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# Evaluating Corporate Loans via a Fuzzy MLMCDM Approach

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**Abstract:** A defuzzification based fuzzy MLMCDM model is proposed for the evaluation of commercial loans, where the importance weights of the criteria in the criteria structure and the ratings of alternatives versus subjective criteria are assessed in linguistic values represented by trapezoidal fuzzy numbers. These fuzzy numbers are defuzzified through the ranking approach of center of area (COA) before applying to the model in order to avoid the problem of multiplying more than two fuzzy numbers. The COA defuzzification for a trapezoidal fuzzy number has the following three situations:

- If area *abg* > area *bdhg*. COA lies between *a* and *b*, i.e., the COA lies in the left side of the core(*B*).
- If area *achg* < area *cdh*. COA lies between *c* and *d*, i.e., the COA lies in the right side of the core(*B*).
- Situation other than (a) and (b). Obviously COA lies in the core(*B*).

The averaged weights and ratings are aggregated going from the lowest-level criteria to the parent criteria and finally the evaluation values of alternatives can be obtained. The proposed model has the advantage of considering both qualitative and quantitative criteria. Moreover, it also allows to deal with a hierarchical criteria structure where parent criteria may have sub-criteria and these sub-criteria may have sub-sub criteria and so on. A numerical example has demonstrated the feasibility of the proposed model. Furthermore, a Monte Carlo simulation is conducted to gain more insight about the behavior of the model. The sensitivity analysis shows that the four most sensitive criteria are the ROE ($f_{1121}$=11.3%), collateral ($f_{134}$=11.2%), competitiveness ($f_{131}$=7.1%) and management's experience ($f_{121}$=7.1%). Moreover, an eight-level risk evaluation or bucketing system is suggested by using the 12.5$^{th}$, 25$^{th}$, 34.5$^{th}$, 50$^{th}$, 62.5$^{th}$, 75$^{th}$, 87.5$^{th}$ and 100$^{th}$ percentiles of the simulation distribution. As a result eight evaluation buckets are defined to facilitate the interpretation of the scores and the final evaluation values are classified into these buckets. The decision on whether to issue a loan or not can then be made based on the risk index of the potential borrower and the risk category in which it falls.

Other than the corporate loans evaluation, the proposed model can also be applied to other fuzzy management problems. Yet the model has some limitations that should be taken into consideration before using it in a real application or as a basis for further research.

- The final evaluation score is dependent on the number of criteria and sub-criteria being considered in the model. The bigger the criteria structure the larger the final evaluation scores.
- Different defuzzification approaches may lead to slightly different results.
- If the definitions for the linguistic values as well as the corresponding fuzzy numbers are different, the final results may also be different.
- Further research may try to demonstrate the feasibility of the model in a case study, applying it to a real company in real situations.
- The consistency and reliability of the model can be contrasted with historical information.
- The model can be computerized and the code could be made available for further improvements or adaptations.

***Keywords:*** *Fuzzy MLMCDM, Corporate loan, Center of area, Monte Carlo simulation, Credit risk*

## 1. INTRODUCTION

In performing activities, banks face a large number of risks, including credit risk, market risk, liquidity risk, operational risk, etc. The true core business of banking is the profitable management of risk (Hempel and Simonson, 1999). Risks are the uncertainties resulting in adverse variations of profitability or in losses (Bessis, 2001). Among the various risks faced by banks, credit risk is the first of all in terms of importance (Bessis, 2001). Credit risk is the event in which customers default, meaning that they fail to comply with their obligations to service debt. It is critical since the default of a small number of important customers can generate large losses potentially leading to insolvency (Bessis, 2001). Banks' survival and ability to compete depend foremost on their ability to profitably manage this sort of risk (Hempel and Simonson, 1999).

To cope and hedge against credit risk effects, banks must gather exhaustive information about their potential borrowers to assess their ability to repay debts in accordance with the loan agreement terms. In fact this process known as credit selection is generally considered one of the banking industry's core activities (Jacobson et al., 2006). Credit selection has both qualitative and quantitative dimensions (Hempel and Simonson, 1999). The quantitative dimension consists of the analysis of historical financial information and the qualitative dimension considers factors that can not be directly quantified and are generally more difficult to assess. There is a plethora of credit selection methodologies that use quantitative criteria to evaluate a borrower's risk. These methods range from analysis of financial ratios using univariate statistical techniques (Courtis, 1978) to the use of techniques such as discriminant analysis (Dimitras et al., 1996), data envelopment analysis (Min and Lee, 2007), and expert systems (Saunders and Allen, 2002). However, all of them are distinguished by the fact that they only rely on measurable information (financial ratios) and, therefore, ignore some qualitative aspects, such as competitiveness and organizational structure that may also have a strong impact on a borrower's capacity to repay a loan (Dimitras et al., 1996). Models that consider both quantitative and qualitative aspects are scarce because of the difficulties faced in operationalizing qualitative variables. Qualitative variables are usually determined by human perception, which is fuzzy or uncertain by nature. Therefore a credit selection model that allows a simultaneous assessment of qualitative and quantitative criteria is needed. Furthermore, many of the criteria used in the borrower's evaluation process may have sub-criteria and theses sub-criteria may in turn have sub-sub-criteria, etc. (Chen and Chiu, 1999). Thus the model has to deal with a hierarchical structure of criteria. To resolve the above problems, this work proposes a fuzzy multiple levels multiple criteria decision making (fuzzy MLMCDM) model.

