

Modeling and evaluating water allocation risks using Value-at-Risk

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Abstract: Research in risk modeling and evaluation in eWater Collaborative Research Centre has two emphases: (a) Integrating risk modeling and evaluation into the common decision making process of eWater so that eWater tools can be used to model and evaluate risks. (b) Providing risk measures that can be integrated into the eWater models. The study presented in this paper demonstrates that the process of risk modeling can be integrated into eWater's decision making framework, where performance data and risk data (variability and likelihood) are generated and used. It also demonstrates that the Value-at-Risk (VaR) method can be used to formulate risk-based objective functions, and that minimising risks can be treated in the same way as minimising values like costs and losses. It is expected that the next generation eWater tools will be extended to incorporate risk data, multiple-objectives assessment, and the Monte-Carlo simulation so that risk modeling and performance modeling can use results from each other.

We conducted the case study in three stages. The first stage was to identify risks that were related to decision making in service delivery in the public domain (water resource assessment). We analysed the risk context in multiple domains. For this we followed the steps of a previously established, system-based risk management framework, to demonstrate the process of identifying risks and relating them to key system elements via the impact chain concept "control → cause/factor → system component → risk" (→ means "influences"). Through these steps, we succeeded in relating risks with their corresponding factors and causes (decision variables) in a high level schematic diagram.

The second stage of the case study was divided into two parts. The first part studied how the historical inflow sequence was used to perform water resource assessment. The second part adopted the historical inflow sequence (historical simulation) approach, but reformulated the problem into that of minimising a loss function $Z(\mathbf{x}, \mathbf{r}_\alpha)$, by adjusting the decision variable \mathbf{x} of general security (GS) water allocation, for the inflow sequence \mathbf{r}_α corresponding to an exceedance probability α . We introduced two measures Value-at-Risk ($\text{VaR}_\alpha(\mathbf{x})$) and Conditional Value-at-Risk ($\text{CVaR}_\alpha(\mathbf{x})$), borrowed from the financial sector, to measure risks. In the water domain, $\text{VaR}_\alpha(\mathbf{x})$ is the α -quantile of the loss function $Z(\mathbf{x}, \mathbf{r}_\alpha)$ for a water system, whereas $\text{CVaR}_\alpha(\mathbf{x})$ is the conditional expected loss for $Z \geq \text{VaR}_\alpha(\mathbf{x})$.

We noted that the approach offers at least two benefits: (a) The measures of VaR and CVaR associate water allocation risk with water inflows, supporting a better understanding and explanation of risks and actions. (b) Minimising risk is treated the same as minimising cost or loss; thus opening up the opportunity of applying stochastic optimisation methods in risk assessment.

In the third stage we set up scenarios for a range of exceedance levels $\alpha = 0.99, 0.95, 0.90, 0.80,$ and 0.60 . The optimised solution \mathbf{x} was contingent on the inflows into the river dam. If the inflow was very low (i.e., α very close to 1), the GS allocation \mathbf{x} could be very small. Conversely, if α was not so close to 1, e.g. 0.90, 0.80, or 0.60, then the GS allocation \mathbf{x} would grow accordingly.

While we appreciated the potential merit of measuring VaR and CVaR in economic and other values, we did not have time to pursue this in the study. Instead, we focused on demonstrating the feasibility of integrating risk management into decision making. In the future, we will expand the idea of setting up risk values as objective functions and applying stochastic optimisation methods to reduce risks.

Keywords: *Integrated decision making, Water resource assessment, Risk, Value-at-Risk, Conditional Value-at-Risk, Exceedance probability, Cumulative inflow, Historical simulation.*

1. INTRODUCTION

The eWater Collaborative Research Centre (eWater 2008) undertakes both strategic and practical research to generate a range of hydrological and ecological models, and to develop decision support systems to be used by urban and rural water managers in Australia. In order to maximise the research impact, the eWater models and related tools are integrated under a generic decision making framework. Risk management is one of the integrated research areas that will affect the successful use of the new eWater models and related tools in the future.

