18th World IMACS / MODSIM Congress, Cairns, Australia 13-17 July 2009
http://mssanz.org.au/modsim09

Meta-heuristic approach for the conceptual design and optimization of multistage interceptor

Q. Zeeshan [1], D. Yunfeng [1], A.F. Rafique [1], K. Nisar [1] and A. Kamran [1]

[1] *School of Astronautics, Beijing University of Aeronautics and Astronautics, Beijing 100191, P.R. China*
Email: qasimzeeshan@sa.buaa.edu.cn, qsmzeeshan@yahoo.com,

Abstract: The design of a missile system capable of intercepting fast moving target(s) is a complex problem that must balance competing objectives and constraints. It involves teams of specialists working separately in their specialized design domains (such as propulsion, aerodynamics, guidance etc), but are also coordinated through a system level set of design requirements such as physical size or weight. This type of segmented design process requires rigorous iterations to ensure that the missile sub-systems are compatible with each other while still meeting the mission specifications. Therefore the need arises for a Multidisciplinary Design Optimization (MDO) approach that can control the design domains concurrently and configure an optimum design within the set design limits and constraints.

Recently the authors have considered the design of ground-launched and air-launched configurations for short range endo-atmospheric interceptors using evolutionary optimization techniques but still the potential of using meta-heuristic search algorithms like Simulated Annealing (SA) for the MDO of a multistage long range exo-atmospheric interceptor have not yet been gauged. In this paper we propose a conceptual design and optimization strategy using Genetic Algorithm (GA) cascaded with Simulated Annealing (SA), for the design of a multistage ground based Interceptor comprised of a three stage solid propulsion system for an exo-atmospheric boost phase intercept. The elite solution from GA is passed on to SA as initial guess. Search Space Reduction (SSR) is used to enhance the convergence of the Hybrid Meta-Heuristic Search Algorithm (HMSA). The SSR is applied on the optimal solution from GA, the upper and lower bounds for SA are then reset based upon the optimal solution from GA.

The mission of the Ground Based Interceptor is to deliver the Kinetic Kill Vehicle (KKV) to an optimal position in space to allow it to complete the intercept. The modules for propulsion characteristics, aerodynamics, mass properties and flight dynamics have been integrated to produce a high fidelity model of the entire vehicle. The Propulsion module is comprised of sub-modules of Solid Rocket Motor (SRM) design, nozzle geometry and performance prediction analysis. Internal ballistics and performance prediction parameters have been calculated using a lumped parameter method. For the present effort, the design objective is to minimize the Gross Lift Off Mass (GLOM) (kg) of the interceptor under the mission constraints of miss distance (m), intercept time (sec), lateral divert (m/sec), velocity at intercept (km/sec), g-loads and stage configuration requirements. SRM envelope constraints comprised length to diameter ratios, nozzle expansion ratios, propellant burn rates and grain geometry constraints such as web fraction, volumetric loading efficiency, etc. Interceptor conceptual design problem was posed to optimizer and it successfully solved these under the given conditions and constraints and satisfied the Interceptor trajectory/ performance objectives. The proposed meta-heuristic design and optimization methodology coupled with the SSR provides the designer with a computationally efficient and powerful approach for the design of interceptor systems.

Keywords: *Genetic Algorithm, Simulated Annealing, Interceptor, Multidisciplinary Design Optimization, Solid Rocket Motor*

1. INTRODUCTION

Significant work has been done in recent years to advance the design, analysis and optimization of launch vehicles. There has been little development of dedicated code and software for the design and analysis of interceptor systems. Hull and Salguero (1994) determined the size of a ground-launched, multistage missile for the ascent phase intercept. For simplicity, the interceptor and the ballistic missile are assumed to operate in the same vertical plane. The optimization technique used is recursive quadratic programming. Anderson (1998), Anderson et al. (1999) and Anderson et al. (2001) used GA as an optimizer to create an objective function that could be used in the design, optimization and analysis the performance of single-stage solid propellant interceptor. In recent years, GA have been used in rocket-based vehicle design optimization (Hartfield et al., 2004, Brown et al., 2005, Riddle et al., 2007, Bayley et al., 2007). In this paper we propose a conceptual design and optimization strategy for a multistage ground-based interceptor (GBI) comprised of a three stage solid propulsion system for an exo-atmospheric boost phase intercept (BPI) (Mantle, 2004). The strategy uses a Meta-heuristic Search Algorithm, cascading the search properties of both the Genetic Algorithm (GA) and Simulated Annealing (SA). Search Space Reduction (SSR) is used to enhance the convergence of HMSA.