Numerous methods have been suggested to investigate fuzzy MCDM problems (Chou, 2007). However, the hierarchical structure for depicting the relationship among criteria in the above papers are analyzed to one level at most. But in some cases such as the one in Chu and Tsao (2002), parent criterion "power" has several sub-criteria such as "fuel consumption", and this sub-criterion again has several sub-sub-criteria such as "city" and "highway". Thus a fuzzy MLMCDM model is needed. In addition, when there is more than one level in the criteria hierarchy, the multiplication of more than three fuzzy numbers will be encountered. Currently there is no solution to produce the membership function for the multiplication of more than three fuzzy numbers. The best way to resolve the above limitations may be to defuzzify all the fuzzy numbers before applying them to the fuzzy MLMCDM model. Thus, a proper defuzzification method is needed.

Many approaches for ranking fuzzy numbers have been studied (Wang and Kerre, 2001). In this work, the method of center of area is suggested to rank trapezoidal fuzzy numbers for its intuitiveness. The suggested ranking method has the merits of equally dividing the area under a trapezoidal membership function. The suggested ranking method is further applied to establish a defuzzified fuzzy MLMCDM model. Finally, a numerical example demonstrates the computational process and the feasibility of the proposed model. Moreover, through a Monte Carlo simulation, the distribution of the risk indices from the MLMCDM model can be obtained and different risk categories ranging from very low risk to very high risk are suggested. The decision on whether to issue a loan or not can then be made based on the risk index of the potential borrower and the risk category in which it falls.

## 2. FUZZY SET THEORY

A fuzzy number *B* is a trapezoidal fuzzy number, denoted by (*a*,*b*,*c*,*d*), if its membership function is given by (Kaufmann and Gupta, 1991): $f_B = (x-a)/(b-a)$ if $a \le x \le b$ ; $f_B = 1$ if $b < x < c$ ; and $f_B = (x-d)/(c-d)$ if $c \le x \le d$ , where $a,b,c,d \in R$, $b \neq c$. The set of elements ($x$'s) having the largest degree of membership in *B* is called the *core* of *B*, denoted by core(*B*).

A linguistic variable is a variable whose values are expressed in linguistic values. Linguistic variable is a very helpful concept for dealing with situations which are not well-defined to be reasonably described by traditional quantitative expressions (Zadeh, 1975-6), e.g., variable "importance" has several values such as "fairly important", which can be represented by a trapezoidal fuzzy number such as (0.3,0.45,0.55,0.7).

The concept of center of area (COA) defuzzification can be found in (Tong, 1978) as early as 1978. Formulae for COA in defuzzifying a trapezoidal fuzzy number (*a*,*b*,*c*,*d*) are presented as the following three situations. Suppose *g* and *h* are the two upper vertices above *b* and *c*, respectively.

If area *abg* > area *bdhg*. Obviously COA lies in the left side of the core(*B*). Thus, *e* is derived from" $I_{\mathrm{L}}(B)=I_{\mathrm{R}}(B)$" as $e=a+\frac{1}{2}\left[2a^2-2b^2+2bc+2bd-2ac-2ad\right]^{1/2}$. (1)

If area *achg* < area *cdh*. Obviously COA lies in the right side of the core(*B*). Thus, *e* is derived from " $I_{\mathrm{L}}(B)=I_{\mathrm{R}}(B)$" as $e=d-\frac{1}{2}\left[2d^2-2c^2+2ac+2bc-2ad-2bd\right]^{1/2}$. (2)

Situation other than (a) and (b). COA lies in the core(*B*). Thus $e=\frac{1}{4}(a+b+c+d)$. (3)

## 3. A FUZZY MULTIPLE LEVELS MCDM MODEL

Suppose the importance weights of criteria and the ratings of alternatives under criteria are assessed in linguistic values represented by positive trapezoidal fuzzy numbers. Further suppose a set of linguistic values represented by positive trapezoidal fuzzy numbers $A_i, i=1\sim n$, are applied by decision makers to evaluate the importance of the criteria. And a set of linguistic values represented by positive trapezoidal fuzzy numbers $B_i, i=1\sim n$, are applied to evaluate the suitability of alternatives versus criteria. By formulas (1)~(3), we obtain the values of COA of these fuzzy numbers as $e(A_i)$ and $e(B_i)$, respectively