The integration of risk management and decision making processes is relatively new to Australian river managers and operators. In order to spread the systemic use of risks to the eWater CRC researchers and the water industry in general, we developed a risk management framework (Yum *et al.* 2007; Yum & Blackmore 2007). The framework aimed to be compatible with eWater’s generic decision making framework.

The work described in this paper was a follow-up of the risk framework development – studying the case of managing the regulated section of the Cudgegong River, and demonstrating how the framework guides the process of risk formulation and evaluation.

The risk framework is applicable to a variety of measures e.g. Bayesian belief measure, classical risk measure of expected impacts, and risk measure for extreme (e.g. drought, flood) events with small probabilities and large impacts. The Cudgegong River’s practice of water resource assessment led us to formulate risk measures for water allocation decisions contingent on extreme low flow events.

2. HIGHLIGHTS OF THE UNDERLYING RISK MANAGEMENT FRAMEWORK

eWater CRC adopts an integrated decision making framework into its product development process. This leads to a systematic integration of eWater tools, allowing performance data and the stochastic data to be used readily for systemic problem solving purposes (Figure 1).

The following is a summary of the risk management framework in Yum & Blackmore (2007):

At the **start phase** of the framework (understanding the risk context), interacting systems and system elements are identified to model the causal relationships of risk.

The **second phase** (risk assessment) identifies quantitative or qualitative relationships among subsystem elements: *risk* (measured as a combination of the consequence and likelihood), *cause* (something or an action that makes some specific thing happen as a result), *factor* (a cause of risk that cannot be manipulated by the river manager), *control* (an existing process, policy, device, practice or other action that acts to minimise negative risk or enhance positive opportunities), and *system component*. The risks are evaluated according to the chains of impact: control → cause/factor → system component → risk, which are derived from domain knowledge or models.

In response to the risks identified and assessed in the previous steps, the **third phase** designs and evaluates both structural and non-structural controls that help prevent or mitigate the risks. The **fourth phase** selects and implements appropriate risk-mitigation or prevention controls. Throughout the whole process, stakeholders are involved in understanding, monitoring and controlling risks.

3. PRELIMINARY STUDY OF RISKS TO CUDGEGONG RIVER OPERATORS

The regulated section of the Cudgegong River lies within the Central West Management Area in the State of New South Wales (NSW). Its upstream limit is Windamere Dam. It is a major tributary of the Macquarie River and flows into Burrendong Dam. In NSW, river water regulation is carried out at two levels. (a) At the policy level, the state regulator NSW Department of Water and Energy (DWE) determines how much water is allocated to the irrigators in the valley. (b) At the operation level, State Water carries out water releases.

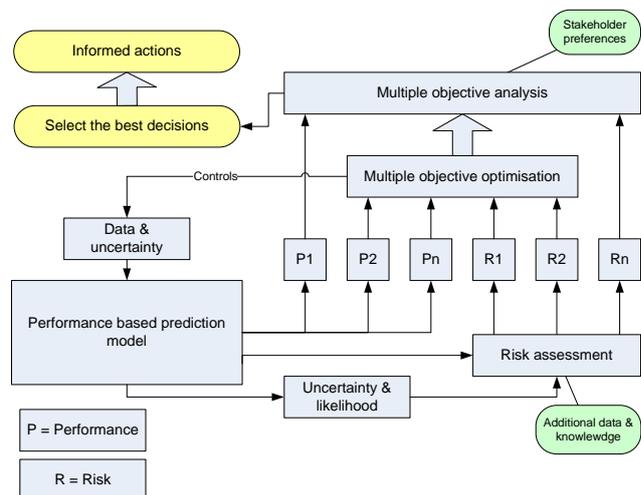


Figure 1. Integrating performance data and risk data under the generic decision support framework.

State Water earns its income by selling water, and looks for full cost recovery on its activities, with the aim of minimising operational losses (e.g. dam seepage, oversupply, over extraction, etc.) In 2008 the price of water was \$150 – 200 per mega litre (ML), and one share (1 ML) was worth \$1500. Water shares were originally attached to the land, but now anyone can own them.

