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Control over Limited-Stroke Suspensions using Jerk

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ABSTRACT

This paper investigates a suspension control that applies Pontryagin's theorem to the problem of returning a system to equilibrium in minimum time. This can be viewed as a special case of a sliding-mode control. Sliding-mode control is known for "chattering" around the switching plane. By using jerk (the rate-of-change of acceleration) as the control variable, chattering on the switching manifold becomes simply small changes in acceleration, rather than highly uncomfortable extreme changes in acceleration.

Evolutionary algorithms were used to evolve suboptimal controls that could be used for a fair test of relative performance. The use of evolutionary algorithms, using computer power running over a large number of models, provides a testing regime where analytical techniques are impossible.

1. INTRODUCTION

Electronically controlled suspensions have been studied since at least the early 1970s (Crosby and Karnopp, 1973). Since the 1990s, accelerometers and microprocessors have become much smaller and cheaper, and they have improved in performance, making electronic control a viable proposition for production vehicles.

There has been an enormous amount of research into the theoretical underpinnings of passive (non electronically controlled) suspensions (Barak, 1991; Bastow et al., 2004; Milliken and Milliken, 1995). However, some aspects of suspension design need to become more rigorous in the face of the flexibility of electronic control. For example, a number of experimental suspensions are subject to suddenly changing forces. Such a force discontinuity is clearly visible in Crosby and Karnopp's control, in their early paper of 1973 (1973, p125). On-off control is very simple to implement, and for some optimization problems it is even optimal. But such controls (using acceleration) result in sudden force changes which are immediately obvious to a passenger and are highly uncomfortable.

In a relatively large study of passenger perceived comfort, involving 18 roads and 78 passengers, Smith et al. recommended RMS acceleration of chassis movements as a predictor of ride comfort (Smith et al., 1978). Such a measure however will not penalize systems that suddenly change force (between moderate values). The use of jerk as a measure of comfort does not have this defect, and this has been used by a number of researchers (Hrovat and Hubbard, 1981; Hrovat and Hubbard, 1987; Paddison et al., 1994; Hashiyama et al., 1995; Ahmadian and Vahdati, 2003; Ahmadian et al., 2004; Ahmadian and Vahdati, 2006; Yamakado and Abe, 2006).

While minimum-time controls using acceleration suffer greatly from the defect mentioned above, this no longer becomes a problem when jerk is used as the control parameter. This paper is a first-order evaluation of the possibility of using minimum-time control over jerk as a basis for suspension control.

Two algorithms are included as bench marks: the truly classical, purely linear passive suspension, and the ubiquitous skyhook control, which has a damping force in proportion to chassis velocity (Karnopp, 1995; Burton, 1993; Paddison et al., 1994; Reichert, 1997; Elbeheiry and Karnopp, 1996; McLellan, 1998; Wagner and Liu, 2000; Goncalves and Ahmadian, 2002; Ahmadian et al., 2004; Donahue, 2001; Song et al., 2003; Song and Ahmadian, 2004; Stembridge et al., 2006; Li et al., 2004; Williams and Best, 1994; Hyvärinen, 2004; Krüger, 2002; Caponetto et al., 2003; Guglielmino et al., 2008, p70). Analysis of the skyhook's transmissibility response implies that it is superior to the passive suspension (Reichert, 1997, p9).

As well as these benchmarks, a number of different suspension control algorithms are also run in evolutionary algorithms (EAs) under the same conditions, to produce suboptimal control parameters. The final weighted performance measure of the final generation, after "cooling", is used as the suspension's overall performance measure. A relatively modest single-DOF numerical physical model is used to represent the control between the chassis and the wheel. There are two types of controls, active and semi-active controls, explained below. All active controls output force to exactly the same numerical package, and similarly all semi-active controls output damping rate to another numerical package. This is similar to a testing method used previously by the authors (Storey et al., 2008).

2. PERFORMANCE MEASURES

In all controls presented here, the chassis vertical movement is designated as *y*, and is a function of time, $y(t)$. Road/wheel movement is similarly a function of time, represented as $r(t)$. The suspension stroke, $s(t)$, is given by,

$$s(t) = y(t) - r(t). \qquad (1)$$

The positive direction is up.

Jerk is defined as the rate-of-change of acceleration (equivalently, the third-order rate-of-change of distance). Chassis jerk is represented as $\dddot{y}(t)$. The integral of the fourth power of jerk is used as the comfort measure,

$$J_C = \int_0^T \dddot{y}(t)^4 \, dt. \qquad (2)$$

The fourth power measure used here gives higher weight to larger jerks than the RMS measure. This is multiplied by a scaling factor and subtracted from 1,000. Thus the highest possible comfort score is 1,000.

