

# Downscaling runoff products using areal interpolation: a combined pycnophylactic-dasymetric method

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**Abstract:** Hydrological models are commonly forced with variables describing climate. These variables are often obtained from climate models, requiring processing via interpolation or downscaling in order to be useful for the hydrological modelling study. Outputs from climate – as well as hydrological – models are generally expressed as spatially explicit fields, but the results from hydrological models are rarely interpolated or downscaled. There can be many reasons for this. For instance, the existing model output may be of incompatible spatial units (raster, when vector needed, or vice versa), or of a different scale (regional, where local scale is needed). Runoff generation is a local process which is determined by the influx of water, landuse, vegetation and soil characteristics, and there have been many applications of its interpolation to watersheds, river lines or as continuous fields. However, most of the methods employed in runoff interpolation are statistical and do not account for process characteristics in runoff generation in the interpolation step.

Here we present a novel spatial interpolation method for the purpose of downscaling coarse resolution runoff products based on areal interpolation. Areal interpolation is a process where a variable from a source zone is reallocated to overlapping target zones. We combine two advanced methods: Dasymetric Mapping (DM), which is simple Area weighted Interpolation (AI) informed by an ancillary variable, and Pycnophylactic Interpolation (PP), which is designed to refine the spatial distribution of a variable within the source zone. Each of these methods preserve mass balance – the volume of runoff from source zone is preserved in the target zones. Our methodology can address the Modifiable Area Unit Problem (MAUP) – a statistical bias caused by the sensitivity of analytical results of spatial data to levels of aggregation (the scale effect), as well as the arbitrary sizes, shapes, and arrangements of zones (the zoning effect). Addressing MAUP enhances usability of existing model results for runoff estimation because the zoning can be modified to fit the needs of a new analysis. The method is also able to take into account the spatial distribution of characteristics which govern runoff generation in the interpolation step.

To test the methodology, we downscale a coarse global runoff product, LORA (Linear Optimal Runoff Aggregate), on to 126 Australian catchments with natural flow regimes, and compare how AI, DM, PP and the combined PP-DM fare against streamflow records. A recently developed topographical index, DUNE (Dissipation Per Unit Length), which is able to distinguish topographies with different runoff regimes, is used as the ancillary variable in DM and PP-DM. We assume that runoff is highly correlated with precipitation and we assume it can be interpolated with a smooth function. We also assume that topography can inform us about the actual distribution of runoff generation within a source zone (the spatial unit in a runoff product).

We find that the simple AI method is more efficient in replicating the runoff profile in arid catchments where potential evapotranspiration is higher than precipitation. However, as precipitation increases and aridity is reduced, DM and PP-DM prove more efficient in replicating the recorded runoff. Using DUNE as the ancillary variable also results in higher performance in catchments with variable topography and performs worse in less variable terrain. In catchments with a high range of slopes, DM and PP-DM utilizing DUNE are consistently better than AI or PP, which do not utilize DUNE. Additionally, we find that in wet catchments which are located entirely within a single source zone of runoff, the performance is higher using DM, PP, and PP-DM than with the simple AI. In catchments which are covered by multiple source zones there is no clear benefit in using the more advanced areal interpolation methods over AI.

Our results show that the method is able capture the spatial variability of runoff generation, but this requires careful selection of the ancillary variable or a combination of ancillary variables. It is also evident from the results that arid and wet catchments require a different approach in runoff downscaling. Further investigation with a larger sample is needed to fully understand the properties of the downscaling methods.

**Keywords:** *Downscaling, runoff, global hydrology, interpolation*

## 1. INTRODUCTION

Hydrological models are commonly forced with climate variables, such as temperature, precipitation, and relative humidity. These variables can be obtained with in-situ or remotely sensed measurements, or they can be obtained from climate models for a specific area of interest. The measurements, however, may not be directly useful for a specific application because they may be sparse in time or space, or the resolution of the climate model where the forcing variables are obtained is too coarse. These often need spatial interpolation or downscaling in order to be useful for the hydrological model application.