2. DESIGN REQUIREMENTS FOR GROUND LAUNCHED BOOST PHASE INTERCEPT

The considerations involved in a GBI design differ from other surface-based systems and space-based systems. GBI must be able to survive the high mechanical and thermal stresses associated with flying through the atmosphere at supersonic speeds. Space-based Interceptors (SBI), by contrast, have little or no interaction with the atmosphere because the intercepts usually occur at very high altitudes. A primary trade-off in designing an interceptor is between speed and acceleration on the one hand and size on the other. A number of characteristics affect the details of the tradeoff between the interceptor speed and size (Wilkening, 2004). The general design requirements and tradeoffs are summarized in Table.1. The Interceptor's Gross Lift off Mass (GLOM) varies as a function of structural mass, payload mass, and speed and acceleration (booster burn time). The system characteristics that provide the desired operational performance should be optimized.

Table 1. General Design Requirements and Trade-offs

Minimize	Maximize
Size, GLOM and Payload (Kill vehicle) mass	Speed and acceleration, burn out velocity
Intercept Time	Thrust, specific impulse, combustion speed
Preparation and Start up time and Burn time	Propellant burning rate
G-Loads	Maneuverability

2.1. Design Objective

In aerospace vehicle design, the development costs tend to vary as a function of gross lift-off mass and is considered a minimum development cost concept (Bayley et al., 2007). For the present effort, the design objective is to minimize the GLOM (kg) of the interceptor under both mission constraints and SRM envelope constraints. In doing so, we try to configure an optimum propulsion system for interceptor missile to achieve our goal i.e. effective intercept of the target in the boost phase. The mission of the interceptor is to deliver a 200kg payload (KKV) to the proximity of the target to complete the effective intercept. The baseline design is for three stages of sequentially stacked SRMs. The payload (KKV) is enclosed in a fairing whose shape is known beforehand. Each SRM has ellipsoidal dome ends. The number of stages is fixed as three. The system design variables for each stage are shown in Table 2. There are 17 variables that govern the interceptor propulsion sizing and one variable to set the effective navigation ratio or gain for proportional navigation. See Table 4.

Table 2. Design Variables Discipline wise

Design Variable	Symbol	Units	Discipline
Relative Mass Coefficient of Grain	μ_{ki}	Ratio	Structure Propulsion
Body Diameter	D_i	m	Structure Propulsion Aerodynamics
Chamber Pressure	p_{ci}	Bar	Structure Propulsion
Exit Pressure	p_{ei}	Bar	Structure Propulsion
Coefficient of Grain Shape	K_{si}		Structure Propulsion
Grain Burning Rate	u_i	mm /sec	Propulsion
Navigation Coefficient	N	Coeff.	Guidance

2.2. Design Constraints

Interceptor design is constrained by physical and/or performance requirements. The constraints can be categorized as Mission Constraints and SRM envelope constraints. Mission Constraints are comprised of miss distance (m), intercept time (sec), lateral divert (m/sec), velocity at intercept (km/sec), G-Loads etc. SRM envelope constraints include stage configuration requirements which are comprised of Length to Diameter Ratios, Nozzle expansion ratios, propellant burn rates and Grain geometry constraints such as web fraction, volumetric loading efficiency etc. Intercept velocity and time are formulated as trajectory constraints. Weight-to-thrust ratios and propellant mass ratio are constrained to be within allowable ranges. Nozzle exit diameters are constrained to be less than stage diameters. A dynamic penalty function is used to handle in flight and terminal constraints. A symbolic problem statement can be expressed as follows (William and Crossley, 1997):

$$\min f(x) = f(x) + h(k)\sum_{i=1}^{m} \max\{0, g_i(x)\} \quad (1)$$

Where *f(x)* is the objective function, *h(k)* is a dynamically modified penalty value; *k* is the current iteration number of the algorithm. The function $g_i(x)$ is a relative violated function of the constraints

3. OPTIMIZATION APPROACH

A meta-heuristic is a heuristic method for solving a very general class of computational problems by combining user-given black-box procedures, usually heuristics themselves, in the hope of obtaining a more efficient or a more robust procedure. The name combines the Greek prefix "meta" ("beyond", in the sense of "higher level") and "heuristic" (from heuriskein, "to find"). Even though heuristic methods do not guarantee the exact optimal solution, they are frequently used because of their fast computation and easy applicability (Reeves, 1993). The purpose is to detect the solutions under time limited situation, when time is more important than quality of problem solution. Barr et al. (1995) identified the cases where the new heuristic methods are accepted, if they are: fast, accurate, robust, simple, high-impact, generalize-able, innovative.