An active suspension that had no vertical chassis movement at all, $y(t) \equiv 0$, is the most comfortable control possible, but it is clearly absurd. Such a control would quickly show its defects when encountering a bump that caused it to hit violently against the limits of the suspension travel, either “bottoming out” or lifting the wheel from the ground. The space within which the suspension stroke can easily move is a natural suspension limit, designated the “rattlespace” by some authors (Burton, 1993; Hrovat and Hubbard, 1981; Hyvärinen, 2004; Takahashi et al., 2000). This constraint can be represented as,

$$|s| \le R, \qquad (3)$$

where R is half the distance covered by the rattlespace. Generally, this constraint is not included in analysis.

Optimization via EAs allows the luxury of applying performance measures that are appropriate to the problem; we are not restricted to measures that are analytically amenable, such as quadratic measures. However, while the constraint of equation 3 is natural to the suspension problem, for the purposes of optimization and numerical stability, the performance measure used in this paper provides penalties for straying close to the rattlespace limits as well as large penalties for excursions outside the rattlespace. There are other factors affecting this decision having to do with the fact that drivers slow down at the approach to large bumps (Storey et al., 2008). The following formula is used as the measure of the capacity of the suspension to resist hitting against the rattlespace limits,

$$J_R = \int_0^T \varphi(s(t))\, dt, \qquad (4)$$

where $\varphi(s)$ is a nonlinear function that penalizes travel outside or near the rattlespace limits. The function has the shape shown in figure 1.

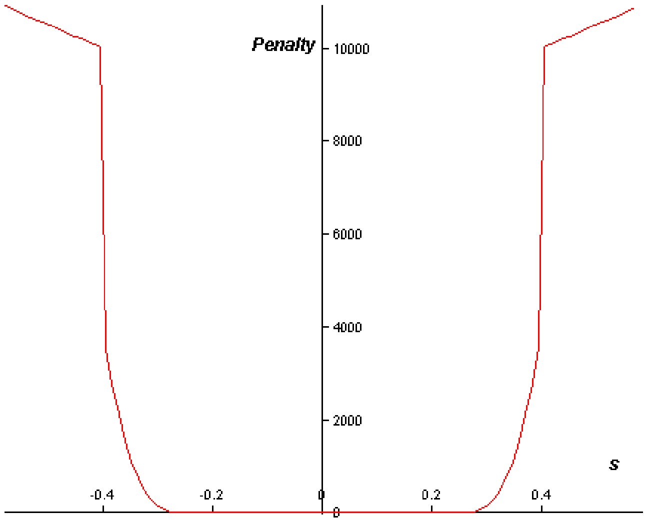

Figure 1. Rattlespace Penalty

The formula used is,

$$\varphi(s) = \begin{cases} 0 & \text{if } |s| < m, \\ P_1 \dfrac{|s| < m}{R - m} & \text{if } m < |s| < R, \\ P_2 + (|s| - R)T & \text{if } |s| > R. \end{cases} \qquad (5)$$

This is a similar function to a “nonlinear filter” used by Lin and Kanellakopoulos (Lin and Kanellakopoulos, 1997, p51), although they used it for a somewhat different purpose. Again, this measure, J_R, is multiplied by a scaling factor and subtracted from 1,000. The two formulas can be weighted and added to give an overall score. Optimization has been performed along Pareto fronts in some early parts of the evolution, to encourage variation, but it has been found that citing the separate scores for comfort and stroke movement, as well as weighted sums, produces a simpler snapshot of the overall performance, at least when a range of algorithms are being compared.

3. ROADS

Large bumps must be included in the road surfaces used in the EAs otherwise the evolutionary process will result in soft suspensions. The method used includes large bumps with a range of heights, randomly determined, and lengths, also randomly determined. Sudden changes in road slope produce a force discontinuity in a damper, since the damping rate changes quickly. In the roads used for testing suspensions in the EAs, a small number of bumps with sudden slope changes are combined with bumps with smooth slope changes.

Furthermore, periods of small corrugations are included to test the capacity of a suspension to handle corrugations without over-reacting to the fast changes in road height and slope. Roads are run for 100 seconds. An example of a random road segment combining all these factors is shown in figure 2.