Climate variables are generally expressed as more or less smoothly varying fields, for which there is an extensive body of literature on spatial interpolation and downscaling and on their use in hydrological models (see e.g. Hu et al., 2019; Lauri et al., 2014; Li and Heap, 2011). Output from hydrological and/or land surface models can likewise be expressed as fields, and there has been significant research advancement in the interpolation techniques for water-related variables during the IAHS decade of Prediction in Ungauged Basins. Reasons for such interpolation rather than running a hydrological model can be, for instance, lack of data, resources, or expertise in hydrological modelling, and readily available model results with incompatible spatial units (raster, when vector needed, or vice versa), or of a different scale (regional, where local scale is needed).

Runoff is generated in a local process involving influx of water, landuse, vegetation and soil characteristics. There have been many applications of runoff interpolation to watersheds, river lines or continuous fields (see e.g. Gottschalk (1993), Lehner and Grill (2013), Parajka et al. (2015), Rochelle et al. (1989), Salas et al. (2018), and Skøien et al. (2006)) to solve the problem of Prediction in Ungauged Basins. However, most of the methods employed in runoff interpolation are statistical and do not account for process characteristics in runoff generation in the interpolation step.

In this study we propose a spatial interpolation method (defined here as estimating an unknown value at a location using known values at nearby locations) for off-the-shelf coarse scale runoff products based on a combination of two areal interpolation methods commonly used in population estimation studies: dasymetric mapping (DM; Eicher and Brewer, 2001), informed by ancillary variables, and pycnophylactic interpolation (PP; Tobler, 1979), which is smooth “volume” preserving interpolation. In areal interpolation, a variable from a source zone is reallocated to target zones of interest. We use gridded reanalysis runoff products from global hydrological and land surface models as source zones and downscale them to higher resolution catchments (target zones). Our methodology can address the Modifiable Area Unit Problem (MAUP) – a statistical bias caused by the sensitivity of analytical results of spatial data to levels of aggregation (the scale effect), as well as the arbitrary sizes, shapes, and arrangements of zones (the zoning effect) (see e.g. Manley (2014) for MAUP in general, or Salmivaara et al. (2015) for MAUP in water assessments). Addressing MAUP enhances usability of existing model results for runoff estimation because the zoning can be modified to fit the needs of a new analysis. The method is also able to take into account the spatial distribution of characteristics which govern runoff generation in the interpolation step.

The rest of the paper is structured as follows. In section two we introduce the relevant concepts of AI, DM and PP followed by the proposed combined Pycnophylactic-Dasymetric (PP-DM) interpolation method in the third section. In the fourth section we describe a case study in Australian catchments and provide results of the experiment. Section five concludes and outlines the next steps.