In the understanding-the-context phase of the framework, we identified the overall objective of managing the regulated section of the Cudgegong River as providing water to the irrigators without harming the environment. We identified the risks of the river operators as making inappropriate releases that deprive the farmers of their livelihood (economic and social domains) and the river managers of their income (operational and political domains), and damage the environment (environment domain).

During the early risk-assessment phase (the second phase of the framework), we identified a set of high level relationships between risks and their causes and factors (Figure 2). All major risks considered were directly influenced by the amount of water released from the dam, and the release was determined by the balance of supply and demand of water.

Water supply was influenced by the state of the storage, minus water lost in operation, which was mainly through evaporation. The state of water storage was a function of past storage, and inflows and outflows during the period under consideration. Other influences (such as inflows from unregulated streams below the dam) were not considered in the study.

Water demand could be influenced by a multitude of causes, e.g. plant types, soil, alternate sources of water supply, and irrigation technology adoption. We confined our water demand to two causes: (a) irrigation demand, and (b) water shares. Finally, the NSW Department of Water and Energy (DWE) had the power to set the percentage allocation to balance supply and demand.

4. WATER RESOURCE ASSESSMENT IN PRACTICE

In the case study, we focused on the risk to operators in times of drought– in terms of volume of water (ML) allocated but unable to deliver to water license holders. The drought cycle of Cudgegong Valley is 7 years long; and the local irrigators wish to safeguard supply of water for 12 years. The DWE/State Water sets up a water resource assessment process to evaluate the maximum percentage allocation that can be sustained over a long period of 12 years. The assessment method hinges on the concept of exceedance probability of cumulative historical inflow.

Exceedance probability is the probability that the random variable in question exceeds some threshold. Figure 3 is a chart showing the cumulative historical inflow sequences with respect to the exceedance probabilities 99% (~minimum), 90%, 60% and 1% (~maximum), starting from February. The chart is obtained by ranking the historical cumulative inflows at each monthly time step. The inflow sequence with exceedance probability 99% (minimum) represents a historical sample of severe drought (1 in 100 years). At the height of the 2007-2008 drought, the minimum drought sequence was used as input.

State Water, the Cudgegong River operator, undertakes resource assessment monthly, or every time there are reasonable inflows into Windamere Dam. Let α be the exceedance probability of the historical *cumulative* inflow sequence $I(\alpha, t)$, measured in ML, and started at the month of assessment (t is the monthly time step).

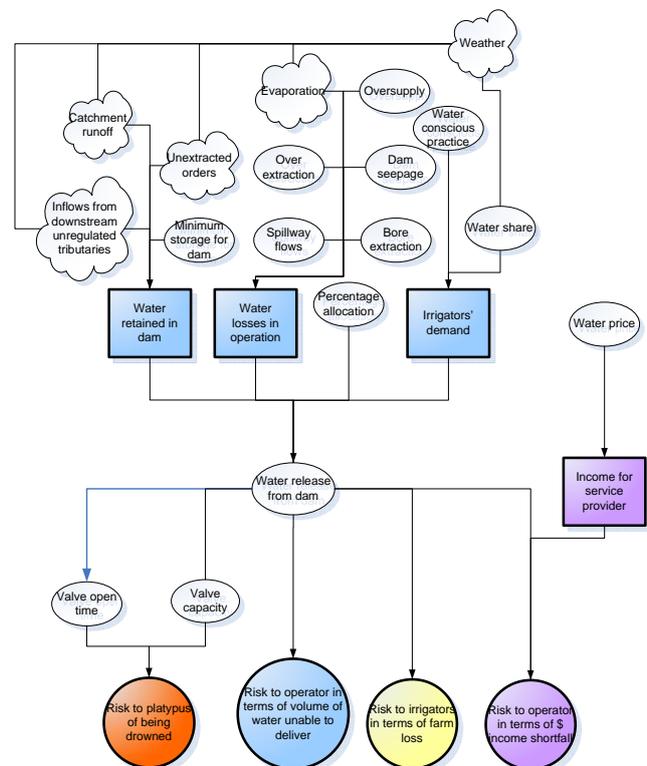


Figure 2. A detailed system view of the risks to the river operator, showing the dependency relationship of risks (circles), factors (clouds), and causes (ellipses).