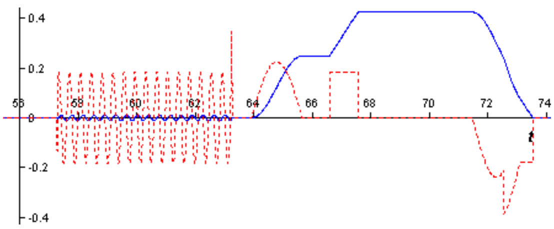

Figure 2. Road Height Example

The vehicle parameters are approximately those quoted for a medium size car, using a quarter vehicle weight of 250Kg.

4. MINIMUM-TIME CONTROL

A variation on a control that has been the subject of research for industrial robotic arm movement is presented here. The control is based around the analytical solution of the problem of returning a system to rest in a minimum time. Applying Pontryagin's principle, the solution to such problems involves bang-bang controls (MacCluer, 2005, p120). A search has found that bang-bang controls over jerk have been proposed for the control of industrial robot arms (Muenchhof and Singh, 2003; Koh et al., 1999; Kyriakopoulos and Saridis, April 1988). However, the final rest position for robotic arm manipulators is not constantly changing, and the environment is quite static. This kind of control has not before been proposed as the basis for suspension systems, to the best of the authors' knowledge.

If acceleration or jerk is unconstrained, we can return a system to rest in an infinitely small time using an infinite force or jerk, highlighting the fact that a badly-framed control problem can produce absurdities. The control for a return to rest using constrained acceleration however is a well-known textbook control, and is elegantly represented via a phase-plane diagram as in figure 3.

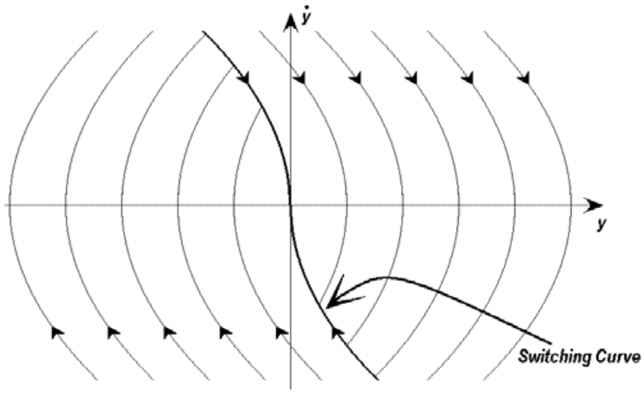

Figure 3. Acceleration Control Phase Plane

The phase-plane diagram for the bang-bang control over jerk on the other hand is three dimensional and is more difficult to visualize. The control generally involves two switches rather than one. An example of a minimum-time control over jerk is shown in figure 4. Note that acceleration, velocity and distance reach equilibrium simultaneously and in a finite time, and the acceleration is continuous. Also the rate-of-change of acceleration switches at two points (excluding the end points).

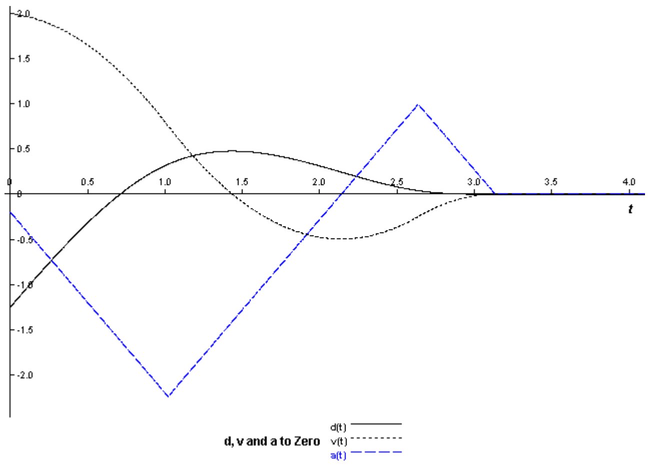

Figure 4. Minimum-Time Jerk Control Example

That this is optimal can be proved in much the same manner used by MacCluer to show that one switch is needed for minimum-time control over acceleration (MacCluer, 2005). A different technique given by Hermes and LaSalle (Hermes and LaSalle, 1969) yields the same result.

To minimize time, minimize the functional,

$$Q = \int_0^T 1 dt, \qquad (6)$$

where the control jerk is constrained,

$$-j \le u \le j. \qquad (7)$$

The equation of motion in vector form is,

$$\dot{\mathbf{x}} = F(\mathbf{x}) = F(x, v, a) = (v, a, u). \qquad (8)$$

That is to say, $u = \dot{a}, a = \dot{v}$ and $v = \dot{x}$. Here *x*, *v* and *a* represent distance, velocity and acceleration respectively.