### 3.1. Develop a *n* level hierarchy structure

$F_{x_i}=\{f_{x_1x_2\cdots x_i\cdots x_n}\}$, where $x_i=0\sim m_{x_1x_2\cdots x_{(i-1)}}\ (\in N)$, $1\le i\le n$, $n\in N$, and $m_{x_1x_2\cdots x_{(i-1)}}=m$, when $i$=1. (4)

### 3.2. Decide the weights

$$w_{x_1x_2\cdots x_i}=\frac{1}{q}\left\{w_{x_1x_2\cdots x_i1}+w_{x_1x_2\cdots x_i2}+\cdots+w_{x_1x_2\cdots x_iv}+\cdots+w_{x_1x_2\cdots x_iq}\right\} \quad (5)$$

where $w_{x_1x_2..x_iv}$ is from $e(A_i)$ and denotes the weight given by the *v*th decision-maker to the $x_1x_2..x_i$th criterion, $1\le v\le q$, $v,q\in N$. $w_{x_1x_2\cdots x_i}$ represents the weight of $f_{x_1x_2\cdots x_i}$, $w_{x_1x_2\cdots x_i}\in W_{x_1x_2\cdots x_{(i-1)}}$, $W_{x_1x_2\cdots x_{(i-1)}}\in M_{1\times m_{x_1x_2\cdots x_{(i-1)}}}$, *M* denotes matrix. $W_{x_1x_2\cdots x_{(i-1)}}=W$, $m_{x_1x_2\cdots x_{(i-1)}}=m$, $M_{1\times m_{x_1x_2\cdots x_{(i-1)}}}=M_{1\times m}$ when *i*=1.

### 3.3. Average alternative suitability

$$r_{x_1x_2\cdots x_it}=\frac{1}{q}\left\{r_{x_1x_2\cdots x_it1}+r_{x_1x_2\cdots x_it2}+\cdots+r_{x_1x_2\cdots x_itv}+\cdots+r_{x_1x_2\cdots x_itq}\right\} \quad (6)$$

where $r_{x_1x_2..x_itv}$ is a defuzzified trapezoidal fuzzy number from $e(B_i)$ and represents the suitability given by the *v*th decision-maker to the $x_1x_2..x_i$th criterion, $1\le v\le q$, $v,q\in N$. $r_{x_1x_2\cdots x_it}$ represents the average suitability of alternative *t* versus criterion $f_{x_1x_2\cdots x_i}$, $1\le i\le n$, $n\in N$.

### 3.4. Normalization of alternative suitability

$$r_{x_1x_2\cdots x_it}=\frac{s_{x_1x_2\cdots x_it}}{\max_t\left\{s_{x_1x_2\cdots x_it}\right\}},\ \text{if } s_{x_1x_2\cdots x_i}\in B;\quad r_{x_1x_2\cdots x_it}=\frac{\min_t\left\{s_{x_1x_2\cdots x_it}\right\}}{s_{x_1x_2\cdots x_it}},\ \text{if } s_{x_1x_2\cdots x_i}\in C. \quad (7)$$

where objective criteria can be classified into benefit (*B*) and cost (*C*) ones. $r_{x_1x_2\cdots x_it}$ denotes the normalized value of $s_{x_1x_2\cdots x_it}$. $s_{x_1x_2\cdots x_it}$ denotes the suitability value of alternative *t* versus criterion $f_{x_1x_2\cdots x_i}$.

### 3.5. Synthetic evaluation

$$M_{x_1x_2\cdots x_{(i-1)}} = W_{x_1x_2\cdots x_{(i-1)}} \times R_{m_{x_1x_2\cdots x_{(i-1)}}\times p}$$

$$= \left[ \sum_{x_i=1}^{m_{x_1x_2\cdots x_{(i-1)}}} w_{x_1x_2\cdots x_i}\cdot r_{x_1x_2\cdots x_i 1} \quad \sum_{x_i=1}^{m_{x_1x_2\cdots x_{(i-1)}}} w_{x_1x_2\cdots x_i}\cdot r_{x_1x_2\cdots x_i 2} \cdots \sum_{x_i=1}^{m_{x_1x_2\cdots x_{(i-1)}}} w_{x_1x_2\cdots x_i}\cdot r_{x_1x_2\cdots x_i t} \cdots \sum_{x_i=1}^{m_{x_1x_2\cdots x_{(i-1)}}} w_{x_1x_2\cdots x_i}\cdot r_{x_1x_2\cdots x_i p} \right]$$

$$= \left\lfloor r_{x_1x_2\cdots x_{(i-1)}1} \; r_{x_1x_2\cdots x_{(i-1)}2} \cdots r_{x_1x_2\cdots x_{(i-1)}t} \cdots r_{x_1x_2\cdots x_{(i-1)}p} \right\rfloor. \tag{8}$$

