Improving the discrete form of the Nash cascade

Barry Croke [a,b], **Georgia McCulloch**[a]

[a] *Mathematical Sciences Institute, Australian National University*
[b] *Institute for Water Futures, Australian National University*
Email: barry.croke@anu.edu.au

Abstract: The Nash cascade (Nash, 1957) is a set of identical linear stores arranged in series, which is represented by a system of identical first-order non-homogeneous linear differential equations. Common applications of the Nash cascade use the discrete-time representations of these equations, leading to a flow peak that is too high, too early and too sharp. This is because the input into the discrete-time formulation of a linear store is an impulse at the start of the time step. This leads to the input to each subsequent store being shifted to the start of each timestep, resulting in a rapid progression through the series of stores. To overcome this, the time constant can be increased. While this can result in the correct time to peak, the dispersion of the peak as it travels through the stores is too large, resulting in underestimating the peak height.

Szöllösi-Nagy (1982) presented an exact solution for the discrete-time representation of a cascade of n stores. Unfortunately, this cannot be employed efficiently for complex river networks as it is designed for a single series of stores. One approach for correcting the discrete-time Nash cascade is to include a fractional delay in the input to each subsequent store, using the flow weighted average time over the first time step for the output of a linear store resulting from an input that is an impulse at the start of the time step. This approach works surprisingly well if the time step is of the order of 10 times the store's time constant. As the time step increases relative to the time constant, the error increases, becoming problematic when the time step is approximately the time constant. Other approaches being explored include an optimised delay and the Muskingum flood routing method (e.g. Overton, 1966).

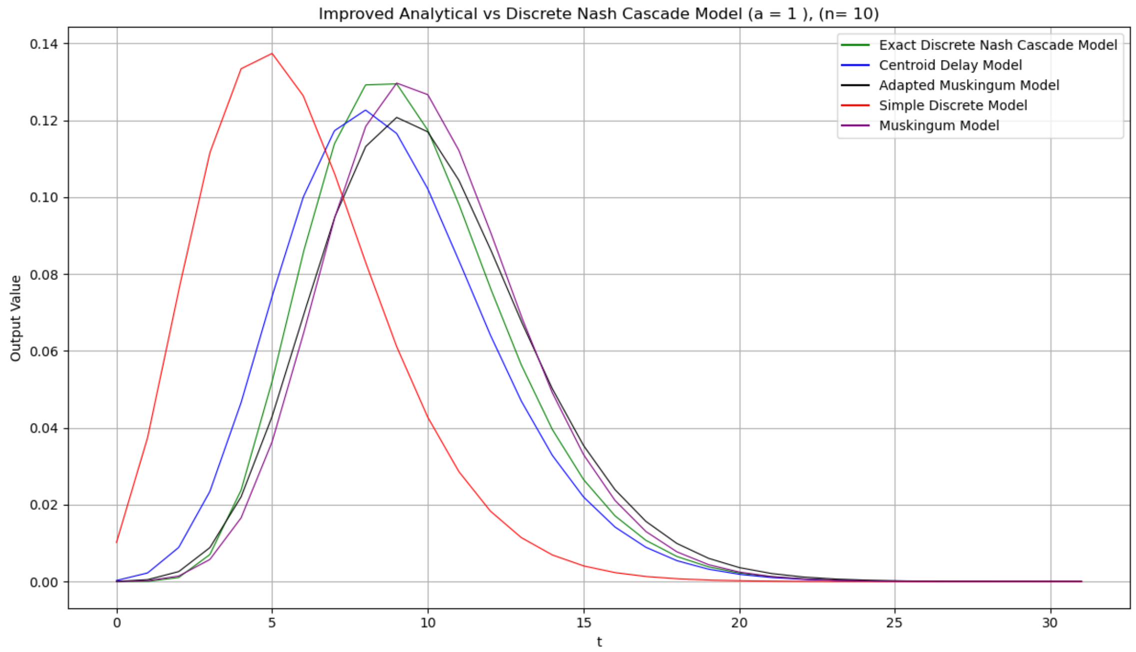

Figure 1. Impulse response for a cascade of 10 stores with time constant = 1 time step.

REFERENCES

Nash JE (1957) The form of the instantaneous unit hydrograph. Comptes Rendus et Rapports Assemblee Generale de Toronto, 3, 114-121.
Overton DE (1966) Muskingum flood routing of upland streamflow. Journal of Hydrology 4, 185-200.
Szöllösi-Nagy A (1982) The discretization of the continuous linear cascade by means of state space analysis. Journal of Hydrology 58, 223-236.

Keywords: *Flow routing, runoff, fractional delay, Muskingum*