## 2. REVIEW OF AREAL INTERPOLATION METHODS

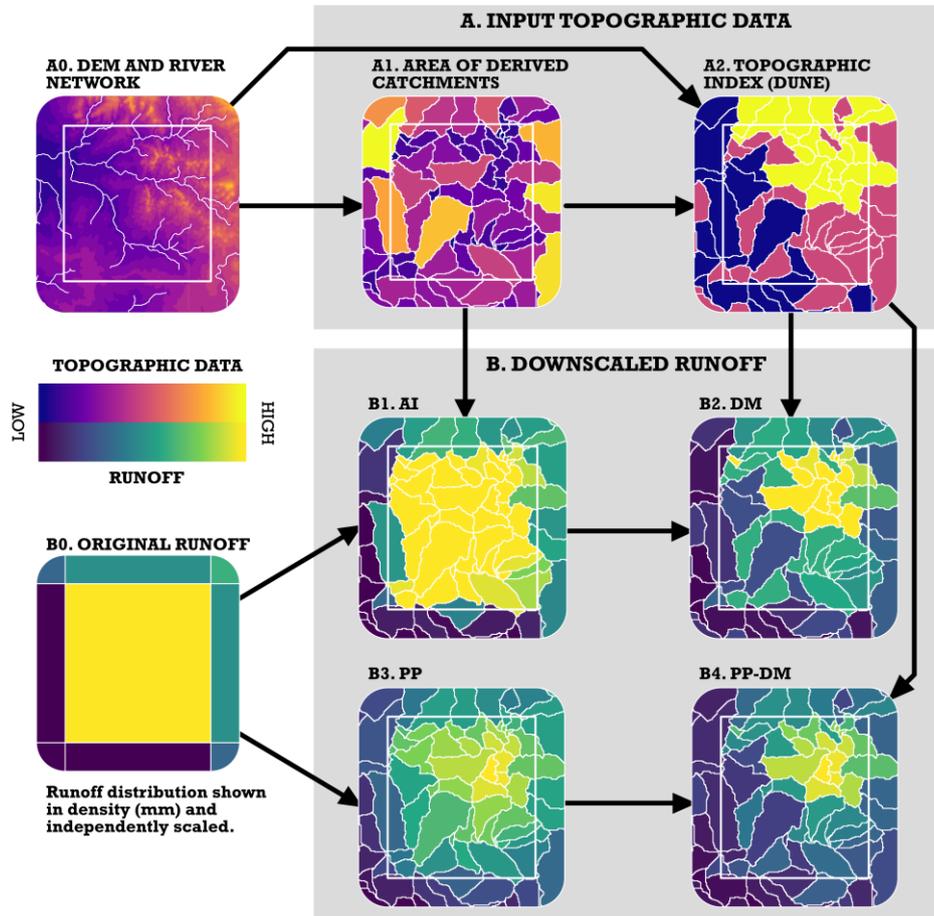
This section gives a brief description of the relevant areal interpolation methods. For a more thorough description of the methods, we invite the reader to review Comber et al. (2008), Eicher and Brewer (2001), Kim and Yao (2010), and Tobler (1979). The methods are visualized in Figure 1.

### 2.1. Area weighted interpolation and dasymetric mapping

Area weighted interpolation (AI) is a method mostly employed in population research to estimate local variations in spatial distribution of a statistic in a different set of spatial units than the original data. It refers to a process where a value of a statistic from a source zone is reallocated to alternative target zones. The simplest form of areal interpolation is area weighted interpolation, which is a simple reallocation where a fraction of the statistic in a source zone is allocated to a target zone by the proportion of their overlapping areas. The reallocation is shown in Equation 1

$$\hat{R}_t = \sum_s^{s \cap t} R_s \frac{A_{t \cap s}}{A_s} \quad (1)$$

where  $\hat{R}_t$  is the estimated statistic in a target zone  $t$ ,  $R_s$  is the statistic in a source zone  $s$ ,  $A_s$  is the area of the source zone, and  $A_{t \cap s}$  is the intersection of the target zone with the source zone. Areal interpolation has a pycnophylactic (volume/mass preserving) property; the count of the statistic in the source zone is preserved within the target zones. If the target zones are smaller than the source zones, the method can be used as a downscaling method.



**Figure 1.** Conceptual presentation of the areal interpolation methods and their result. Panel A presents the catchment areas (A1) and ancillary variable (A2) derived from a Digital Elevation Model (A0). Panel B shows the result of applying Areal Interpolation (B1), Dasymetric Mapping (B2), Pycnophylactic Interpolation (B3) and combined Pycnophylactic-Dasymetric Interpolation (B4) to input runoff data (B0).

Area weighted interpolation, however, assumes that the statistic is evenly distributed within the source zone, which is generally a false assumption. In Dasymetric Mapping (DM), the reallocation process can be enhanced using an ancillary variable, which provides additional information about the distribution within the source zone. In principle, runoff estimated with AI is first rescaled using the ancillary variable, and then rescaled runoff is adjusted to equal the original runoff in the source zone. In population mapping, land use is commonly used as the ancillary variable e.g. as a boolean filter (no population to rivers, lakes, forests) or as multilevel information (urban areas high, suburbs medium and rural areas low density). Equation 1 can be modified to include an ancillary variable (Equation 2)

$$\hat{R}_t = \sum_s^{s \cap t} R_s \frac{A_{t \cap s} V_t}{\sum_t^{t \cap s} A_{t \cap s} V_t} \quad (2)$$

where  $V_t$  is the value of the ancillary variable for the target zones.