where $M_{x_1x_2\cdots x_{(i-1)}}$ denotes the additive weighted evaluations of $m_{x_1x_2\cdots x_{(i-1)}}$'s sub-criteria of the $x_1x_2\cdots x_{(i-1)}$th criteria from $p$ products. $R_{m_{x_1x_2\cdots x_{(i-1)}}\times p}$ denotes a $m_{x_1x_2\cdots x_{(i-1)}} \times p$ matrix. $w_{x_1x_2\cdots x_i}$ and $r_{x_1x_2\cdots x_i t}$ are elements in $W_{x_1x_2\cdots x_{(i-1)}}$ and $R_{m_{x_1x_2\cdots x_{(i-1)}}\times p}$, respectively. $w_{x_1x_2\cdots x_i}$ is derived by Eq. (5), $t$ denotes alternative $t$. $r_{x_1x_2\cdots x_i t}$ is derived from Eq. (6) if $f_{x_1x_2\cdots x_i}$ is a subjective criterion, from Eq. (7) if $f_{x_1x_2\cdots x_i}$ is an objective criterion, or from $\sum_{x_{(i+1)}=1}^{m_{x_1x_2\cdots x_i}} w_{x_1x_2\cdots x_{(i+1)}}\cdot r_{x_1x_2\cdots x_{(i+1)}t}$ if $x_{i+1}\neq 0$. $\sum_{x_i=1}^{m_{x_1x_2\cdots x_{(i-1)}}} w_{x_1x_2\cdots x_i}\cdot r_{x_1x_2\cdots x_i\cdot t}$ denotes the element in $R_{m_{x_1x_2\cdots x_{(i-2)}}\times p}$. The final evaluation matrix can be derived by Eq. (8) based on the rule of backpropogation as follows.

$$M = W \times R_{m\times p} = \left[ \sum_{x_1=1}^{m} w_{x_1}\cdot r_{x_11} \; \sum_{x_1=1}^{m} w_{x_1}\cdot r_{x_12} \cdots \sum_{x_1=1}^{m} w_{x_1}\cdot r_{x_1t} \cdots \sum_{x_1=1}^{m} w_{x_1}\cdot r_{x_1p} \right] = \left\lfloor r_1 \; r_2 \cdots r_t \cdots r_p \right\rfloor. \tag{9}$$

where $M$ denotes the final additive weighted evaluations of $m$ major criteria from $p$ product. $R_{m\times p}$ represents a $m\times p$ matrix. $w_{x_1}$ and $r_{x_1t}$ are elements in $W$ and $R_{m\times p}$, respectively. $r_{x_1t}$ is derived from Eq. (6) when $f_{x_1}$ is a subjective criterion, from Eq. (7) when $f_{x_1}$ is an objective criterion, or from $\sum_{x_2=1}^{m_{x_1}} w_{x_1x_2}\cdot r_{x_1x_2t}$ if $x_2\neq 0$. $\sum_{x_1=1}^{m} w_{x_1}\cdot r_{x_1t}$ denotes the final additive weighted evaluation value ($r_t$) of the $x_1$th major criterion ($f_{x_1}$) from product $t$. The better the alternative, the higher the evaluation value.

## 4. NUMERICAL EXAMPLE

Suppose three decision makers ($D_v$, $v$=1,…,3) are responsible for evaluating the risk of lending money to twelve companies ($A_t$, $t$=1,…,12) under the criteria of a four level hierarchical structure, where criteria are categorized to objective and subjective ones. Objective criteria in the final level of the structure include current ratio ($f_{1111}$, B), profit margin ($f_{1121}$, B), debt ratio ($f_{1131}$, C) and inventory turnover ($f_{1132}$, B). Subjective criteria in the final level of the structure include ROE ($f_{1122}$, B), inventory turnover ($f_{1141}$, B), receivables turnover ($f_{1142}$, B), management's experience ($f_{121}$, B), stockholders' structure type ($f_{122}$, B), organizational structure ($f_{123}$, B), competitiveness ($f_{131}$, B), equipment and technology ($f_{132}$, B), product marketability ($f_{133}$, B), collateral ($f_{134}$, B), potential of the industry ($f_{211}$, B), competition ($f_{212}$, C), economic growth ($f_{221}$, B), inflation rate ($f_{222}$, B), and exchange rate ($f_{223}$, B) (Velásquez, 2008). Suppose decision-makers use the linguistic rating set $S$={VP (very poor), P (poor), F (fair), G (good), VG (very good)}, where VP=(0.0,0.0,0.1,0.3), P=(0.0,0.2,0.3,0.5), F=(0.3,0.45,0.55,0.7), G=(0.5,0.7,0.8,1.0), and VG=(0.7,0.9,1.0,1.0), to evaluate the suitability of companies under qualitative criteria. Moreover, the decision-makers employ a linguistic weighting set $W$={UI (unimportant), SI (slightly important), FI (fairly important), I (important), VI (very important)}, where UI=(0.0,0.0,0.1,0.3), SI=(0.0,0.2,0.3,0.5), FI=(0.3,0.45,0.55,0.7), I=(0.5,0.7,0.8,1.0), and VI=(0.7,0.9,1.0,1.0), to assess the importance of criteria. Further suppose importance weights assigned by decision makers for the criteria are presented in Table 1. These linguistic values are defuzzified by the COA through Eqs. (1)~(3) and the average weight of each criterion can be obtained by Eq. (5) as also presented in Table 1. The suitability ratings of alternatives versus qualitative criteria are presented in Table 2 and these values are defuzzified by the COA through Eqs. (1)~(3). The average ratings can be obtained by Eq. (6) as also presented in Table 2. Ratings of alternatives versus quantitative criteria are displayed in Table 3 and normalized values can be produced through Eq. (7). By Eqs. (8)~(9), the final additive weighted evaluation values of all the companies can be obtained as also presented in Table 3. Table 3 shows company $A_4$ obtained the highest evaluation value whereas $A_2$ produced the lowest evaluation score. This denotes $A_4$ has the lowest level of risk and $A_2$ has the highest risk.