Calculus-based optimization (CBO) e.g. gradient descent methods, schemes use sensitivity derivatives in the immediate vicinity of the current solution and can therefore easily fall into local optima from which they cannot recover. To avoid these local optima and to increase the odds of obtaining an acceptable solution these CBO methods require a reasonable starting scheme (initial design). Population based, non-gradient, stochastic direct search optimization methods (heuristic) are therefore attractive choices for our design problem as they are effective for highly nonlinear problems and allow global search of entire the design space. Furthermore, heuristic optimization (GA, SA) methods require neither sensitivity derivatives nor a reasonable starting solution. The optimization problem is solved by using a combination of Meta-Heuristic Search Algorithms. The Hybrid Meta-heuristic search algorithm (HMSA) aims to combine the GA and SA (GASA) in order to blend their advantages and minimize their disadvantages. GASA allows global search to be performed using a cascaded architecture (Figure 1). A set of design variables (*X*) with lower bound (LB) and upper bound (UB) is passed to the optimizer which creates an initial random population and then performs further operations. These candidates *X* are then passed to multidisciplinary design and analysis modules. The constraints are calculated and handled by an external penalty function. The algorithm runs in a closed loop via the optimizer until an optimal solution is obtained. Search Space Reduction (SSR) is implemented to utilize the most promising solutions during the first optimization phase. The lower bound (LB_R) and upper bound (UB_R) in the second optimization phase (SA) are selected in the vicinity of the of the optimal solution form the first optimization result (GA), thus reducing the design search space (Table 4).

3.1. Genetic Algorithm (GA)

Almost every discipline in aerospace from Guidance, Navigation, Control, Propulsion and Structures has utilized the power of GA (Murray, 1997). GA are relatively easy to implement and effective for highly nonlinear problems. GA requires neither sensitivity derivatives nor a reasonable starting solution. GA allows the global search of our design space. For detail on GA see Goldberg (1989).

3.2. Simulated Annealing (SA)

The term of Simulated Annealing (SA) is derived from metallurgy where annealing techniques use heating and cooling of material to enhance its strength. The algorithm starts with an initial design state; new states are randomly generated in the neighborhood of the current state. The change of objective function value,

(ΔE), between the new and the current design state is calculated as a measure of the energy change of the system. At the end of the search, when the temperature is low the probability of accepting worse designs is very low. Thus, the search converges to an optimal solution. For details on SA see Chattopadhyay et al., (1994) and Aarts et al., (1988). SA can be applied regardless of the conditions of differentiability, continuity, and convexity that are normally required in conventional optimization methods. For applications of SA on missile design see Tekinalp et al. (2000, 2004).

