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A volatility impulse response analysis applying multivariate GARCH models and news events around the GFC

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Abstract: This paper features an application of the Hafner and Herwartz (2006) approach to the analysis of multivariate GARCH models using volatility impulse response analysis. The data set used features ten years of daily return series for the New York Stock Exchange Index and the FTSE 100 index from the London stock Exchange, taken from 3rd January 2005 to January 31st 2015. This period captures both the Global Financial Crisis (GFC) and the subsequent European Sovereign Debt Crisis (ESDC). The attraction of the Hafner and Kerwartz (2006) approach is that it involves a novel application of the concept of impulse response functions, tracing the effects of independent shocks on volatility through time, whilst avoiding typical orthogonalization and ordering problems. Volatility impulse response functions (VIRF) provide information about the impact of independent shocks on volatility.

Hafner and Herwartz's (2006) VIRF extends a framework, provided by Koop et al. (1996), for the analysis of impulse responses. This approach is novel because it explores the effects of shocks to the conditional variance, as opposed to the conditional mean. Hafner and Herwartz (2006) utilise the fact that GARCH models can be viewed as being linear in squares, and that multivariate GARCH models are known to have a VARMA representation with non-Gaussian errors. They use this particular structure to calculate conditional expectations of volatility analytically in their VIRF analysis. Hafner and Herwartz (2006) use a Jordan decomposition of Σ_t in order to obtain independent and identically defined (hence i.i.d.) innovations. One general issue in the approach is the choice of baseline volatilities. Hafner and Herwartz (2006) define VIRF as the expectation of volatility conditional on an initial shock and on history, minus the baseline expectation that only conditions on history. This makes the process endogenous, but the choice of the baseline shock within the data set still obviously makes a difference. We explore the impact of three different shocks, the first marks the onset of the GFC, which we date as 9th August 2007, (GFC1). This began with the seizure in the banking system precipitated by BNP Paribas announcing that it was ceasing activity in three hedge funds that specialised in US mortgage debt. It took a year for the financial crisis to come to a head, but it did so on 15th September 2008, when the US government allowed the investment bank Lehman Brothers to go bankrupt (GFC2). Our third shock point is May 9th 2010, which marked the point at which the focus of concern switched from the private sector to the public sector.

A further contribution of this paper is the inclusion of leverage, or asymmetric effects, after Engle and Ng (1993). Our modelling is undertaken in the context of a multivariate GARCH model featuring pre-whitened return series, which are then analysed via a BEKK model using a t-distribution. A key result is that the impact of negative shocks is larger, in terms of the effects on variances and covariances, but shorter in duration, in this case a difference between three and six months, in the context of our particular return series. An effect previously reported by Tauchen et al., (1996), who use a different theoretical set up.

Keywords: *Volatility Impulse Response Functions, BEKK, asymmetry, GFC, ESDC*

1. INTRODUCTION

The similarities between GARCH and VARMA-type models provide a foundation for the approach to generalize impulse response analysis, as introduced by Sims (1980), to the analysis of shocks in volatility. Various previous approaches in the literature, have been made towards tracing the impact of various types of shocks through time, see Koop et al., (1996); Engle and Ng, (1993), Gallant et al., (1993), and Lin, (1997). Koop et al. (1996) defined generalized impulse response functions for the conditional expectation using the mean of the response vector conditional on history and a present shock, compared with a baseline that only conditions on history.

Hafner and Herwartz's (2006) VIRF extends the framework provided by Koop et al. (1996). Their approach is novel in fact is explores the conditional variance rather than the conditional mean. Given that GARCH models can be viewed as being linear in squares, and that multivariate GARCH models are known to have a VARMA representation with non-Gaussian errors. Hafner and Hewartz (2006) adopt this particular structure to calculate conditional expectations of volatility analytically in their VIRF analysis.