Windamere storage $s(\alpha, t+1)$, is related to the storage, inflow and percentage allocation at each time step t through the following water balance equations:

$$s(\alpha, t+1) = s(\alpha, t) + I(\alpha, t) - I(\alpha, t-1) - e(t) - d(\alpha, t) - er(\alpha, t) - b(\alpha, t), t = 1, 2, \dots, 143 \text{ (months)} \quad (1)$$

where $s(\alpha, t) \geq 20000\text{ML}$ (dead storage of Dam) to make the Dam operational at every time step t ;

$I(\alpha, t) - I(\alpha, t-1)$ is the inflow in the month t ;

$e(t)$, measured in ML, is estimated monthly evaporation loss, which can be approximated by converting the maximum monthly evaporation (in mm) to volume in ML;

$d(\alpha, t)$, measured in ML, is the estimated demand as constrained by the percentage allocation to be determined – the annual demand is about 12000 – 15000 ML/year; monthly release is spread out to reflect some seasonal pattern;

$er(\alpha, t)$, measured in ML, is essential requirements to keep the river flowing; the annual total is 8200 to 8500 ML/year; and monthly released is spread out to reflect the seasonal pattern;

$b(\alpha, t)$, measured in ML, is provision for bulk transfer from Windamere to Burrendong Dams.

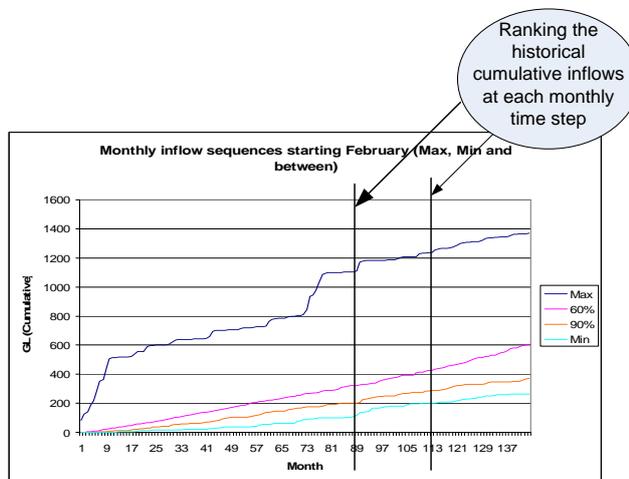


Figure 3. Cumulative monthly inflow sequence.

The input of cumulative inflow sequence into water balance equations (1) offers a non-parametric method to model the impact of stochastic events without any assumed knowledge of the distribution function of risks. We can sample inflows from historic records and substitute them into Eqs (1) to predict their impacts into the future. This method is also called historical simulation. For a given exceedance probability α , $d(\alpha, t)$ is the only decision variable (percentage allocation) that the resource assessment wanted to maximise. As a result the solution (percentage allocation) is contingent on the quantification of low inflows (exceedance probability).

5. RISK MEASURES FOR IMPACTS CONTINGENT ON EXTREME EVENTS WITH SMALL PROBABILITIES

Cumulative inflow sequence with a given exceedance probability is a key input into models that provide dry inflow contingency planning (e.g. DEWHA 2007). Like the water resource assessment for the Cudgegong River, many such models are formulated as stochastic optimisation problems. This paper demonstrates that it is possible to formulate the dry inflow problem as a risk-based stochastic optimisation problem such that minimising risks is no different from minimising costs or losses.