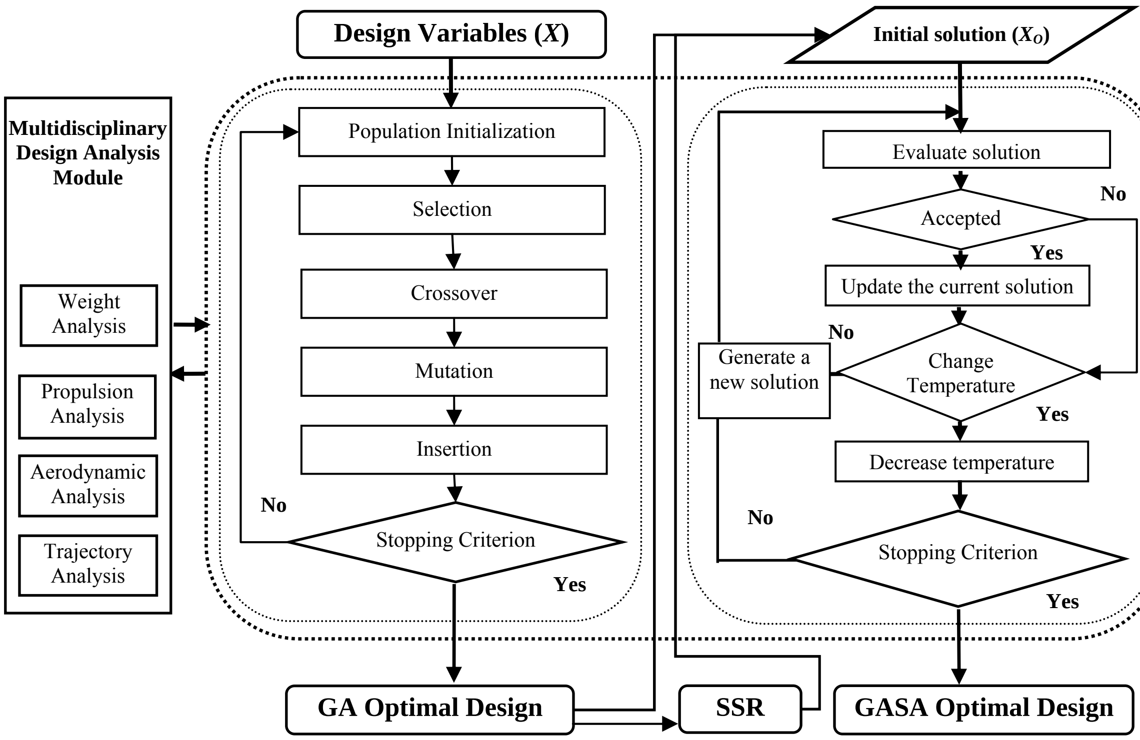

Figure 1. Overall Design and Optimization Strategy

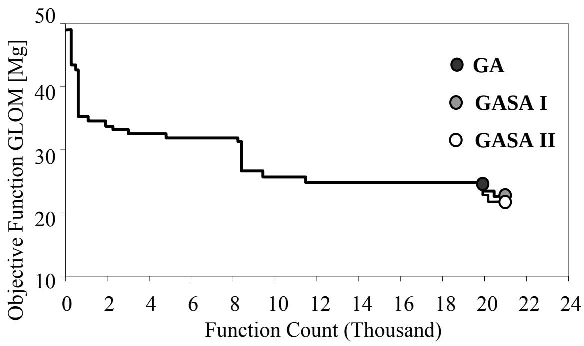

Figure 2. Convergence of Design Objective

Table 3. Parameters of Meta-heuristic Algorithm

GA	SA
Maximum generations:200	Optimization Type:
Population size: 100	Fast Annealing
Population type: Double Vector	Maximum Iteration: 1000
Selection: Stochastic uniform	Function Tolerance: 10^{-6}
Crossover: Single point, pc = 0.8	Temperature Function:
Mutation: Uniform, pm = 0.25641	Exponential
Fitness Scaling: Rank	Maximum Function
Reproduction: Elite count=2	Evaluations: 5000
Function Evaluations: 20000	Initial Temperature 100

4. MULTIDISCIPLINARY DESIGN ANALYSIS

MDA Strategy is envisaged for multi-stage interceptor analysis which includes: (a) Weight Analysis (b) Propulsion Analysis (c) Aerodynamic Analysis, and (d) Intercept Trajectory analysis and (e) Optimization Techniques (Section 3) are included in which the configurations are "optimized" to maximize the performance and minimize the GLOM. Using a combination of physics-based methods and empirical relations, the weight of the SRM components and propulsion analysis for solid stages is determined from LinShu (2004). The mass equation for a multi-stage interceptor can be written as:

$$m_{0i} = m_{pi} + m_{ki} + m_{0(i+1)} \tag{2}$$

Where m_{0i} is gross mass of i^{th} stage, m_{pi} is mass of propellant of i^{th} stage, m_{ki} is mass of structure of i^{th} stage, $m_{0(i+1)}$ is payload of the i^{th} stage. The gross lift-off mass m_{01} of a multistage solid interceptor is calculated as:

$$m_{01} = m_{PAY} + \sum_{i=1}^{n}(m_{gni} + m_{sti} + m_{svi} + m_{asi} + m_{fei} + m_{fsi}) \quad (3)$$