In our GVIRF we consider three major news events which act as shocks to the volatility of our two series. The onset of the GFC, which we date as 9th August 2007, (GFC1) which began with the seizure in the banking system precipitated by BNP Paribas announcing that it was ceasing activity in three hedge funds that specialised in US mortgage debt. It took a year for the financial crisis to come to a head but it did so on 15th September 2008 when the US government allowed the investment bank Lehman Brothers to go bankrupt (GFC2). May 9th 2010 marked the point at which the focus of concern switched from the private sector to the public sector. By the time the IMF and the European Union announced they would provide financial help to Greece, the issue was no longer the solvency of banks but the solvency of governments, and this marks the onset of the European Sovereign Debt Crisis (ESDC).

2. RESEARCH METHOD AND DATA

Hafner and Herwartz (2006) develop their model by letting ε_t denote an *N* dimensional random vector so that:

$$\varepsilon_t = Pt\xi_t, \qquad (1)$$

where $P_t P_t^{'} = \sum_t$ and ξ_t denotes an *i.i.d.* random vector of dimension *N*, with independent components, mean zero and identity covariance matrix. They assume that $\sum_t$ is measurable with respect to the information set available at time *t-1,* F_{t-1}. Equation (1) implies that $E[\varepsilon_t | F_{t-1}] = 0,$ and $Var[\varepsilon_t | F_{t-1}] = \sum_t$. They note that ε_t could be the error of a VARMA process. If ε_t is a multivariate GARCH process then equation (1) may be called a strong GARCH model, according to Drost and Nijman (1993). This is convenient because it permits the modelling of news events as appearing in the *i.i.d.* innovation ξ_t. They identify ξ_t by assuming that P_t is a lower triangular matrix which permits the use of a Choleski decomposition of $\sum_t$. They further use the fact that independent news can often be identified by means of a Jordan decomposition which will permit identification is when the innovation vector is non-normal.

They adopt a multivariate GARCH(p,q) model framework, given by:

$$vech(\textstyle\sum_t) = c + \sum_{i=1}^{q} A_i vech(\varepsilon_{t-i}\varepsilon_{t-i}^{'}) + \sum_{j=1}^{p} B_j vech(\textstyle\sum_{t-i}). \qquad (2)$$

They then adopt the BEKK model, as discussed by Engle and Kroner (1995) which is a special case of equation (2) specified as:

$$\textstyle\sum_t = C_0 C_0^{'} + \sum_{k=1}^{K}\sum_{i=1}^{q} A_{ki}^{'}\varepsilon_{t-i}\varepsilon_{t-i}^{t} A_{ki} + \sum_{k=1}^{K}\sum_{i=1}^{p} G_{ki}^{'} \textstyle\sum_{t-i} G_{ki}. \qquad (3)$$

In (3) C_0 is a lower triangular matrix and A_{ki} and G_{ki} are $N \times N$ parameter matrices.

2.1. Volatility impulse response functions

Hafner and Herwartz (2006) proceed by assuming that at time *t*, some independent news is reflected by ξ_0 and it is not specified whether it is good or bad. The conditional covariance matrix $\sum_t$ is a function of the innovations $\xi_1,.....,\xi_{t-1}$, the original shock ξ_0 and $\sum_0$. Hafner and Herwartz (2006) define VIRF as the expectation of volatility conditional on an initial shock and on history, minus the baseline expectation that only conditions on history, as set out in equation (4):

$$V_t(\xi_0) = E\left[vech(\textstyle\sum_t)\middle|\xi_0, F_{-1}\right] - E\left[vech(\textstyle\sum_t)\middle|F_{-1}\right] \quad (4)$$

In equation (4) $V_t(\xi_0)$ is an N^* dimensional vector.