Out of the four risks identified in Figure 2, we selected the “risk to operator in terms of volume of water unable to deliver” to demonstrate how to formulate water resource assessment in terms of risk. We defined risk as a loss function $Z = Z(\mathbf{x}, \mathbf{r})$ whose value is determined by a decision vector \mathbf{x} (volume of water in ML, equivalent to percentage allocation) and a random vector \mathbf{r} (corresponding to a cumulative inflow sequence in Figure 3.) We can see the similarity between $Z(\mathbf{x}, \mathbf{r})$ and Figure 2: the decision vector \mathbf{x} is our risk control variable (which we manipulate) and risk vector \mathbf{r} is our risk factors (which we cannot manipulate). The dependence relationships between the values of Z and \mathbf{x} , \mathbf{r} reflect the impact chain relationship “cause (\mathbf{x} percentage allocation) / factor (\mathbf{r} inflow sequence) \rightarrow system components \rightarrow risk Z ”. Details are in Section 6.

For any value of α ($0 \leq \alpha \leq 1$) and any fixed value of decision variable \mathbf{x} , we adopted two risk measures:

(a) Value-at-Risk $VaR_\alpha(\mathbf{x}) = \alpha$ -quantile of $Z(\mathbf{x}, \mathbf{r})$ (Jorion 2007) (2)

(b) Conditional Value-at-Risk $CVaR_\alpha(\mathbf{x}) =$ conditional mean value of Z , given that $Z \geq VaR_\alpha(\mathbf{x})$

$$= \int_{Z(\mathbf{x}, \mathbf{r}) \geq VaR_\alpha(\mathbf{x})} Z(\mathbf{x}, \mathbf{r}) dF / \int_{Z(\mathbf{x}, \mathbf{r}) \geq VaR_\alpha(\mathbf{x})} dF = \frac{1}{1 - \alpha} \int_{Z(\mathbf{x}, \mathbf{r}) \geq VaR_\alpha(\mathbf{x})} Z(\mathbf{x}, \mathbf{r}) dF \quad (3)$$

where F is the cumulative density function (CDF) of Z (Rockafeller & Uryashev 2000, 2002)

The risk measures of VaR and CVaR have been used in evaluating finance instruments extensively (e.g. <http://www.gloriamundi.org/> and Jorion 2007). In recent years, they have been used in predicting the

breeding values of cattle (Pruzzo *et al.* 2003), in optimising the drawdown of water for environmental flow release in Lake Burley Griffin (Webby *et al.* 2006), and in evaluating the economic loss of fishery in the Mekong River with respect to the variations of flood volume (Webby *et al.* 2007). In our case study, the measures VaR and CVar (Eqs 2 & 3) were used as objective functions on constraints (1). Our problem was to find a suitable water allocation \mathbf{x} to minimise the VaR and CVar measures.

The relationship between the exceedance probability \mathbf{r} and VaR (and CVar) can be rationalised using the following intuitive explanation. For a more rigorous proof, the reader is referred to Rockafeller & Uryashev (2002). For any fixed decision vector \mathbf{x} , let ζ and $\boldsymbol{\rho}$ be the values of Z and \mathbf{r} such that $\zeta = Z(\mathbf{x}, \boldsymbol{\rho})$. Z is a decreasing mapping with respect to \mathbf{r} , because for any $Z = Z(\mathbf{x}, \mathbf{r})$, $Z \leq \zeta$ if and only if $\mathbf{r} \geq \boldsymbol{\rho}$ (e.g. an inflow sequence with 90% exceedance probability always gives rise to a smaller loss than an inflow sequence with 99% exceedance probability.) Hence we have

$$\text{Probability } \{Z \mid Z \leq \zeta\} = \text{Probability } \{\mathbf{r} \mid \mathbf{r} \geq \boldsymbol{\rho}\} \quad (4)$$

If the common value of the above probabilities is α , the right hand side of the equation says that the inflow sequence $\boldsymbol{\rho}$ has exceedance probability α . The left hand side of (4) says that α is also the α -quantile of Z .