Where m_{gni} is mass of the i^{th} stage SRM grain; m_{sti} is mass of the i^{th} stage SRM structure; m_{svi} is mass of control system, safety self-destruction system, servo, and cables inside the i^{th} stage aft skirt; m_{asi} is mass of the i^{th} aft skirt including shell structure, equipment rack, heating protect structure, and directly subordinate parts for integration; m_{fei} is mass of equipment and cables inside the i^{th} stage forward skirt; m_{fsi} is mass of the i^{th} stage forward skirt including shell structure, equipment rack, and directly subordinate parts for integration. Relative mass coefficient μ_{ki} as given below in Equation (4) is function of range or burnout velocity. It is a design parameter which should be optimized.

$$\mu_{ki} = \frac{m_{gni}^{e}}{m_{oi}} \quad (4)$$

We have not restricted to a particular shape of grain at preliminary design level, rather a variable k_{si} is used to represent the burning surface area S_{ri} of grain as a function of grain length L_i and diameter D_i.

$$m_{gni} = \frac{\pi}{4}\rho_{gni}\psi_i\lambda_{gni}D_i^3 \quad (5)$$

Where D_i is stage diameter, ρ is the density and ψ_i is grain volumetric efficiency. Propulsion analysis describes important parameters like thrust, burn time, mass flow rate and nozzle parameters (Sutton and Oscar, 2001). The aerodynamic analysis incorporates USAF Missile DATCOM 1997 (digital) (Blake, 1998). A 3D model is developed for both interceptor and target with a boost phase acceleration profile that depends on total mass, propellant mass and the specific impulse in the gravity field (Zarchan, 1997). It is assumed that we have a priori knowledge of the target launch site and target launch (Zeeshan et al., 2008). The guidance algorithm used is proportional navigation Figure 3. The system commands accelerations (n_C) normal to Line of Sight (LOS) between the interceptor and the target, proportional to the closing velocity (V_C) and the LOS rate. Mathematically, the guidance law can be stated as

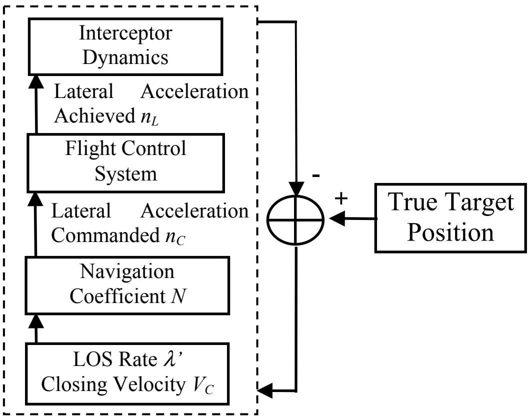

Figure 3. Guidance Algorithm

$$n_C = NV_C\dot{\lambda} \quad (6)$$

Where N is the effective navigation ratio or gain. Typical ranges for N are 3- 5 (unit less) (Zarchan, 1997). It is assumed that there is a perfect seeker and a perfect radar system so that the target position and velocity are known exactly. For preliminary design studies these two assumptions are appropriate.

5. OPTIMIZED CONFIGURATIONS

The performance is depicted in Figure 4 and the optimized configurations are shown in Table 4 and Figure 5.