A Monte-Carlo simulation is conducted to gain some insight into the proposed model. The simulation is done with the software Crystal Ball 7.2, the number of trials is 100,000, the confidence level is 99%. For simplicity the importance weights are assumed to be constant and only the ratings of alternatives versus qualitative and quantitative criteria are assumed to vary. Variables (criteria) involved in the simulation are assumed to move independently. The distribution of the four quantitative criteria is assumed to be normal. Suppose each average alternative suitability versus each qualitative criterion follows a truncated triangular distribution with a minimum of 0.1 (lowest deffuzzified average ratings), a maximum of 0.9 (highest deffuzzified average ratings) and a likeliest value obtained from the average of the twelve alternatives. The distribution of the final evaluation values may range from 2.26 to 4.68 with a mean and a medial of 3.49. It is also shown that the results of the simulation can be fit to a beta distribution with alpha = 97.35, minimum = -0.77 and maximum = 7.28. The sensitivity analysis shows that the four most sensitive criteria are the ROE ($f_{1121}$=11.3%), collateral ($f_{134}$=11.2%), competitiveness ($f_{131}$=7.1%) and management's experience ($f_{121}$=7.1%). Moreover, an eight-level risk evaluation or bucketing system is suggested by using the 12.5$^{th}$, 25$^{th}$, 34.5$^{th}$, 50$^{th}$, 62.5$^{th}$, 75$^{th}$, 87.5$^{th}$ and 100$^{th}$ percentiles of the simulation distribution. The evaluation buckets are presented in Table 4. The first level corresponds to the less risky companies whereas the eighth one corresponds to those that are on the brink of bankruptcy. The final evaluation values for the twelve firms obtained in Table 3 can be placed into the buckets as displayed in Table 4. Clearly companies like $A_4$, $A_5$ and $A_8$ that have very high evaluation values in this model can be classified in the first level and ranked as companies with a high payback capacity. Companies that had a low ranking such as $A_2$, $A_6$, $A_7$ and $A_{12}$, can be ranked as lacking payback capacity. However, the final decision of whether to grant a loan or not can not be determined by the model because different commercial banks have different risk policies and loan evaluators have also different perceptions about the potential clients that should be accepted and those that should be rejected. Large banks rarely grant loans to customers classified lower than the average level and smaller financial institutions might be more risk-taking and grant lower quality credits to attract more customers.

**Table 1. The importance weights of the criteria and the aggregated weights**

| Criteria | Decision-Makers | | | Average weights |
|---|---|---|---|---|
| | $D_1$ | $D_2$ | $D_3$ | |
| $f_1$ | VI | I | VI | 0.8500 |
| $f_2$ | FI | FI | I | 0.5833 |
| $f_{11}$ | VI | VI | VI | 0.9000 |
| $f_{12}$ | I | FI | FI | 0.5833 |
| $f_{13}$ | I | FI | I | 0.6667 |
| $f_{21}$ | FI | I | I | 0.6667 |
| $f_{22}$ | I | I | I | 0.7500 |
| $f_{111}$ | VI | VI | I | 0.8500 |
| $f_{112}$ | VI | I | VI | 0.8500 |
| $f_{113}$ | VI | VI | I | 0.8500 |
| $f_{114}$ | I | FI | I | 0.6667 |
| $f_{121}$ | I | I | I | 0.7500 |
| $f_{122}$ | FI | SI | FI | 0.4167 |
| $f_{123}$ | FI | UI | FI | 0.3667 |
| $f_{131}$ | I | I | I | 0.7500 |
| $f_{132}$ | FI | FI | FI | 0.5000 |
| $f_{133}$ | I | I | FI | 0.6667 |
| $f_{134}$ | VI | VI | VI | 0.9000 |
| $f_{211}$ | I | I | I | 0.7500 |
| $f_{212}$ | I | I | FI | 0.6667 |
| $f_{221}$ | I | I | I | 0.7500 |
| $f_{222}$ | I | I | VI | 0.8000 |
| $f_{223}$ | VI | VI | VI | 0.9000 |
| $f_{1111}$ | VI | VI | VI | 0.9000 |
| $f_{1121}$ | VI | VI | VI | 0.9000 |
| $f_{1122}$ | VI | VI | VI | 0.9000 |
| $f_{1131}$ | VI | VI | VI | 0.9000 |
| $f_{1132}$ | VI | VI | VI | 0.9000 |
| $f_{1141}$ | I | VI | I | 0.8000 |
| $f_{1142}$ | I | I | I | 0.7500 |

