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Towards robust estimation of hydrological parameters focusing on flood forecasting in small catchments

Krauße, Thomas[1] and **Johannes Cullmann**[2]

[1]Institute of Hydrology and Meteorology, Department of Hydrology,
Dresden University of Technology, Würzburger Str. 46, 01187 Dresden, Germany
Email: thomas.krausse@tu-dresden.de

[2]IHP/HWRP Secretariat, Federal Institute of Hydrology
Am Mainzer Tor 1, 56068 Koblenz, Germany

Abstract

Modelling the rainfall-runoff task is a challenging task, especially if flood forecasting in small catchments is considered. In this context, hydrological models are not yet able to equally well describe the full range of processes that drive the runoff generation. This holds both for simple concept models and detailed process models with physically based components. One of the main reasons for this lack of "process fidelity" are the highly dynamic characteristics of such events. This situation, amongst other reasons, requires that models have to be adapted to a specific catchment by a parameter vector, i.e. a set of hydrological parameters to be calibrated. The lack of "process fidelity" can be partly compensated for by adapting various parameter vectors according to the actual dominant driving forces of the rainfall-runoff processes. A number of approaches address this way forward. Cullmann et al. (2008) proposes an event specific classification method to enable the application of an adequate parameter vector to different classes of flood patterns. Along these lines Fenicia et al. (2007) propose the combination of local models each best describing a specific range of processes and tested it for distributed physically based models in small catchments describing a specific range of processes.

One of the keys to successful modelling of rainfall-runoff processes in a specific catchment is the calibration itself. In a classical way this task is formulated as a mathematical optimization problem for a given single or multi-objective function. The result is a single best performing parameter vector, or a set of equally well performing parameter combinations. This equifinality and the missing consideration of measurement errors can lead to over-fitted models, which consequently lack the robustness required for operational purposes. To overcome this problem a new approach has been developed, the robust parameter estimation algorithm ROPE, first introduced by Bárdossy and Singh (2008). We further developed this algorithm with advanced depth and tested it for a distributed physically based model in small catchments with high runoff dynamics at a smaller time scale (hourly instead of daily modelling time-step).

Keywords: *Automatic parameter estimation; model calibration; flood forecasting; robust modelling; parameter uncertainty; Monte Carlo methods; data depth*

INTRODUCTION

This paper presents two new techniques to be applied for flood forecasting in small and fast responding catchments. Flash floods represent one of the most common and dangerous natural hazards. However, the menace arising from flash floods worldwide is often not clearly addressed in the media and lacks public awareness. Flash floods are characteristic for small to medium sized catchments. Usually, they are a consequence of severe rainstorms. Regarding the total volume, flash floods are often much smaller than inundations. Nonetheless, due to the immense flow velocities and steep gradients, flash floods pose the most serious threat to human life (see Cullmann, 2006).

All commonly accepted approaches in flood forecasting, even data-driven ones, make use of a rainfall-runoff model to simulate the dominant runoff processes within the considered basin. An arbitrary rainfall-runoff model m with current internal model storage s_i is a function which takes a vector of meteorological observations $\vec{x_i}$, representing the driving forces, and a model specific parameter vector θ to simulate the actual runoff q_i.

$$q_i \leftarrow m(\vec{x_i}, \theta, s_i) \tag{1}$$

The internal storage is updated as follows:

$$s_{i+1} \leftarrow u(\vec{x_i}, \theta, s_i) \tag{2}$$

By the help of θ the model can be adapted to different catchment areas. Additionally parameter and model uncertainty can be expressed by a set of parameter vectors $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$ whose members describe the range of uncertainty. A robust estimation of θ is the base of most flood forecasting approaches. Both for simple concept models and detailed process models with physically based components, one single parameter vector θ is not capable to describe the full range of processes that drive the runoff generation equally well (Fenicia et al., 2007). The lack of "process fidelity" can be partly compensated for by adapting various parameter vectors according to the actual dominant driving forces of the rainfall-runoff processes. Furthermore an estimation of robust parameters can be improved by a paradigm change in model calibration. The calibration algorithm ROPE, firstly presented by Bárdossy and Singh (2008) which relies on Monte Carlo methods and the definition of data depth is one possible method to find robust and reject non-robust parameter vectors.