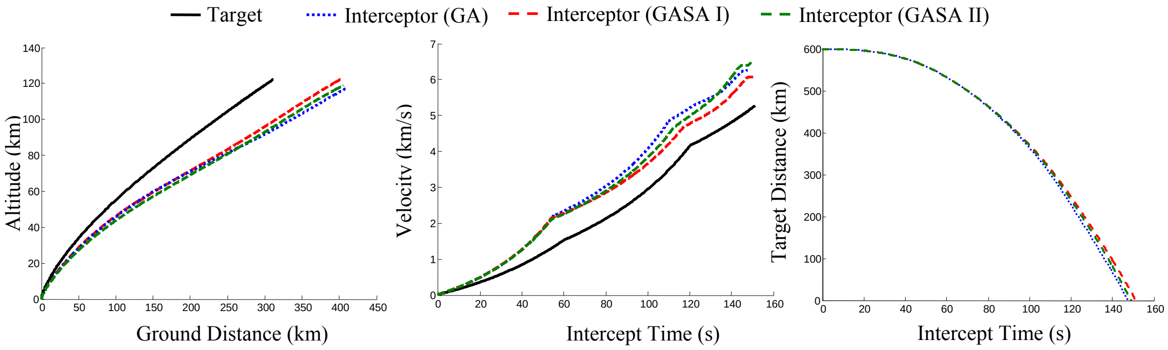

Figure 4. Performance of Optimized Configurations

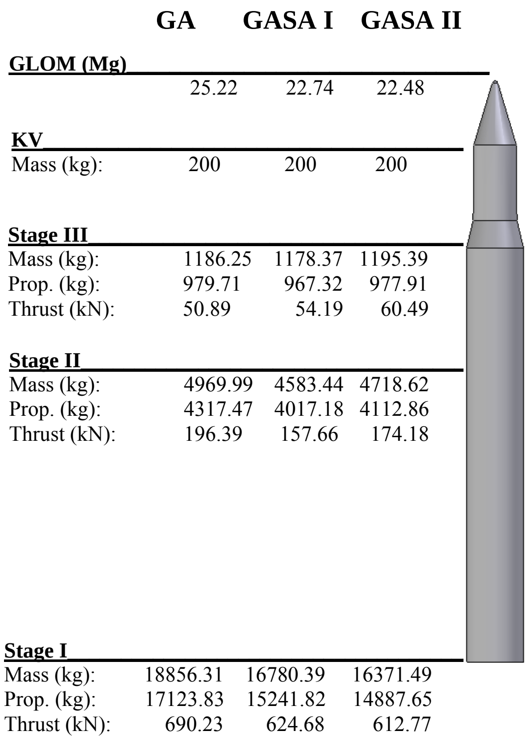

Figure 5. Optimized Configurations

Table **4**. Optimum Values of Design Variables (*X*)

X	**Sym-bol**	**Units**	**Initial**		**GA**	**GASA I**	**SSR**		**GASA II**
			LB	**UB**			**LB_R**	**UB_R**	
1	μ_{k1}	Ratio	0.6	0.7	0.6792	**0.6702**	0.66	0.68	**0.6621**
2	μ_{k2}/μ_{k1}	Ratio	1	1.04	1.0001	**1.0054**	1	1.04	**1.0160**
3	μ_{k3}/μ_{k2}	Ratio	1	1.08	1.0405	**1.0415**	1	1.08	**1.0418**
4	D_1	m	1.2	1.8	1.2903	**1.2609**	1.25	1.3	**1.2938**
5	D_3	m	0.7	1.0	0.9534	**0.9514**	0.9	1.0	**0.9561**
6	p_{c1}	bar	50	70	57.433	**56.32**	50	60	**54.33**
7	p_{c2}	bar	40	60	56.338	**51.63**	50	60	**53.35**
8	p_{c3}	bar	40	60	51.458	**53.26**	50	60	**54.14**
9	p_{e1}	bar	0.5	0.9	0.6252	**0.6171**	0.5	0.9	**0.7598**
10	p_{e2}	bar	0.15	0.35	0.2804	**0.3194**	0.2	0.35	**0.3082**
11	p_{e3}	bar	0.1	0.25	0.2351	**0.2232**	0.2	0.25	**0.2128**
12	u_1	mm/s	5	10	7.086	**7.134**	5.5	8	**7.257**
13	u_2	mm/s	5	10	7.536	**6.334**	5.5	8	**6.667**
14	u_3	mm/s	5	10	5.859	**6.792**	5.5	8	**7.263**
15	k_{s1}		1.5	2.3	2.1857	**2.1632**	1.8	2.2	**2.1818**
16	k_{s2}		1.5	2.3	1.9664	**1.9917**	1.8	2.2	**2.0884**
17	k_{s3}		1.5	2.3	2.0366	**1.9429**	1.8	2.2	**1.9493**
18	N		3	5	5	**5**	4	5	**5**

6. CONCLUSION

Simulation experiments and results show that GASA proved able for the MDO of interceptor and fulfilled the mission objectives and performance constraints. The Search Space Reduction (SSR) enhances the convergence of the hybrid search algorithm. The inclusion of SSR module in GASA II further reduces the GLOM. The reduction in GLOM achieved by using the GASA I and GASA II was around 2.4 and 2.7kg respectively i.e. about 8% which is quite significant. Though, the performance of the solution is approximately the same in GASA I and II but the solution converges rapidly in GASA II. The optimization results and performance are to be considered as preliminary (proof-of-concept) only, but they can be compared to existing systems (Isakowitz, 1999) and used for the conceptual design and optimization of interceptors and launch vehicles.