**Table 2. Ratings of alternatives versus qualitative criteria and aggregated ratings**

| *C.* | *A.* | $D_1$ | $D_2$ | $D_3$ | $x_{ij}$ | *C.* | *A.* | $D_1$ | $D_2$ | $D_3$ | $x_{ij}$ | *C.* | *A.* | $D_1$ | $D_2$ | $D_3$ | $x_{ij}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f_{1122}$ | $A_1$ | P | P | P | 0.25 | $f_{123}$ | $A_1$ | VG | VG | VG | 0.90 | $f_{211}$ | $A_1$ | F | P | P | 0.33 |
| | $A_2$ | VP | P | P | 0.20 | | $A_2$ | P | P | P | 0.75 | | $A_2$ | F | F | F | 0.50 |
| | $A_3$ | VP | P | VP | 0.15 | | $A_3$ | F | G | VG | 0.72 | | $A_3$ | VG | G | G | 0.80 |
| | $A_4$ | G | VG | VG | 0.85 | | $A_4$ | G | VG | VG | 0.85 | | $A_4$ | VG | VG | VG | 0.90 |
| | $A_5$ | F | F | F | 0.50 | | $A_5$ | G | G | VG | 0.80 | | $A_5$ | VG | VG | VG | 0.90 |
| | $A_6$ | VP | P | P | 0.20 | | $A_6$ | G | F | G | 0.67 | | $A_6$ | P | P | F | 0.33 |
| | $A_7$ | VP | P | P | 0.20 | | $A_7$ | G | G | G | 0.75 | | $A_7$ | G | F | F | 0.58 |
| | $A_8$ | F | G | F | 0.58 | | $A_8$ | VG | VG | VG | 0.90 | | $A_8$ | VG | VG | VG | 0.90 |
| | $A_9$ | VP | P | P | 0.20 | | $A_9$ | F | F | G | 0.58 | | $A_9$ | VG | G | G | 0.80 |
| | $A_{10}$ | VP | P | VP | 0.15 | | $A_{10}$ | VG | G | G | 0.80 | | $A_{10}$ | F | G | G | 0.67 |
| | $A_{11}$ | VP | P | P | 0.20 | | $A_{11}$ | F | G | G | 0.67 | | $A_{11}$ | G | F | F | 0.58 |
| | $A_{12}$ | VP | P | P | 0.20 | | $A_{12}$ | F | F | P | 0.42 | | $A_{12}$ | G | F | F | 0.58 |
| $f_{1141}$ | $A_1$ | G | VG | G | 0.80 | $f_{131}$ | $A_1$ | G | G | G | 0.75 | $f_{212}$ | $A_1$ | VG | G | G | 0.20 |
| | $A_2$ | F | F | F | 0.50 | | $A_2$ | P | F | F | 0.42 | | $A_2$ | VG | G | G | 0.20 |
| | $A_3$ | P | P | VP | 0.20 | | $A_3$ | F | G | F | 0.58 | | $A_3$ | F | P | G | 0.50 |
| | $A_4$ | G | G | G | 0.75 | | $A_4$ | VG | VG | VG | 0.90 | | $A_4$ | P | VP | VP | 0.85 |
| | $A_5$ | F | G | G | 0..67 | | $A_5$ | G | G | G | 0.75 | | $A_5$ | F | P | P | 0.67 |
| | $A_6$ | F | F | F | 0.50 | | $A_6$ | P | F | P | 0.33 | | $A_6$ | VG | G | G | 0.20 |
| | $A_7$ | P | P | P | 0.25 | | $A_7$ | F | F | F | 0.50 | | $A_7$ | F | F | F | 0.50 |
| | $A_8$ | G | G | G | 0.75 | | $A_8$ | VG | G | G | 0.80 | | $A_8$ | P | VP | VP | 0.85 |
| | $A_9$ | F | F | F | 0.50 | | $A_9$ | F | F | F | 0.50 | | $A_9$ | G | F | F | 0.42 |
| | $A_{10}$ | G | VG | VG | 0.85 | | $A_{10}$ | G | VG | G | 0.80 | | $A_{10}$ | G | P | G | 0.33 |
| | $A_{11}$ | F | F | F | 0.50 | | $A_{11}$ | F | F | F | 0.50 | | $A_{11}$ | G | F | F | 0.42 |
| | $A_{12}$ | G | F | G | 0.67 | | $A_{12}$ | F | G | G | 0.67 | | $A_{12}$ | F | F | F | 0.75 |
| $f_{1142}$ | $A_1$ | G | G | G | 0.75 | $f_{132}$ | $A_1$ | VG | G | G | 0.80 | $f_{221}$ | $A_1$ | F | F | F | 0.50 |
| | $A_2$ | P | F | P | 0.33 | | $A_2$ | G | G | G | 0.75 | | $A_2$ | F | G | F | 0.58 |
| | $A_3$ | G | G | G | 0.75 | | $A_3$ | G | G | G | 0.75 | | $A_3$ | P | F | P | 0.33 |
| | $A_4$ | F | G | F | 0.58 | | $A_4$ | VG | VG | VG | 0.90 | | $A_4$ | F | G | G | 0.67 |
