the original GR4J model serves as a benchmark. The simulations run at a daily timestep from 1968 to 2023 and can be visualised in Figure 1. The corresponding NSE performance metrics computed from power-transformed flows over the full simulation period are shown in Table 1.

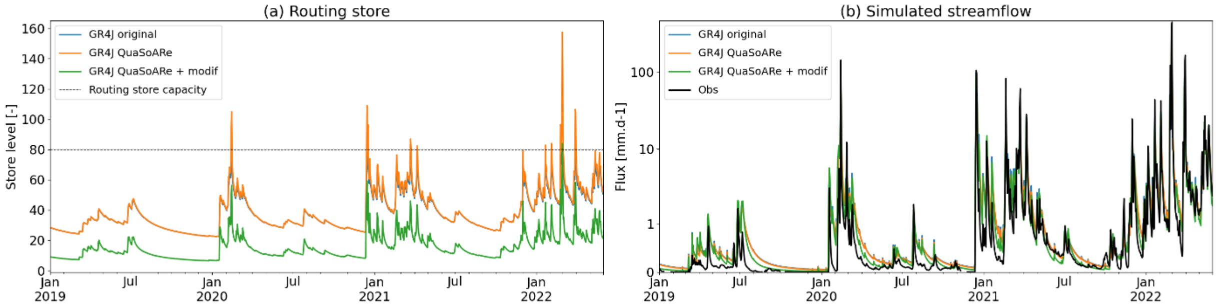

Figure 1 Simulations generated from three variants of the GR4J model

In Figure 1, the QuaSoARe-based GR4J (orange) can be seen to follow closely the original GR4J (blue) despite the differences in their algorithmic solutions. This can be explained by the fact that the original GR4J implements a special type of analytical approximation called "operator splitting", which preserves the main feature of the simulation. A major difference between the two simulations is the fact that the store level in the QuaSoARe implementation, shown in Figure 1.a, can exceed the store capacity (X_3), a behaviour that is not possible in the original GR4J. This phenomenon occurs because Equation 2 does not enforce a cap on the store level. Consequently, the solution of this equation generated by QuaSoARe can freely exceed this threshold. Interestingly, the store levels in the modified model based on Equation 3 (green) remain below the store capacity.

The performance of the three models presented in Table 1 suggests that the modified model consistently improves simulation for the low, medium and high flow regimes. The low-flow simulation improvement is also visible in Figure 1.b, where the modified simulation remains closer to the observations compared to the two other models.

Table 1: Nash-Sutcliffe efficiencies of three variants of the GR4J model

Transformation exponent	**GR4J original**	**GR4J + QuaSoARe**	**GR4J + QuaSoARe + modified routing**
0.0 (log)	0.60	0.61	0.68
0.5	0.79	0.80	0.84
1.0 (no transform)	0.72	0.73	0.75

3. CONCLUSION

This abstract demonstrates the application of the QuaSoARe algorithm to a well-established run rainfall-runoff model. The algorithm facilitates the investigation of changes in the model structure that lead to performance improvement. Further work is required to determine the significance of these improvements via an application on a large number of catchments and a proper cross-validation scheme.

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