REFERENCES

Aarts, E.H.L., Korst, J.H.M. and Van Laarhoven, P.J.M. (1988), A Quantitative analysis of the simulated annealing algorithm, *Statistical Physics*, 50(1–2), 187–206.

Anderson, M.B. (1997), Genetic algorithms in aerospace design: Substantial progress, tremendous potential, Sverdrup Technology Inc./TEAS Group Eglin Air Force Base, FL, USA.

Anderson, M.B. (1998), Design of a missile interceptor using genetic algorithms, Dissertation, Auburn University, Alabama.

Anderson, M.B., Burkhalter, J.E. and Jenkins, R.M. (1999), Design of an air to air interceptor using genetic algorithms, AIAA Guidance, Navigation and Control Conference and Exhibit.

Anderson, M.B., Burkhalter, J.E. and Jenkins, R.M. (2001), Intelligent systems approach to designing an interceptor to defeat highly maneuverable targets, 39th AIAA Aerospace Sciences Meeting and Exhibit.

Barr, R.S., Golden, B.L., Kelly, J.P., Resende, M.G.C. and Stewart, W.R. (1995), Designing and reporting computational experiments with heuristic methods. *Journal of Heuristics*, 1:9–32.

Bayley, D.J., Hartfield, R.J., Burkhalter, J.E. and Jenkins, R.M. (2007), Design optimization of a space launch vehicle using a genetic algorithm, 3rd AIAA Multidisciplinary Design Optimization Specialist Conference.

Blake W.B. (1997), Missile DATCOM: User's manual-1997 FORTRAN 90 revision, U.S. Air Force Research Lab/ Air Vehicle Directorate.

Brown, N. and Olds, J. (2005), Evaluation of multidisciplinary optimization (MDO) techniques applied to a reusable launch vehicle, 43rd AIAA Aerospace Sciences Meeting and Exhibit.

Chattopadhyay A. and Seeley C.E. (1994), Simulated annealing technique for multiobjective optimization of intelligent structures, *Smart Materials and Structures*, 3, 98–106.

Crossley, W.A. and Williams, E.A. (1997), A study of adaptive penalty functions for constrained genetic algorithm based optimization, Purdue University West Lafayette.

Goldberg D.E. (1989), Genetic algorithms in search, optimization, and machine learning, 1st Edition, Addison Wesley Longman, Reading.

Hartfield, R.J., Jenkins, R.M. and Burkhalter, J.E. (2004), Ramjet powered missile design using a genetic algorithm, 42nd AIAA Aerospace Sciences Meeting and Exhibit.

Hull, D.G. and Salguero D.E. (1994), Missile sizing for ascent-phase intercept, AIAA Atmospheric Flight Mechanics Conference.

Isakowitz, S. J. (1999), International reference guide to space launch systems, 3rd Edition, Reston, AIAA.

LinShu, H. (2004), Launch vehicle design, Beijing University of Aeronautics and Astronautics, China.

Mantle, P.J. (2004), The missile defense equation: factors for decision making, Reston, AIAA.

Wilkening, D.A. (2004), Airborne boost-phase ballistic missile defense, *Science and Global Security*, 12(1-2), 1–67.

Reeves, C. R. (1993), Modern heuristic techniques for combinatorial problems, John Wily & Sons, NY.

Riddle, D.B., Hartfield, R.J., Burkhalter, J.E. and Jenkins, R.M. (2007), Genetic algorithm optimization of liquid propellant missile systems, 45th AIAA Aerospace Sciences Meeting and Exhibit.

Sutton, P. and Oscar, B. (2001), Rocket propulsion elements, 7th Edition, John Wiley & Sons, New York.

Tekinalp, O. and Utalay, S. (2000), Simulated annealing for missile trajectory planning and multidisciplinary missile design optimization, 38th AIAA Aerospace Sciences Meeting and Exhibit.

Tekinalp, O. and Bingol, M. (2004), Simulated annealing for missile optimization: Developing method and formulation techniques, *Guidance, Control and Dynamics*, 27(4), 616–626.

Zarchan, P. (1997), Tactical and strategic missile guidance, 3rd Edition, Volume 176, Reston, AIAA.

Zeeshan, Q., Momtaz, M. and Yunfeng, D. (2008), Conceptual design architecture modeling and simulation for boost phase ballistic missile defense, *Computer Aided Design, Drafting and Manufacturing*, 18(1), 47–65.