The remainder of the paper is organized as follows. First an enhanced version of the robust parameter estimation algorithm ROPE briefly presented together, followed by a short description of the case study-area, the experimental set-up and the results. This is completed by a discussion and a short outlook on future work.

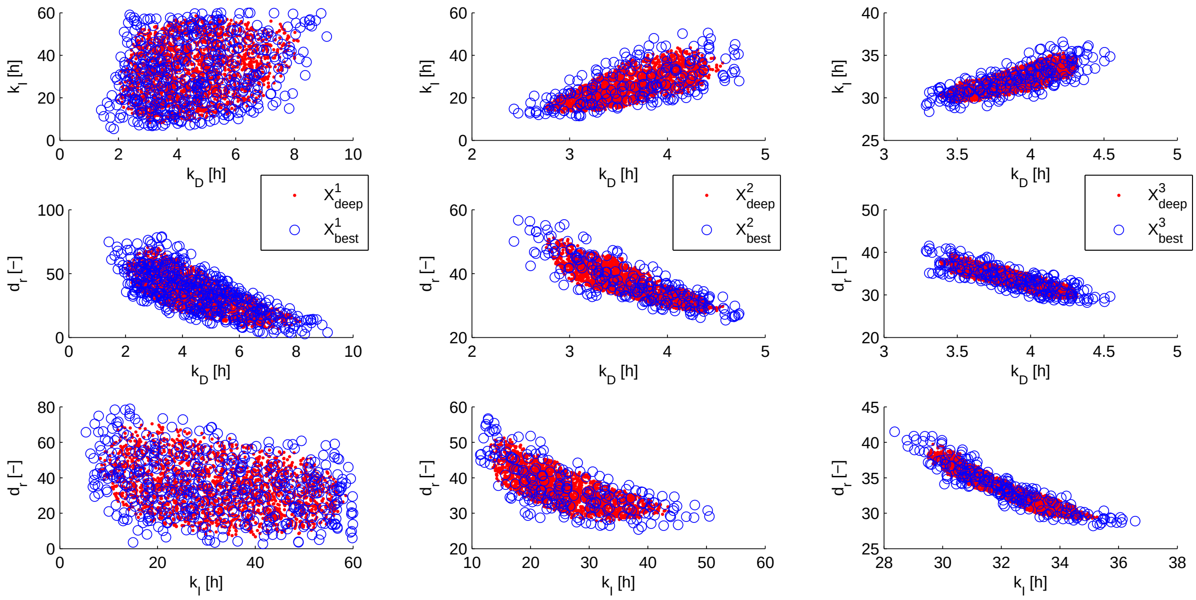
1. ROBUST PARAMETER ESTIMATION

A new approach to deal with the mentioned problems of model parametrization and calibration is the robust parameter estimation algorithm (ROPE). The main idea of this approach is a paradigm shift in model calibration in hydrology. No more is the calibration procedure understood as a pure optimization procedure (according to a given objective). A geometric search of a set of robust performing parameter vectors Θ_{robust} by the help of Monte Carlo methods and a definition of data depth has replaced this philosophy. The basic assumption is that robust parameter vectors are located deep within a set of good performing parameter vectors (see Bárdossy and Singh, 2008). In this context, depth is related to the definition of data depth, which is used to estimate the degree of centrality of a point with respect to a set of points within a n-dimensional space. Tukey (1975) introduced depth functions first to identify the centre of a multivariate dataset. ROPE bases upon the halfspace depth which can be calculated with respect to arbitrary sets and is one of the most robust depth measures developed.