Both of the methods are pycnophylactic by definition; they preserve the volume of the original source zone. The difference between the methods can be seen in Figure 1. AI (sub-plot B1) makes use of only overlap between the two zonations and as a result, the density (mm) of runoff is unchanged for those target zones which do not cross source zone boundaries. In sub-plot B2, the effect of using additional information is evident; a larger portion of the runoff is assigned to the northeast corner of the white square due to the distribution of the ancillary variable.

## 2.2. Pycnophylactic interpolation

Pycnophylactic interpolation is another method which aims to overcome the assumption of uniform distribution within source zones. A source zone is broken into sub-areas (commonly a raster, but it can equally well be a fine-scale polygon network), and a statistic is smoothed by a weighted average of nearest neighbouring source zones (this requires solving a 2D differential equation). The smooth surface is adjusted so that the statistic count in the source zone is preserved, resulting in a continuous surface without sharp discontinuities between source zones. Plot B3 in Figure 1 shows an example where runoff is reallocated to sub-areas (catchments) from a source zone. The runoff distribution within the original grid cell is smoothed so that the high runoff density in the center of the grid gradually falls towards the values of neighbouring grid cells.

## 3. COMBINED PYCNOHYLACTIC- DASYMETRIC INTERPOLATION

A few studies have explored a combination of DM and PP to estimate population (Kim and Yao, 2010; Mohammed et al., 2012), or agricultural data (Comber et al., 2008). In this process, DM is first employed to divide the population count from source zones to target zones where population is assumed to live. Second, PP is employed to estimate within-target zone distribution because population density within a zone cannot be assumed constant. The result is a more realistic smooth population density surface.

For the purpose of runoff interpolation or downscaling, the assumptions that hold true for population mapping are not adequate. We propose that the order should be reversed when interpolating runoff. Suppose we have a runoff product in a coarse spatial resolution, and we are interested in runoff generated in higher resolution catchments. We assume that runoff generation is highly correlated with rainfall in the source zones (spatial units in the runoff product). Runoff from source zones can be smoothed using PP with the catchments as target zones. If we know of any of the runoff generation characteristics, those can be used to further refine the allocation of runoff from the smoothed surface. The process is the same as in DM: runoff distribution estimated by PP is rescaled using the ancillary variable, and adjusted so that sum of runoff within a source zone matches its runoff volume. The concept is illustrated in Figure 1 sub-plot B4. The combined method has the desired property of preserving mass balance (it is pycnophylactic) within the source zone.

## 4. CASE STUDY

We evaluate the usability of the method by downscaling runoff from the Linear Optimal Runoff Aggregate product (LORA; Hobeichi et al., 2019), a  $0.5^\circ$  resolution global optimised dataset. Downscaling is performed to catchments provided in the WIRADA dataset (Zhang et al., 2013) of 780 Australian catchments with natural flow regimes. The downscaled runoff is routed down a DEM generated river network in order to estimate streamflow at the gauging stations. An instantaneous routing scheme is applied, because of its simplicity and the small size of most catchments compared to the monthly timestep. We use the full available streamflow record between 1980 and 2012 for the comparison, and aggregate it to monthly mean values to match the temporal resolution of LORA. We compare the performance of the four methods introduced in previous sections (AI, DM, PP, and PP-DM). To minimize effects of boundary conditions, we perform downscaling for the full source zones each WIRADA catchment intersects, with an added  $0.1^\circ$  buffer. The original runoff value of a source zone is used as boundary condition in PP. To facilitate the DM and PP-DM methods, we use a recently developed topographic index DUNE (Dissipation Per Unit Length: Loritz et al., 2019), which combines the potential energy of elevation difference between area of rainfall and the nearest stream, and the flow path distance to the nearest stream. DUNE is able to distinguish between topographies producing different runoff profiles. By using DUNE we assume that higher runoff is generated in catchments which have high elevation differences and which are compact, than in large flat catchments.