| | $A_5$ | VG | VG | VG | 0.90 | | $A_5$ | VG | G | G | 0.80 | | $A_5$ | F | G | G | 0.67 |
| | $A_6$ | F | G | F | 0.58 | | $A_6$ | G | VG | G | 0.80 | | $A_6$ | P | F | G | 0.50 |
| | $A_7$ | F | G | G | 0.67 | | $A_7$ | F | F | F | 0.50 | | $A_7$ | P | F | F | 0.42 |
| | $A_8$ | F | F | F | 0.50 | | $A_8$ | VG | VG | VG | 0.90 | | $A_8$ | F | F | F | 0.50 |
| | $A_9$ | P | F | P | 0.33 | | $A_9$ | G | F | F | 0.58 | | $A_9$ | P | F | F | 0.42 |
| | $A_{10}$ | P | F | F | 0.42 | | $A_{10}$ | VG | VG | G | 0.85 | | $A_{10}$ | F | G | G | 0.67 |
| | $A_{11}$ | F | G | G | 0.67 | | $A_{11}$ | F | F | F | 0.50 | | $A_{11}$ | F | G | G | 0.67 |
| | $A_{12}$ | G | F | G | 0.67 | | $A_{12}$ | G | F | F | 0.58 | | $A_{12}$ | P | F | P | 0.33 |
| $f_{121}$ | $A_1$ | G | G | G | 0.75 | $f_{133}$ | $A_1$ | F | F | F | 0.50 | $f_{222}$ | $A_1$ | P | VP | P | 0.20 |
| | $A_2$ | G | VG | G | 0.80 | | $A_2$ | F | G | F | 0.58 | | $A_2$ | P | VP | P | 0.20 |
| | $A_3$ | F | F | G | 0.58 | | $A_3$ | G | G | F | 0.67 | | $A_3$ | P | F | F | 0.42 |
| | $A_4$ | VG | G | VG | 0.85 | | $A_4$ | VG | VG | VG | 0.90 | | $A_4$ | P | F | F | 0.42 |
| | $A_5$ | G | G | G | 0.75 | | $A_5$ | F | G | G | 0.67 | | $A_5$ | P | P | F | 0.33 |
| | $A_6$ | VG | VG | G | 0.85 | | $A_6$ | P | P | F | 0.33 | | $A_6$ | P | F | P | 0.33 |
| | $A_7$ | F | G | G | 0.67 | | $A_7$ | G | G | F | 0.67 | | $A_7$ | F | F | F | 0.50 |
| | $A_8$ | VG | VG | G | 0.85 | | $A_8$ | VG | VG | VG | 0.90 | | $A_8$ | P | F | F | 0.42 |
| | $A_9$ | F | G | G | 0.67 | | $A_9$ | F | F | F | 0.50 | | $A_9$ | P | F | F | 0.42 |
| | $A_{10}$ | G | G | G | 0.75 | | $A_{10}$ | F | G | F | 0.58 | | $A_{10}$ | P | P | P | 0.25 |
| | $A_{11}$ | F | F | F | 0.50 | | $A_{11}$ | F | F | G | 0.58 | | $A_{11}$ | F | F | F | 0.50 |
| | $A_{12}$ | F | P | F | 0.42 | | $A_{12}$ | F | G | F | 0.58 | | $A_{12}$ | F | F | F | 0.50 |
| $f_{122}$ | $A_1$ | G | VG | G | 0.80 | $f_{134}$ | $A_1$ | VG | VG | VG | 0.90 | $f_{223}$ | $A_1$ | VP | VP | VP | 0.10 |
| | $A_2$ | G | G | G | 0.80 | | $A_2$ | G | VG | G | 0.80 | | $A_2$ | VP | P | VP | 0.15 |
| | $A_3$ | G | G | G | 0.75 | | $A_3$ | G | G | G | 0.75 | | $A_3$ | F | G | G | 0.67 |
| | $A_4$ | G | G | VG | 0.80 | | $A_4$ | VG | VG | VG | 0.90 | | $A_4$ | F | F | G | 0.58 |
| | $A_5$ | G | G | G | 0.75 | | $A_5$ | VG | VG | VG | 0.90 | | $A_5$ | G | G | G | 0.75 |
| | $A_6$ | P | F | P | 0.33 | | $A_6$ | P | P | P | 0.25 | | $A_6$ | VP | VP | VP | 0.10 |
| | $A_7$ | G | G | F | 0.67 | | $A_7$ | G | G | G | 0.75 | | $A_7$ | F | F | F | 0.50 |
| | $A_8$ | VG | VG | VG | 0.90 | | $A_8$ | VG | VG | VG | 0.90 | | $A_8$ | F | F | G | 0.58 |
| | $A_9$ | G | G | G | 0.75 | | $A_9$ | G | G | G | 0.75 | | $A_9$ | F | F | F | 0.50 |
| | $A_{10}$ | G | G | G | 0.75 | | $A_{10}$ | G | G | G | 0.75 | | $A_{10}$ | F | F | F | 0.50 |
| | $A_{11}$ | G | G | G | 0.75 | | $A_{11}$ | VG | VG | VG | 0.90 | | $A_{11}$ | G | G | G | 0.75 |
| | $A_{12}$ | G | F | G | 0.67 | | $A_{12}$ | P | P | VP | 0.20 | | $A_{12}$ | G | F | G | 0.67 |