We implemented and further developed the algorithm in a MATLAB® framework. Additionally we further developed the algorithm by addition of further depth measures, e.g. convex-hull peeling depth, L_1 depth, zonoid depth and weighted halfspace depth (see Hugg et al., 2006; Vencálek, 2008). This was done to be able to apply problem specific depth functions according to the form of the clouds to sample from. The definition of weighted halfspace depth even enables the sampling from non-convex clouds. A further problem can occur in the sampling procedure for higher dimensions. With increasing dimension the ratio of the volume of the unit cube with respect to the unit sphere increases rapidly. This is why the volume of the bounding box around the best performing parameter vectors can increase in a way that simple sampling techniques cannot be applied anymore. We implemented importance sampling successfully to tackle this problem.

Note that not all implemented improvements are used in the studies presented in this paper. For instance for reasons of comparison with previous results from the study of Bárdossy and Singh (2008) we still use the halfspace depth exclusively. At the moment our framework is in a steady development process. The framework and the full bandwidth of its depth functions will be presented and studied in future papers. The implementation procedure for the (enhanced) ROPE algorithm is as follows:

1. Define a set of model parameters to be calibrated and their feasible range.
2. Draw a set of parameter vectors Θ that are within the hypercube defined by the parameter boundaries. We consider the latin hypercube sampling for this task to yield a good and uniform coverage of the hypercube by a small number of samples; if there is a prior distribution for this parameter, this can be used instead.
3. The (hydrological) model is run for each parameter vector $\theta \in \Theta$ and the corresponding model performance is calculated by a task adequate objective function.
4. A subset $\Theta_{best} \subset \Theta$ of the best performing parameter vectors in Θ is identified, for instance such that Θ_{best} comprises the best 10% of all parameters in Θ.
5. A set of parameter vectors Θ_{deep} is generated by the help of importance sampling out of the parameter space defined by Θ_{best}, such that $\forall \theta \in \Theta_{deep} : D(\theta, \Theta_{best}) \geq l$ with $l \geq 1$ (Figure 1).
6. Set $\Theta = \Theta_{deep} \cap \Theta_{best}$ and repeat steps 3-6 while the model performance of Θ and Θ_{deep} differ more than one would expect from the observation errors or the maximum number of iterations is not reached; otherwise assign $\Theta_{robust} = \Theta_{deep}$ and terminate the algorithm with result Θ_{robust}.



(a) *(b)* *(c)*

Figure 1. Visualization of the clouds of best (blue) deep (red) parameters for an application of ROPE for the flood event nr. 14 in June 1994; after first (a), second (b) and third iteration (c)

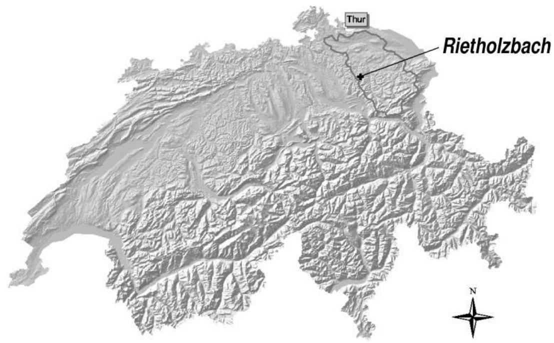
It is worthwhile noting that an arbitrary objective function can be used in step 3. Generally, this opens the possibility to address specific characteristics of the considered processes and/or catchments. In the context of flood forecasting additional data can be used to support calibration, e.g. a hydrograph separation to yield a robust representation of the runoff processes in the catchment.

2. CASE STUDY-AREA AND DATA

The Rietholzbach catchment is analysed in this section. It has a long record of hourly data sets and the perturbing impact of data heterogeneity is relatively small in this catchment. The basin has been observed as a research catchment by the ETH Zurich since 1975. The outlet drains a 3.18 km^2 hilly pre-alpine watershed with an average precipitation of 1600 mm per year, generating a mean annual runoff of 1046 mm. As a sub-basin of the Thur catchment it is located in the north-east of Switzerland. Its basic geographical and land-use characteristics are listed in Table 2. A significant number of studies have been conducted in this basin. Further information can be found in Gurtz et al. (1999), Zappa (2002) and on the web under `http://www.iac.ethz.ch/research/rietholzbach`. The data we based our study upon is a timeseries consisting 27 years of meteorological[1] and discharge measurements. Out of this timeseries we selected a set of 24 flood events in the time from May until October to avoid the problem of modelling snow accumulation and melting processes.