### 4.1. Pre-processing

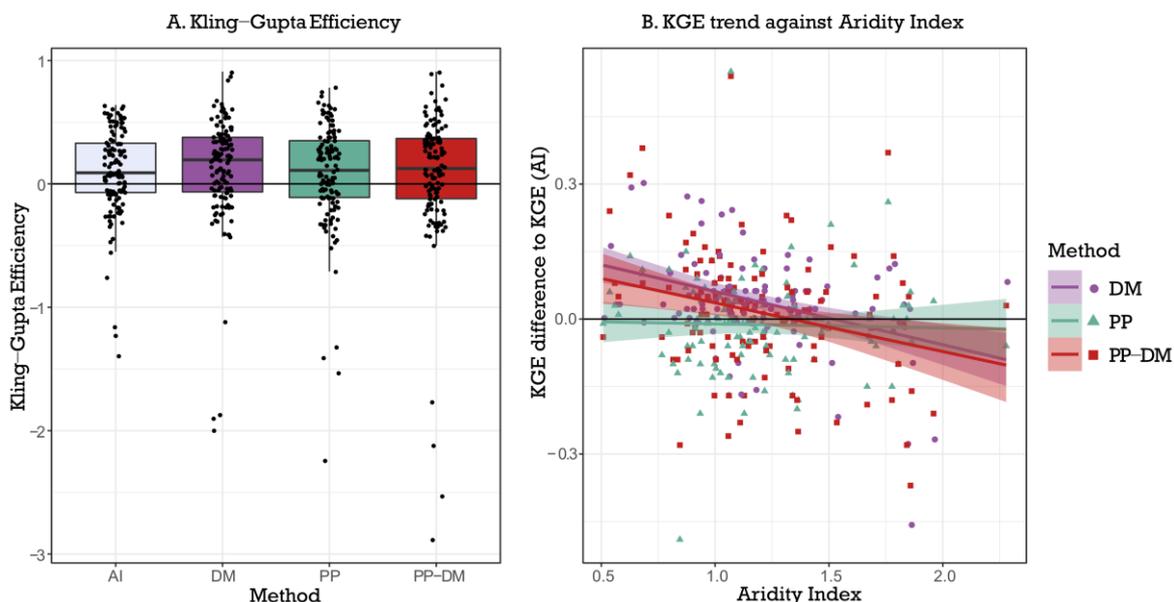
River network and catchment areas for each river segment were delineated using an Australian DEM developed specifically for drainage networks (Stein et al., 2014). The existing river network, with the most important

streams from the drainage network dataset, relative to catchment outlet size, were selected and burned to the DEM followed by sink filling, network extraction and catchment delineation. River segments with corresponding gauge locations in the WIRADA dataset were selected by intersecting WIRADA catchments with the river network and selecting the segment with largest upstream area as the location for the gauge. For each target zone we used the mean of DUNE, computed using the DEM and extracted river networks, within the catchment. Values of DUNE are high for flat areas with long distance to streams, and low for catchments with high elevation differences, so to accommodate our method, we finally invert DUNE.

The number of WIRADA catchments considered in this study were subset in three steps. First, we only select those catchments which are entirely covered by the LORA runoff product ( $n = 678$  from the original collection of 780). Next, we discard those catchments for which our derived basin areas deviate more than 10 % from the original WIRADA catchment (excluded  $n = 171$ ). In the third step, due to the nature of DUNE, we only consider those catchments with large enough topographic variation. We therefore exclude catchments with mean slope smaller than 5 degrees, bringing the final count to 126 catchments.