For a fixed value \mathbf{x} , Equation (4) enables us to plot the CDF of Z by simulation as follows. First, we generate sample inflow sequences $\boldsymbol{\rho}$ with corresponding exceedance probabilities α . Then we substitute the corresponding inflow sequences $\boldsymbol{\rho}$ to get values $\zeta = Z(\mathbf{x}, \boldsymbol{\rho})$. Finally, the CDF can be plotted (where \mathbf{x} is held constant):

$$\text{CDF of } Z = \{(\zeta, \alpha) \mid \alpha = \text{the exceedance probability of any inflow sequence } \boldsymbol{\rho}, \zeta = Z(\mathbf{x}, \boldsymbol{\rho})\}$$

We can compare the three risk measurements $E(Z)$, $\text{VaR}_\alpha(\mathbf{x})$ and $\text{CVar}_\alpha(\mathbf{x})$ as follows:

$$\begin{aligned} \text{Risk measure of expected loss } E(Z) &\leq \text{VaR}_\alpha(\mathbf{x}) = \alpha\text{-quantile} \\ &\leq \text{Conditional expected loss of } Z \text{ where } Z \text{ exceeds } \text{VaR}_\alpha(\mathbf{x}) \end{aligned}$$

The traditional risk measure of expected loss $E(Z)$ has the smallest value. $E(Z)$ is not sensitive to any value of α because $E(Z)$ is the mean loss over all possible α ; hence $E(Z)$ is a constant when α varies. $\text{VaR}_\alpha(\mathbf{x})$ provides a more sensitive and conservative (meaning larger) estimation of risk that is contingent on the exceedance probability α – as α approaches 1, the α -quantile increases. $\text{CVar}_\alpha(\mathbf{x})$ is the largest loss contingent on the exceedance probability α , hence it offers the most conservative risk estimation of the three.

6. DATA

Generally the loss in \$ terms has a non-linear relationship to water units. Due to time constraints we were not able to incorporate economic values into the analysis. Instead we took the simple assumption that 1 water unit (ML) has value 1, and the total loss is the sum of all water units that incur losses. The worthwhile study of mapping loss to economic and other values will be left for the next phase of the research.

Our total loss function $Z(\mathbf{x}, \mathbf{r})$ is the total loss accumulated over 144 months, constrained by equations (1), with the following components:

$$Z(\mathbf{x}, \mathbf{r}) = Z_{\text{WaterStorage}}(\mathbf{x}, \mathbf{r}) + Z_{\text{GS-Allocation}}(\mathbf{x}, \mathbf{r}) + Z_{\text{Non-operation}}(\mathbf{x}, \mathbf{r}) + Z_{\text{Water-transfer}}(\mathbf{x}, \mathbf{r}) \quad (5)$$

where: **Loss incurred in reduction in storage volume**, $Z_{\text{WaterStorage}}(\mathbf{x}, \mathbf{r}) = 360000 - s_f(\mathbf{x}, \mathbf{r})$ where 360000 (ML) is the storage capacity of Windamere Dam, and $s_f(\mathbf{x}, \mathbf{r})$ is the final storage in the Dam at the end of 12 years (Equations (1).) This means that when the storage Dam is full, the loss is zero. The less water is the Dam, the more loss is incurred.

Loss of general security water allocation, $Z_{\text{GS-Allocation}}(\mathbf{x}, \mathbf{r}) = (22000 - \mathbf{x}) * (12 - y_{s \leq 20000}(\mathbf{r})) + 22000 * y_{s \leq 20000}(\mathbf{r})$, where 22000 (ML) is the annual water entitlement of the Cudgegong River Valley, \mathbf{x} is the annual allocation of water, and $y_{s \leq 20000}(\mathbf{r})$ is the number of years when the storage in Windamere Dam is less than or equal to 20000 ML (minimal operative storage of the Dam). For those years when the Dam is functioning (i.e. Dam storage is above dead storage), the annual loss is $22000 - \mathbf{x}$, meaning that the loss is computed by the amount of water that cannot be allocated but entitled. For those years when the Dam is not functioning, the annual loss is 22000, meaning that the loss is maximised.