Hafner and Herwartz (2006) consider a VARMA representation of a multivariate GARCH(p,q) model in order to find an explicit expression for $V_t(\xi_0)$ and define $\eta_t = vech(\varepsilon_t \varepsilon_t^{'})$. They then define the multivariate GARCH(p,q) model as a VARMA(max(p,q)p) model:

$$\eta_t = \omega + \sum_{i=1}^{\max(p,q)} (A_i + B_i)\eta_{t-i} - \sum_{j=1}^{p} B_j u_{t-j} + u_t, \quad (5)$$

where $u_t = \eta_t - vech(\sum_t)$ is a white noise vector. From expression (5) they derive the VMA(∞) specification:

$$\eta_t = vech(\textstyle\sum) + \displaystyle\sum_{i=0}^{\infty} \phi_i u_{t-i}, \quad (6)$$

Where the $N^* \times N^*$ matrices ϕ_i can be determined recursively. The General expression for VIRF is:

$$V_t(\xi_0) = \phi_t D_N^{+} (\sum_0^{1/2} \otimes \sum_0^{1/2}) D_N vech(\xi_0 \xi_0^{'} - I_N). \quad (7)$$

Hafner and Herwartz (2006) consider a variety of specifications for the baseline shock. The behavior implied by expression (7) is different from traditional impulse response analysis. In (7) the impulse is an even, not odd, function of the shock, it is not linear in the shock, and the VIRF depends on the history of the process, although this is via the volatility state at the time the shock occurs. The decay or persistence is given by the moving average matrices ϕ_t, similar to traditional impulse response analysis.

Further complications arise from the choice of baseline, as no natural baseline exists for ε_0^0 in VIRF, because any given baseline deviates from the average volatility state. For example, a zero baseline would represent the lowest volatility state and volatility forecasts would increase from this baseline. After discussing various alternatives, Hafner and Herwartz (2006) adopt the definition set out in expression (4). In their original study of exchange rates they look at the impact of particular historical shocks that fall in their sample as well as considering random shocks for their estimated model.

We follow suit, in this study of US and UK indices, and consider the onset of the GFC, which we date as 9th August 2007, (GFC1), then the date when the financial crisis came to a head, 15th September 2008, when the US government allowed the investment bank Lehman Brothers to go bankrupt (GFC2). May 9th 2010 marked the point at which the focus of concern switched from the private sector to the public sector, and this marks the onset of the European Sovereign Debt Crisis (ESDC). We also consider random shocks.

3. RESULTS

Summary statistics for the two index return series are shown in Table 1. Both the NYSE and the FTSE return series display excess kurtosis and are negatively skewed. Plots of the index values are shown in Figure 1.

Table 1: Summary Statistics, using the observations 2005-01-03 - 2014-12-31			
for the variable NYSERET (2608 valid observations)			
Mean	Median	Minimum	Maximum
0.000154204	0.000431926	-0.102321	0.115258
Std. Dev.	C.V.	Skewness	Ex. kurtosis
0.0133989	86.8909	-0.417694	10.8634
5% Perc.	95% Perc.	IQ range	Missing obs.
-0.0202854	0.0179030	0.0103402	0
Summary Statistics, using the observations 2005-01-03 - 2014-12-31			
for the variable FTSERET (2608 valid observations)			
Mean	Median	Minimum	Maximum
3.92100e-005	0.000475224	-0.105381	0.122189
Std. Dev.	C.V.	Skewness	Ex. kurtosis
0.0148037	377.549	-0.110113	9.87695
5% Perc.	95% Perc.	IQ range	Missing obs.
-0.0227705	0.0205110	0.0132403	0

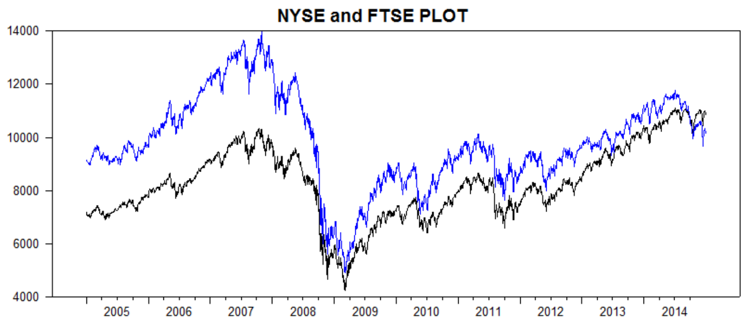


Figure 1. NYSE and FTSE Plot.