## 4.2. Results and discussion

Over the entire range of catchments in the subset ( $n=126$ ), only small differences in Kling-Gupta Efficiency can be identified. Figure 2A shows the distribution of KGE obtained using the four areal interpolation methods. From this plot we can see that there is a slight improvement using an ancillary variable with respect to AI and PP. For the subsequent analysis, we exclude the catchments with KGE outliers visible in Figure 2A ( $n=4$ ). However, differences between the simplest AI and the more advanced methods become evident when comparing the efficiencies against descriptive statistics in the WIRADA dataset. Figure 2B shows that in areas of high Aridity Index (defined as the ratio of evapotranspiration against precipitation), AI performs better than the more advanced techniques (DM, PP-DM), whereas the performance is poorer in catchments with low aridity. There does not seem to be a large difference between AI and PP. The results are, however, noisy and a lot of unexplained variation remains. We assumed that runoff generation is a function of rainfall, and that it can be areally smoothed in a similar vein to rainfall. The results are not conclusive, however when comparing against mean precipitation in catchments, the pattern is similar to DM and PP-DM (not shown), hinting that the assumption may have merit when rainfall exceeds evapotranspiration.

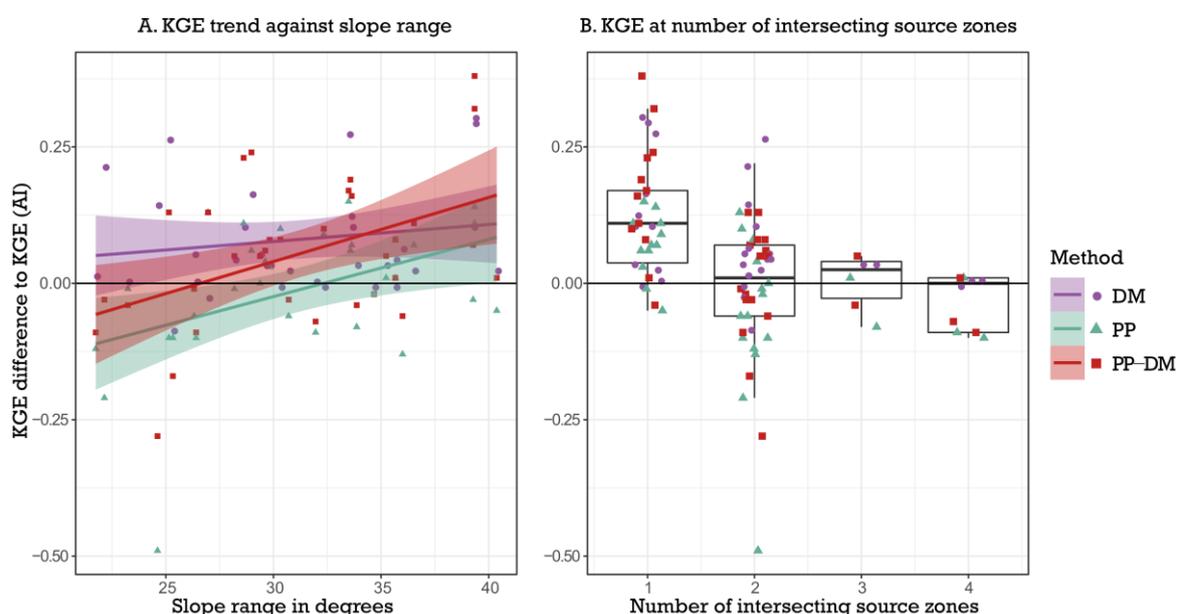


**Figure 2.** Efficiency of the downscaled runoff against streamflow record (A), and the trend in Kling-Gupta Efficiency (KGE) against Aridity Index of catchments (B). KGE in B) shown as the difference to KGE of the simplest areal interpolation method AI. A positive value in relative KGE signifies that the method performs better than AI.

The second assumption made in the study design, that the DUNE index gives meaningful information about runoff generation in catchments, is evaluated in Figure 3A, for a subset of 33 catchments with Aridity Index

smaller than 1 ( $PET < P$ ). AI performs worse than the other methods in catchments with a higher range of slopes, while at lower slope ranges there is no significant difference between the methods. The same trend is observed also using the full subset ( $n=126$ , not shown). The figure also shows that in this subset of catchments with low aridity, DM and PP-DM perform consistently better than their counterparts AI and PP. This provides us with some evidence that DUNE can be a meaningful ancillary variable in catchments with variable terrain.