area	3.31 km^2
maximum altitude	938 m a.s.l.
altitude at basin outlet	681 m a.s.l.
mean altitude	796 m a.s.l.
mean slope	12.5°
pasture land	67 %
forest	25 %
wetland	4 %
bushes	2 %
roads	2 %

(a)



(b)

Figure 2. Overview of the most important basin characteristics[2](a); the Rietholzbach catchment is located in the north-east of Switzerland (b)

3. EXPERIMENTAL SET-UP

In the following, we discuss the experimental set-up and the focused research goals. Firstly we want to show that one single parameter vector is not able to represent the full range of runoff dynamics within one catchment. Base of this study was a parameter vector θ_{wb} , estimated by semi-automatic calibration and successfully validated for water-balance simulations within the Rietholzbach catchment (see Pompe, 2009). Furthermore the set of 24 flood events was divided into 4 classes according to their peakflow values (Table 1) and the hydrological model was calibrated for one event per class by a state-of-the-art optimization algorithm, the Levenberg–Marquardt algorithm (LMA). As a kind of control measure for selected events we also applied the global optimization algorithm (SCE) which estimated nearly the same results as LMA. Both algorithms are implemented within the well known PEST framework. Afterwrds the estimated parameter vector θ_{flood} was validated for all events of the class in comparison with θ_{wb}.

Table 1. Classification of the flood events according to peakflow

class	description	peakflow $[mm/h]$	return period $[y]$
1	low events	1... < 2	1
2	medium events	2... < 3	2
3	high events	3... < 5	< 8
4	extreme events	≥ 5	> 8

Independently from the results of the first study the found parameter vectors are not estimated to be robust. Therefore within a second study we compared the developed robust calibration algorithm ROPE with the previously used optimization algorithm LMA.

[1]Temperature, precipitation, global radiation, humidity, wind speed and vapour pressure
[2]adapted from `http://www.iac.ethz.ch/research/rietholzbach/overview`

3.1. Rainfall-runoff model WaSiM–ETH

The used hydrological model is WaSiM–ETH. It is a spatial distributed physically based rainfall-runoff model and was developed by Schulla and Jasper (2007) at the ETH Zürich. WaSiM–ETH has been used successfully for modelling the rainfall-runoff processes in several studies in catchments located within mid mountain ranges and especially also in the pre-alpine Rietholzbach catchment Gurtz et al. (1999, 2003a,b). Additionally WaSiM–ETH has been used for extrapolation of extreme flood events by Cullmann (2006). The model has a modular structure. Specific modules can be switched on and off depending on the actual task. Within the modules one can use different algorithms, e.g. Topmodel vs. Richards for modelling the water movement within the unsaturated soil zone. For this study we used WaSiM–ETH/6.4. with the Richards approach. The model parameters considered for calibration are the storage coefficients of direct runoff and interflow, k_D and k_I, and the drainage density d_r which is a scaling parameter of interflow generation (Table 2). In previous studies (Cullmann, 2006) these three parameters have been proven to be sensitive with respect to modelling flood events.

Table 2. Overview of the used model parameters considered for calibration

parameter	reference value (θ_{wb}	range	description
$k_D[h]$	7	0.01..25	storage coefficient of direct runoff
$k_I[h]$	20	0.01...60	storage coefficient of interflow
$d_r[-]$	2.1	0.01...60	drainage density

3.2. Objective criteria

Within this study commonly used objective functions are applied. The efficiency criterion NS according to Nash and Sutcliffe (1970) has been widely used to asses the performance of hydrological models. The relative deviation in peakflow is a simple criterion to asses the model ability to predict correct peakflow values. More advanced objective functions as for instance the deviation DEVS according to Schulz (1968) or approaches based on weighting different parts of the hydrograph will be considered in future studies.