Note: NYSE - Blue, FTSE – Black.

Table 2 provides tests of skewness, kurtosis and whether the return series for the two index series are normally distributed. The Jarque-Bera test rejects this at better than the 1% level. We utilize the Student T

Table 2. Tests of skewness, excess kurtosis, and conformation to a normal distribution.

NYSERET(*100)			
Skewness	-0.417934	Signif Level (Sk=0)	0.000000
Kurtosis (excess)	10.886570	Signif Level (Ku=0)	0.000000
Jarque-Bera	12954.814995	Signif Level (JB=0)	0.000000
FTSERET(*100)			
Skewness	-0.110176	Signif Level (Sk=0)	0.021693
Kurtosis (excess)	9.898215	Signif Level (Ku=0)	0.000000
Jarque-Bera	10651.855632	Signif Level (JB=0)	0.000000

distribution in our subsequent analysis. We filter the return series through an AR(1) and GARCH(1,1) processes, before proceeding to use the residuals from this in a BEKK analysis to generate the VIRF, following Hafner and Herwartz (2006). Table 3 shows the results of the application of the BEKK model. We can forecast the two series' volatility and correlations using the BEKK model. We forecast for 100 days at the end of our time series and use a window of 400 daily observations to fit the model. The results are shown in Figure 2.

Table 3. BEKK model

Variable	Coeff	Std Error	T-Stat	Signif
Constant	0.094673045	0.015120103	6.26140	0.000
LNYSERET{1}	-0.252211378	0.018119393	-13.91942	0.000
Constant	0.077323881	0.019894664	3.88666	0.000
LFTSERET{1}	-0.168032092	0.016587251	-10.13020	0.00
C(1,1)	-0.097175963	0.044805916	-2.16882	0.030
C(2,1)	-0.264611585	0.034032404	-7.77528	0.00
C(2,2)	-0.000000180	0.149309283	-1.20715e-006	0.999
A(1,1)	0.021678144	0.041879070	0.51764	0.605
A(1,2)	-0.383455482	0.052098541	-7.36020	0.000
A(2,1)	-0.222393062	0.035195693	-6.31876	0.000
A(2,2)	-0.063023626	0.046314167	-1.36079	0.173
B(1,1)	1.202152703	0.015121227	79.50100	0.000
B(1,2)	0.450960714	0.027752985	16.24909	0.000
B(2,1)	-0.354541888	0.021500835	-16.48968	0.000
B(2,2)	0.591348452	0.024731239	23.91099	0.000
Shape	7.670707369	0.748939459	10.24209	0.000

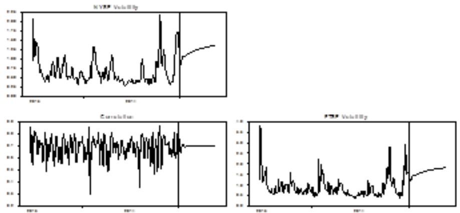
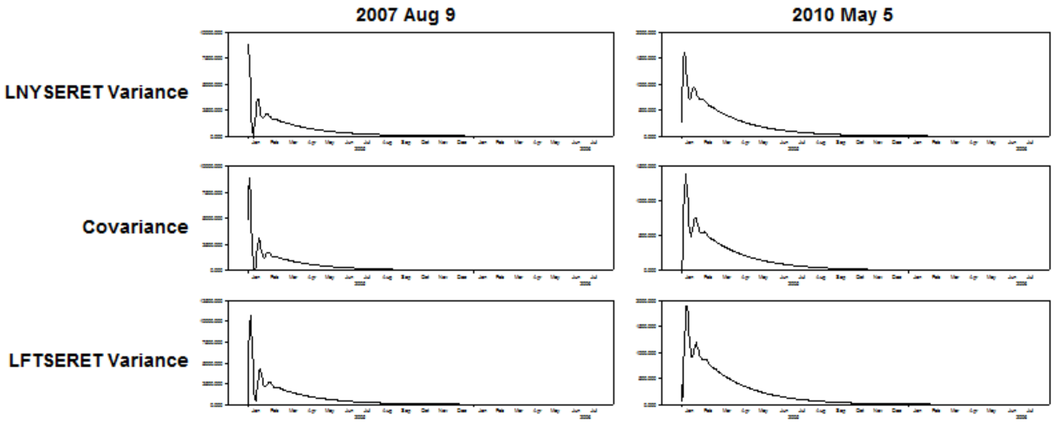