The downscaling methods can only be meaningful if they can improve the representation of spatial variability in the source zones. The case where the entire catchment is within a single source zone is a special case where AI assigns the catchment the same runoff density as the source zone (compare Figure 1 sub-plots B0 and B1). If PP, DM or PP-DM improve performance against AI, it provides us evidence that distribution of downscaled runoff represents the true distribution better. Figure 3B confirms that this is indeed the case. The performance difference is smaller, or is lost as the number of intersecting source zones increase. This suggests that if the catchment of interest is large enough, the more advanced methods show no advantage over the simpler AI (at least with the current instantaneous routing solution) – the volume preserving property means that runoff falling within the WIRADA catchment is varying only near the boundaries of catchments.



**Figure 3.** Kling-Gupta Efficiency difference against slope range of catchments (A), and the relative efficiency in catchments with varying number of source zones (of runoff) intersecting the target basin (B). Both plots show results for a subset of catchments with Aridity Index smaller than 1 ( $n = 33$ ).

## 5. CONCLUSION

We proposed a novel areal interpolation method for downscaling runoff based on smooth, volume preserving, interpolation (Pycnophylactic Interpolation, PP) and Dasymetric Mapping (DM) – Areal Interpolation informed by ancillary variables. We tested the methodology in 126 unregulated Australian catchments by downscaling LORA, a global low-resolution runoff product, and assessed the performance of the downscaled runoff timeseries against recorded streamflow. A comparison of the methods was performed having the simple Area weighted Interpolation (AI) as the baseline.

We found that, across the entire catchment dataset, each of the methods show a similar distribution of performance. However, we find that there is an apparent relationship between the advanced interpolation methods in terms of rainfall and aridity. Simple AI performs better in catchments of high aridity, while the other methods fare better under low aridity. Our assumption that the DUNE topographical index, can inform areal interpolation is confirmed - we find that the performance is improved as topographical variability of catchments increases. However, alternative ancillary variables – or combinations of variables – are needed to apply the methodology in catchments where topography is similar, or where land use and soil characteristics are highly variable. With the current routing approach, the advanced areal interpolation methods, however, do not add value when target catchment is large enough to be covered by several source zones of runoff information.

This study gives preliminary results and suggests that the methodology is promising, but further studies with a larger sample of catchments with wide variety of characteristics are required to investigate the properties of the method.