Table 3. Objective functions used in this study, where x_i is the observed value at time-step i and $y_i(\theta)$ is the simulated value estimated by parameter vector θ

name	description	formula
NS	Nash-Suttcliffe efficiency	$1 - \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - y_i(\theta))^2}{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$
DPEAK	Relative peak deviation	$\frac{\lvert x_{max} - y_{max}(\theta)\rvert}{x_{max}} \cdot 100\%$

4. RESULTS

4.1. Event classification

As previously supposed, the parameter vector θ_{wb} is not able to simulate the rainfall-runoff processes of all flood events with a sufficient performance. Besides a high relative peakflow error, the dynamics in the observed hydrograph are not sufficiently well portrayed by the simulation, especially for larger events. The model calibration with LMA yields promising results. The performance of θ_{wb} was outperformed both for peakflow error and Nash-Suttcliffe efficiency. An overview of the model performance over all events, classified into 4 groups as mentioned before (see 4) is given in Table 4. Further studies (Pompe, 2009) with the same dataset showed that a coarse event classification according to peakflow into 2 classes and additionally according to the type of the rainfall event is the most reasonable for the chosen set of parameters. Classification according to pre-moisture was not proven to be significant. However, this must be seen in the light that we did not consider any soil-hydraulic parameters in calibration process. Further studies will help to solve this question.

Table 4. Validation of parameter vector used for water-balance simulations θ_{wb} and process specific parameters θ_{flood}

class	DPEAK [%]		NS [-]	
	θ_{wb}	θ_{flood}	θ_{wb}	θ_{flood}
1	25.3	1.3	0.66	0.67
2	34.0	7.9	0.64	0.49
3	46.9	15.3	0.59	0.56
4	66.6	20.8	0.41	0.34

4.2. Optimization vs. robust calibration

The calibration results for the robust parameter estimation algorithm ROPE yielded a calibration performance in the range of the results estimated by the optimization algorithm LMA (Table 5). That is not disappointing, because the objective of ROPE is primarily not better calibration performance but to outperform the validation performance of parameter vectors estimated by classical optimization. The validation results confirm the supposed advantages of the approach. The parameter vectors estimated by ROPE outperform the ones estimated by LMA for both objectives within all classes of runoff events.

Table 5. Calibration results of 4 selected flood events (one per class) estimated by LMA and ROPE and validation of the estimated parameter vectors for all events of the respective class

Calibration

id	event	class	DPEAK [%]		NS [-]	
			LMA	ROPE	LMA	ROPE
19	June 1995	1	0.1	0.3	0.73	0.81
14	June 1994	2	0.5	2.7	0.58	0.90
9	August 1982	3	0.3	4.4	0.52	0.69
4	August 2007	4	7.0	5.3	0.86	0.77

Validation

class	DPEAK [%]		NS [-]	
	LMA	ROPE	LMA	ROPE
1	1.3	0.9	0.67	0.87
2	7.9	3.1	0.49	0.85
3	15.3	5.2	0.56	0.71
4	20.8	6.1	0.34	0.59

5. CONCLUSION AND FUTURE WORK

Within this study we presented two approaches to improve the modelling of flood events within small and fast responding catchments. First of all it was shown that a single parameter vector is not able to represent the full range of runoff dynamics within a specific catchment. An adequate classification of runoff events according to their dominant processes can help to solve this problem. Such a classification should limit its number to its possible minimum to keep it as simple as possible. Furthermore we presented the application of an enhanced implementation of the robust parameter estimation algorithm (ROPE). Eventhough the robustness of the estimated parameter vectors seems to be impressing, the development and understanding of ROPE is still at its beginning. Further studies will analyze the impact of different depth functions and advanced objective criteria. Furthermore the approach will be applied to problems with higher dimensions.

The proposed approaches improve the search of robust parameter vectors of hydrological models. This will help to improve operational flood forecasting techniques, based upon a coupling of physically based hydrological models and artificial neural networks (see Cullmann, 2006; Krauße, 2007). Furthermore ROPE can be easily adapted for robust model calibration in other fields.

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