Loss due to non-operation, viz. after Dam storage drops to or below the dead storage, $Z_{\text{Non-operation}}(\mathbf{x}, \mathbf{r})$ is zero when Dam storage is above 20000 ML (operational state). When the storage in Windamere Dam is less than or equal to 20000 ML within the 12 year period, i.e. $y_{s \leq 20000}(\mathbf{r}) < 12$ (non-operational state), we assume that there is no water supply from then onward until the end of the 12 year period. The total loss for this period will be: $Z_{\text{Non-operation}}(\mathbf{x}, \mathbf{r}) = (6500 + 8500) * y_{s \leq 20000}(\mathbf{r})$, where 6500 ML is the *high security*

entitlement, and 8500 ML is the *essential release* amount, meaning that loss is equal to water volume that is entitled but cannot be allocated. (The loss of non-operation for general security allocation has been considered in the previous component.)

Loss due to bulk water transfer to Burrendong Dam, $Z_{\text{Water-transfer}}(x, r) = - \text{accumulated_bulk_water_transfer_volume}$. In our evaluation, water transfer from Cudgegong River to another water system (downstream Burrendong Dam) is considered only when there is enough water to cover the local use. As a result any bulk water transfer is considered as a credit (negative loss) because it is a surplus that adds value to the Cudgegong-Macquarie Rivers system as a whole.

For any cumulative inflow exceedance level α ($\alpha = 0.99, 0.95, 0.90, 0.80$, etc.), let r_α be the corresponding exceedance inflow sequence. The value of the loss function $Z(x, r_\alpha)$ (i.e. $\text{VaR}_\alpha(x)$) is found by historical simulation, i.e. feeding the corresponding exceedance inflow sequence r_α and the decision parameter x (General Security allocation) into the formula (5) (Figure 4).

For a fixed α value, the risk measures $\text{VaR}_\alpha(x)$ and $\text{CVaR}_\alpha(x)$ can be minimised by selecting an appropriate x , which leads to percentage allocation to be announced. For example, in Figure 5, at the exceedance level $\alpha = 0.90$, the flow sequence can be used to compute the loss values for various x values. When x is increasing from zero, the loss value drops steadily; because the more water is released to irrigators, the less is the loss. VaR is minimum at $Z = 426492$ ML, and $x = 15600$ ML. When x moves to slightly larger than 15,800, the loss value jumps because at this point, according to the result of historical simulation, the storage of Windamere Dam drops below its dead storage (20000 ML) in the month of October 2012 (evaluation years are from 2008 to 2019). When the dam storage is below 20000 ML, it triggers the event of no water delivery to downstream and thus increases the value of loss.

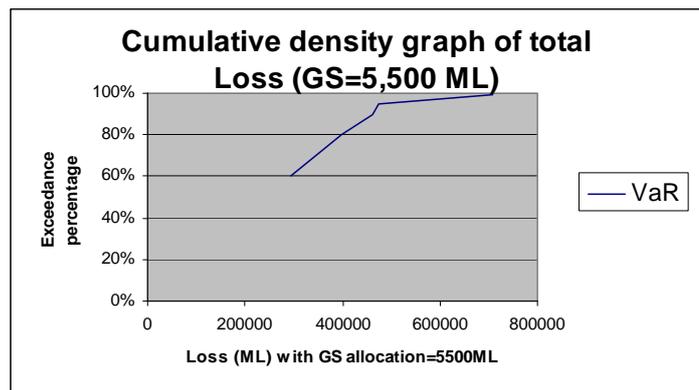


Figure 4. The CDF of loss function $Z(x,r)$ where $x = 5500$ ML.

The CVaR values in Figure 5 were simulated by using equation (3). CVaR values have three local minimums. The highest local minimum corresponds to $\text{VaR}_{90\%}$ ($x=15800$ ML, $Z=629318$ ML), the middle one corresponds to $\text{VaR}_{95\%}$ ($x= 10700$ ML, $Z=529314$ ML) and the smallest one corresponds to $\text{VaR}_{99\%}$ ($x= 2000$ ML, $Z=491499$ ML). This means that CVaR can give us good risk warnings. When we choose the largest local minimum $x=15800$ ML, we need to prepare for drought that is more severe than that with 90% exceedance probability. If there are indications that the drought is more severe, we should perhaps move to the next CVaR local minimum ($x= 10700$ ML, $Z=529314$ ML) or even the smallest CVaR local minimal ($x= 2000$ ML, $Z=491499$ ML). Note that when we move to lower CVaR risks, x is traded for smaller values (meaning that when we choose a lower conditional risk, we opt for less water allocation, i.e. a more conservative solutions.)