The Hamiltonian is,

$$\begin{aligned} H &= -1 + \lambda . F = -1 + (\alpha, \beta, \gamma).(v, a, u) \\ &= -1 + \alpha v + \beta a + \gamma u. \end{aligned} \qquad (9)$$

In order to optimize the Hamiltonian we require,

$$u = \begin{cases} -j & \text{if } \gamma < 0 \\ j & \text{if } \gamma > 0 \end{cases}. \qquad (10)$$

The adjoint equation is,

$$\begin{aligned} (\dot{\alpha}, \dot{\beta}, \dot{\gamma}) &= \dot{\lambda} = -H_{\mathbf{x}} \\ &= -\left(\frac{\partial H}{\partial x}, \frac{\partial H}{\partial v}, \frac{\partial H}{\partial a} \right) = (0, -\alpha, -\beta). \end{aligned} \qquad (11)$$

Solving this differential equation produces,

$$\begin{aligned} \alpha &= \alpha_0, \\ \beta &= -\alpha_0 t + c_1, \\ \gamma &= -\tfrac{1}{2}\alpha_0 t^2 + c_1 t + c_2, \end{aligned} \qquad (12)$$

where α_0, c_1 and c_2 are constants. Since the equation for γ is quadratic and changes sign at most twice we can deduce from equations 9 and 10 that there are at most two switches for the control, *u*. That is to say, there are at most two points at which the jerk switches between *j* and $-j$ or vice versa. A different form of argument is used by Ben-Itzhak and Karniel, albeit for a slightly different purpose; they optimized RMS acceleration over a given finite time period, with constrained jerk as the control (Ben-Itzhak and Karniel, 2008).

To find the optimal trajectory, we deduce the control from the end condition. In order to reach rest, the control after the first switch must bring velocity and acceleration to zero in exactly the same way that acceleration brings velocity and distance to zero in the control over acceleration. Thus we define a two-dimensional plane made up of the controls that bring the system back to rest using just one switch. The projection of the control trajectories on this surface are exactly the same as the phase-plane diagram over acceleration, figure 3, except with the distance axis replaced by velocity, and the velocity axis replaced by acceleration. This surface contains all the points that can reach the origin using just one switch. The sliding-mode analogue to this surface could be called the “switching surface” or “switching manifold”, but it has also been called the “landing domain” in the literature on time-optimal control for robotic manipulators (Koh et al., 1999).

The distance to the landing plane does not need to be calculated however to use the algorithm if it is applied on a moment-to-moment basis. We simply calculate the distance of the point on the switching curve with current velocity and acceleration values, which is easily calculated. This distance is compared with the current displacement, and jerk is applied in a direction that projects back to the plane. This method has been used in numerical experiments aimed at application with robotic manipulators (Koh et al., 1999).

The minimum-time control over acceleration is similar to sliding-mode suspension control (Ashari, 2004; Donahue, 2001; Yagiz, 2005; Dixit and Buckner, 2005; Yokoyama et al., 2001). It has been recognized that the major problem facing pure sliding mode is the same problem faced by minimum-time control over acceleration: sudden changes in acceleration. This may happen repeatedly and has been referred to as “chattering” (Ashari, 2004). Note that this problem is completely invisible to the comfort measure of RMS acceleration.

In sliding-mode controls, techniques are employed to smooth the movement around the sliding surface (Ashari, 2004, p371; Dixit and Buckner, 2005, p93; Liu et al., 2005, p1029; Yokoyama et al., 2001, p2654; Young et al., 1999). Instead of applying the same techniques to minimum-time control over acceleration, we can employ a minimum-time control over jerk. Minimum-time control over jerk does not suffer from a spike in jerk. The acceleration changes smoothly with changes in jerk between moderate values.

To adapt the minimum-time control to a suspension, we estimate the road equilibrium height. This is complicated by the fact that the road is constantly changing, particularly over corrugated surfaces. For this experiment, we use the current road height, and some constant fraction of road velocity (we have assumed that current road acceleration is next to worthless as a predictor), and we let the EA determine the strength of the velocity multiplier.

Thus, at any time, t_0, we use the following estimates of distance, velocity and acceleration:

$$s = y(t_0) - r(t_0),$$
$$\dot{s} = \dot{y}(t_0) - \alpha\dot{r}(t_0), \quad (13)$$
$$\ddot{s} = \ddot{y}(t_0),$$

where α is a parameter to be determined by the EA. The jerk direction is calculated using the minimum-time method with these values as the initial values. This is repeatedly computed for each small step time; 10ms was used in the simulations.

This method can be tested using flat road surfaces. This has been thoroughly tested and gives close agreement with the pure minimum-time method returning to rest (see figure 5). The approximation is near perfect with smaller numerical step sizes.