**Table 3. Ratings of alternatives and evaluation values**

| A. | Quantitative criteria | | | | Evaluation values |
|---|---|---|---|---|---|
| | $f_{1111}$ | $f_{1121}$ | $f_{1131}$ | $f_{1132}$ | |
| $A_1$ | 1.8000 | 0.0130 | 0.7220 | 1.2000 | 3.4961 |
| $A_2$ | 1.2000 | -0.0830 | 0.7160 | -0.1000 | 2.8068 |
| $A_3$ | 1.7000 | 0.0070 | 0.7150 | 1.1000 | 3.4193 |
| $A_4$ | 2.1000 | 0.3070 | 0.2560 | 20.8000 | 5.9847 |
| $A_5$ | 1.4000 | 0.0780 | 0.4440 | 6.7000 | 4.5955 |
| $A_6$ | 3.1000 | 0.0000 | 0.8950 | 1.0000 | 2.8138 |
| $A_7$ | 0.8000 | -0.0370 | 0.7010 | 0.6000 | 3.0458 |
| $A_8$ | 1.2000 | 0.1420 | 0.5720 | 2.6000 | 4.6243 |
| $A_9$ | 2.7000 | 0.0160 | 0.5780 | 1.3000 | 3.4386 |
| $A_{10}$ | 0.4000 | 0.0290 | 0.4710 | 1.2000 | 3.5329 |
| $A_{11}$ | 0.9000 | 0.0130 | 0.6130 | 2.0000 | 3.4489 |
| $A_{12}$ | 1.5000 | -0.0610 | 0.7790 | -0.4000 | 2.9846 |

**Table 4. Risk buckets**

| Level | Values | A. and values |
|---|---|---|
| 1 | > 3.87 | $A_4$, $A_5$, $A_8$ (5.9847, 4.5955, 4.6243) |
| 2 | 3.71 – 3.87 | |
| 3 | 3.59 – 3.70 | |
| 4 | 3.50 – 3.58 | $A_{10}$ (3.5329) |
| 5 | 3.41 – 3.49 | $A_1$, $A_{11}$, $A_9$, $A_3$ (3.4961, 3.4489, 3.4386, 3.4193) |
| 6 | 3.30 – 3.40 | |
| 7 | 3.13 – 3.29 | |
| 8 | < 3.13 | $A_7$, $A_{12}$, $A_6$, $A_2$ (3.0458, 2.9846, 2.8138, 2.8068) |

## 5. CONCLUSIONS

A defuzzification based fuzzy multiple levels MCDM model is proposed for the evaluation of commercial loans, where the importance weights and the ratings of subjective criteria are assessed by linguistic values represented by fuzzy numbers. These fuzzy numbers are defuzzified through COA before applying to the model to avoid the multiplying more than two fuzzy numbers. The averaged weights and ratings are aggregated going from the lowest-level to the highest to obtain the final evaluation values of alternatives. The proposed model deals with a multiple level hierarchical criteria structure and has the advantage of considering qualitative and quantitative criteria. A numerical example has demonstrated the feasibility of the proposed model. A Monte Carlo simulation is conducted to gain more insight about the model behavior.

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