## REFERENCES

- Comber, A., Proctor, C. and Anthony, S. (2008). The Creation of a National Agricultural Land Use Dataset: Combining Pycnophylactic Interpolation with Dasymetric Mapping Techniques. *Transactions in GIS*, 12(6) 775-791. doi:10.1111/j.1467-9671.2008.01130.x, 2008.
- Eicher, C. L. and Brewer, C. A. (2001). Dasymetric Mapping and Areal Interpolation: Implementation and Evaluation. *Cartography and Geographic Information Science*, 28(2), 125–138, doi:10.1559/152304001782173727, 2001.
- Gottschalk, L. (1993). Interpolation of runoff applying objective methods. *Stochastic Hydrology and Hydraulics*, 7(4), 269–281. doi:10.1007/BF01581615.
- Hobeichi, S., Abramowitz, G., Evans, J. and Beck, H. E. (2019). Linear Optimal Runoff Aggregate (LORA): a global gridded synthesis runoff product. *Hydrology and Earth System Sciences*, 23(2), 851–870. doi:10.5194/hess-23-851-2019, 2019.
- Hu, Q., Li, Z., Wang, L., Huang, Y., Wang, Y. and Li, L. (2019). Rainfall Spatial Estimations: A Review from Spatial Interpolation to Multi-Source Data Merging. *Water*, 11(3), 579. doi:10.3390/w11030579.
- Kim, H. and Yao, X. (2010). Pycnophylactic interpolation revisited: integration with the dasymetric-mapping method, *International Journal of Remote Sensing*, 31(21), 5657–5671. doi:10.1080/01431161.2010.496805.
- Lauri, H., Räsänen, T. A. and Kumm, M. (2014). Using Reanalysis and Remotely Sensed Temperature and Precipitation Data for Hydrological Modeling in Monsoon Climate: Mekong River Case Study. *Journal of Hydrometeorology*, 15(4), 1532–1545. doi:10.1175/JHM-D-13-084.1.
- Lehner, B. and Grill, G. (2013). Global river hydrography and network routing: Baseline data and new approaches to study the world's large river systems. *Hydrological Processes*, 27(15), 2171–2186. doi:10.1002/hyp.9740.
- Li, J. and Heap, A. D. (2011). A review of comparative studies of spatial interpolation methods in environmental sciences: Performance and impact factors. *Ecological Informatics*, 6(3), 228–241. doi:10.1016/j.ecoinf.2010.12.003.
- Loritz, R., Kleidon, A., Jackisch, C., Westhoff, M., Ehret, U., Gupta, H. and Zehe, E. (2019). A topographic index explaining hydrological similarity by accounting for the joint controls of runoff formation, *Hydrology and Earth System Sciences Discussions*, 1–22. doi:10.5194/hess-2019-68.
- Manley, D. (2014). Scale, Aggregation, and the Modifiable Areal Unit Problem. In: Fischer M., Nijkamp P. (eds) *Handbook of Regional Science*, pp 1157-1171. Springer, Berlin, Heidelberg.
- Mohammed, I. J., Comber, A. and Brunson, C. (2012). Population estimation in small areas: combining dasymetric mapping with pycnophylactic interpolation. Paper presented at GIS Research UK 20<sup>th</sup> Annual Conference (GISRUK 2012), Lancaster, UK, April 11-13.
- Parajka, J., Merz, R., Skøien, J. O. and Viglione, A. (2015). The role of station density for predicting daily runoff by top-kriging interpolation in Austria. *Journal of Hydrology and Hydromechanics*, 63(3), 228–234. doi:10.1515/johh-2015-0024.
- Rochelle, B. P., Stevens, D. L. and Church, M. R. (1989). Uncertainty analysis of runoff estimates from a runoff contour map. *Journal of the American Water Resources Association*, 25(3), 491-498. doi:10.1111/j.1752-1688.1989.tb03084.x.
- Salas, F. R., Somos-Valenzuela, M. A., Dugger, A., Maidment, D. R., Gochis, D. J., David, C. H., Yu, W., Ding, D., Clark, E. P. and Noman, N. (2018). Towards Real-Time Continental Scale Streamflow Simulation in Continuous and Discrete Space. *Journal of the American Water Resources Association*, 54(1), 7–27. doi:10.1111/1752-1688.12586.
- Salmivaara, A., Porkka, M., Kumm, M., Keskinen, M., Guillaume, J. H. A. and Varis, O. (2015) Exploring the Modifiable Areal Unit Problem in Spatial Water Assessments: A Case of Water Shortage in Monsoon Asia. *Water*, 7(3), 898-917. doi: 10.3390/w7030898
- Skøien, J. O., Merz, R. and Blöschl, G. (2006). Top-kriging – geostatistics on stream networks. *Hydrology and Earth System Sciences*, 10, 277–287.
- Stein, J. L., Hutchinson, M. F. and Stein, J. A. (2014). A new stream and nested catchment framework for Australia. *Hydrology and Earth System Sciences*, 18(5), 1917–1933. doi:10.5194/hess-18-1917-2014.
- Tobler, W. R. (1979). Smooth Pycnophylactic Interpolation for Geographical Regions. *Journal of the American Statistical Association*, 74(367), 519–530. doi:10.2307/2286968.
- Zhang, Y., Viney, N., Frost, A., Oke, A., Brooks, M., Chen, Y. and Campbell, N. (2013). Collation of Australian modeller's streamflow dataset for 780 unregulated Australian catchments. *Water for a Healthy Country Flagship Report series*. CSIRO, Australia. doi:10.4225/08/58b5baad4fcc2.