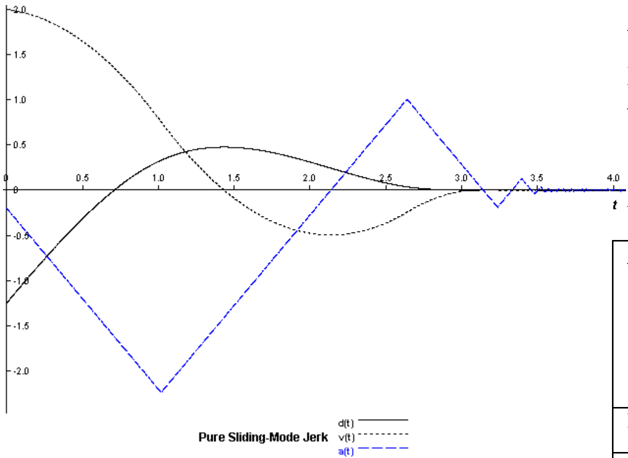

Figure 5. Continuous Control

We also experimented with a control over jerk in which unnecessary increases of acceleration on the switching plane were limited, as shown in figure 6. The control is otherwise the same as the minimum-time control over jerk. The acceleration limit is an extra parameter of the method.

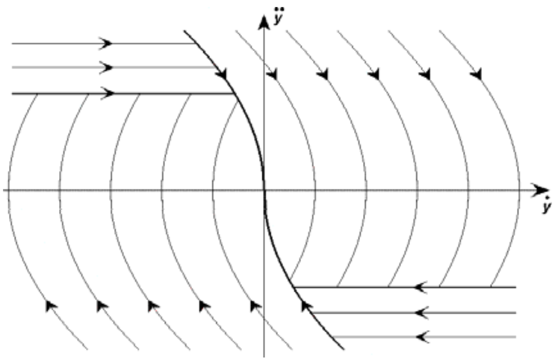

Figure 6. Acceleration Limit

Furthermore, the skyhook and the minimum-time control over jerk were adapted for semi-active suspensions using simple clipping of the control within the constraints of the controlled damper. Maximum and minimum damping rates were included as parameters for the EAs.

5. RESULTS

The overall performance measures for the various controls are given in table 1. The minimum time control over acceleration performs extremely poorly because of “chattering”. The minimum-time control over jerk performs worse than the purely linear passive control. To some extent this may be due to the fact that the method exhibits chattering even over smooth portions of road. This was ameliorated somewhat by the use of the acceleration limit on the switching plane.

Because of the differences in the ways the two performance indexes were calculated, the following values were weighted with a jerk factor of 0.005 and a rattlespace factor of 5,000. This has been found to give representative performance in previous experiments.

What is interesting is that the semi-active version of the clipped version of the minimum-time control (without acceleration limit) performs better than the semi-active skyhook.

Algorithm	**Type**	**Performance Measures**		
		Median Weighted Sum	**Median Jerk**	**Median Rattlespace**
Linear Skyhook	Active	992.17	984	1,000
Minimum Time, clipped semi-active	Semi Active	949.48	989	1,000
Minimum Time, acceleration limit	Active	880.9	847	914
Skyhook, clipped for semi-active	Semi Active	844.47	689	1,000
Linear Passive	Semi Active	841.49	715	968
Minimum Time, single jerk level	Active	37.6	869	-793.98
Minimum Time for acceleration	Active	-27,024,583	-22,390,017	-31,659,148

Table 1. Overall Results

The jerk applied by the highest performing gene using the basic minimum-time control over jerk is $3.7 ms^{-3}$. For the minimum-time jerk control with limited acceleration, the value was similar: $3.3 ms^{-3}$. The acceleration limit on the sliding plane is very low: $0.0012 ms^{-2}$.

6. CONCLUSIONS

These results show that the minimum-time control over jerk may be a viable basis for a suspension control. The control when modified for limited acceleration performs quite well.

The minimum-time control over jerk uses a very simple estimate of future equilibrium road height. It has been noted that the system performs relatively badly over corrugations. Perhaps a method of smoothing road corrugations could provide a better estimate of future road heights.

The method also suffers from jerk chattering, and while this is not as bad as acceleration chattering for comfort, it decreases the jerk performance indicator.

The semi-active version of the simple version of the minimum-time control performed better than the clipped skyhook. This may be due to the fact that when the suspension is moving away from equilibrium and is approaching the rattlespace limit, the minimum-time control is attempting to bring the suspension back to its centre in minimum time and is less subject to clipping. This seems to be one of those happy cases in which physics favours the engineer.

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