Figure 2. 100 day forecasts based on a BEKK model

The recent experience of relatively high volatilities cause the increase in the two forecast volatilities, whilst the correlation tends towards the average over the sub-sample.

Plots of the VIRFs are shown in Figure 3, panels A and B. The VIRF impulse responses for August 9th 2007, shown in Panel A, use the variance at that point in time as the baseline. The initial response for the NYSE is scaled at just under 10000, and when this is compared to the impulse response of the FTSE in the UK, the response is even larger at just over 10000. These have been computed using a baseline of the estimated volatility state, so they are excess over the predicted covariance. They can be contrasted to the impact of the EU debt crisis on May 5th 2010, in which the NYSE initial response is just over 1500, whilst the FTSE response at the same point in time is nearly 2000, suggesting that, as might be expected, the EU debt crisis had a larger impact in London than in New York. These shocks have been predicted using a baseline of zero. The 2007 shocks take a period of about 6 months to work through, whilst the 2010 shocks take a longer period of 8-9 months, but this may well reflect the choice of a lower baseline. The covariances show a dramatic spike in response to both shocks but remain higher longer, in relation to the 2010 shock, again, perhaps in response to the choice of baseline. Thus, the choice of baseline remains a key issue in the implementation of VIRF analysis.

Panel A: Baseline 9th August 2007 and May 5th 2010



Panel B: Baseline 2008 September 15th and May 5th 2010

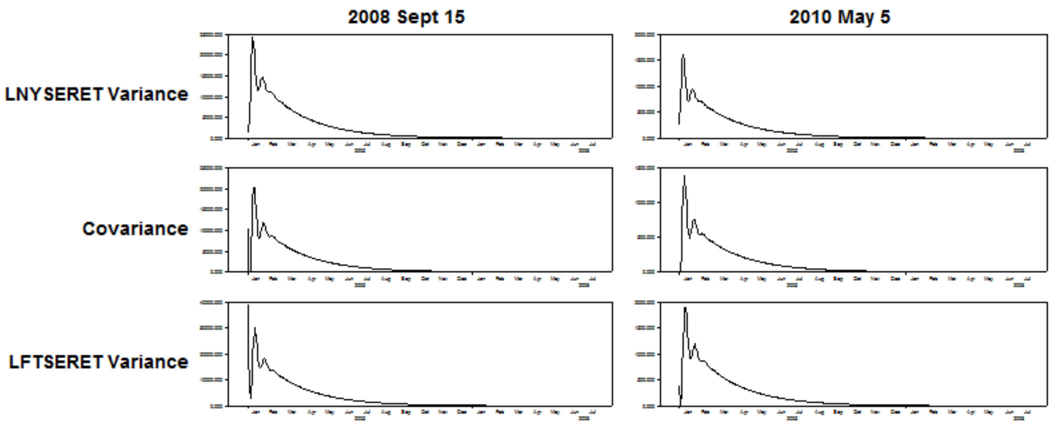


Figure 3. VIRF

Panel B of Figure 3 contrasts the September 15th 2008 GFC impact with the May 5th 2010 EU debt crisis once again, and the choice of baselines mirrors that made in Panel A.