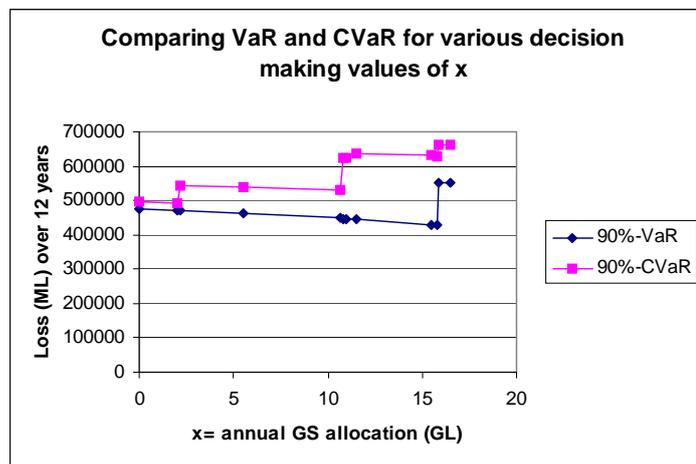


Figure 5. Using VaR_α and CVaR_α as objective functions.

7. SCENARIOS SETUP AND EVALUATION

In our case study, we evaluated $\text{VaR}_\alpha(x)$ and $\text{CVaR}_\alpha(x)$ for a range of exceedance levels $\alpha = 0.99, 0.95, 0.90, 0.80, 0.60$, each of which was regarded as a scenario of water allocation. We adopted a simple assumption

that the percentage allocation was fixed over most of the 12 years. The first water year (July to next June) was perhaps an exception; it was always a mixture of past data and future prediction because the evaluation might not start in July. We assumed that State Water had only one decision variable x to combat low inflows and. The decision variable was kept constant throughout the evaluation period of 12 years. However, as conditions change (water resource assessments are taken as frequently as monthly), its value may change accordingly.

In summary the optimised solution x was contingent on the inflows into the Dam. When the inflow was very low (i.e., $\alpha = 0.99$), the GS allocation x was very small (2100ML/year). Conversely, when α was not so close to 1, e.g. 0.90, 0.8, or 0.6, the GS allocation x increased accordingly ($x = 15800$ ML/year, $x = 21800$ ML/year, $x = 31200$ ML/year, respectively.)

8. DISCUSSION AND CONCLUSIONS

We adopted a systematic approach toward risk modeling, integrating risk consideration into the decision making process (Section 2). As a result, risk modeling would be able to use data from performance modeling, and vice versa. In our case study the set of water balance equations (1) mixed hydrological domain data with risk data – almost all terms came from hydrology, except the monthly inflow term (viz. $I(\alpha, t) - I(\alpha, t-1)$) which was related to risk data. If the inflow timeseries $I(\alpha, t) - I(\alpha, t-1)$ is known, water balance constraints like equations (1) can be readily handled by eWater tools such as River Manager and River Operator.

Traditionally water resource assessment was a stochastic optimisation method using the historical simulation method (Section 4). We reformulated the stochastic optimisation problem as the problem of minimising risks $VaR_\alpha(x)$ and $CVaR_\alpha(x)$ for a given α (Figure 5). We noted that the approach offered at least two benefits: (a) The measures VaR and CVaR associated water allocation risk with water inflows, supporting stakeholders to better understand and explain risks and actions. (b) Minimising risks was treated the same as minimising other values such as costs and losses, thus opening up the opportunity of using stochastic optimisation methods in risk assessment.

We recognised the potential merit of putting VaR and CVaR in economic or other type of values, but were not able to pursue this in this study. In the future, we will revisit the idea of setting up risk measurements as economic or other objective functions and applying stochastic optimisation methods to reduce risks.

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