The impact of the shock in 2008, at the height of the GFC, is relatively higher than previously, in both New York and London. On the NYSE it approaches 25000, whilst on the FTSE it is even higher, approaching 40000, and the shocks in both markets take longer to die out than in 2007, taking 9 months to get back to equilibrium. The covariance approaches 20000 and remains at high levels for 6-7 months. The 2010 May 5th graphs are the same as in Panel A and included for the purposes of direct comparison.

Given that we are considering VIRF in the context of stock market indices it seemed appropriate to consider leverage effects via the introduction of the separate consideration of the impact of negative shocks. The asymmetric BEKK model estimated is shown in Table 4 (for the sake of brevity only the multivariate GARCH and asymmetric terms are reported). Figure 4 shows the VIRF (Again, for the sake of brevity only September 2008 and May 2010 are considered). The key difference in the results, when compared to the previous analysis, is that the VIRFS are larger and of shorter duration. For example, the NYSE variance increases to 8000 and the LSE variance increases to 15,000 in September 2008. The duration of the response for both 2008 and 2010 is reduced to 3 months for both the variances and covariances.

4. CONCLUSION

In this paper we have applied the Hafner and Herwartz (2006) VIRF analysis to ten years of daily return series taken from the New York Stock Exchange Index, and the London Stock Exchange FTSE 100 index, for a period from 3rd January 2005 to January 31st 2015. An attractive feature of VIRF analysis of the effects of shocks on volatility through time, is that the shocks are treated as being endogenous. However, we also note that the choice of the baseline for the shock makes a difference. A contribution of this paper is to consider leverage effects, which are well documented in the empirical analysis of stock markets, see e.g. Engle and Ng (1993). We show that the impact of negative shock is larger, but of shorter duration, than that implied by a symmetric treatment of shocks.

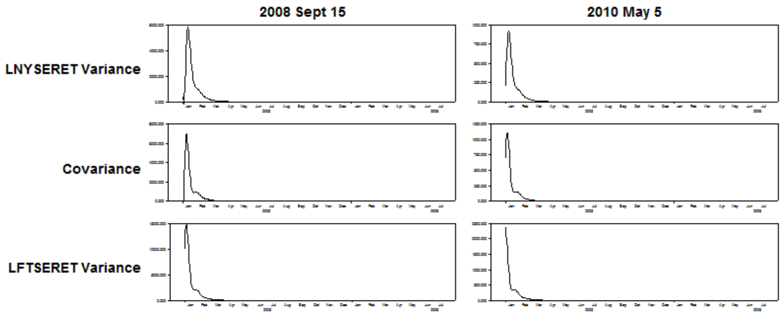


Figure 4. VIRF based on Asymmetric BEKK (responses to negative price movements).

Table 4. Asymmetric BEKK model based on t distribution.

Variable	Coeff	Std Error	T-Stat	Signif
A(1,1)	-0.022753722	0.060798967	-0.37425	0.708
A(1,2)	-0.405700847	0.065933722	-6.15316	0.000
A(2,1)	0.148631275	0.035519302	4.18452	0.000
A(2,2)	0.296233075	0.041308360	7.17126	0.000
B(1,1)	0.812855262	0.026787787	30.34425	0.000
B(1,2)	-0.151242974	0.031493570	-4.80234	0.000
B(2,1)	0.161414758	0.030535132	5.28620	0.000
B(2,2)	0.997063705	0.025611106	38.93091	0.000
D(1,1)	-0.469369500	0.036937131	-12.70725	0.000
D(1,2)	-0.393521072	0.089578341	-4.39304	0.000
D(2,1)	0.211373660	0.061407304	3.44216	0.000
D(2,2)	-0.083147397	0.085927903	-0.96764	0.333
Shape	8.904691765	0.951329821	9.36026	0.000

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