Investigation of the Thermodynamic Component of Penman's Method for Estimating Evaporation

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EXTENDED ABSTRACT

The study presented here was initially commissioned by the Sydney Catchment Authority (SCA) in 2004, in response to the independent recommendations for the improvement of evaporation estimates being used in SCA reservoir water balance models. A study by Sinclair Knight Merz (SKM) independently reviewed the WATHNET model used by the SCA, and suggested improvement of the basic "Pan Conversion" evaporation estimates used in the model. Best estimates suggest that evaporation can cause as much as 14 billion litres per month (14Ml/month) to be lost from water supply reservoirs in Sydney in the worst of conditions. This accounts for as much as 20% for worst months of all water lost from the main water supply reservoirs in Sydney, such as Warragamba Dam (Lake Burragorang) and Woronora Dam. These estimates are based on observations of historical Pan Evaporation data.

Evaporation is the largest loss component of water balance in open water reservoirs. However, it is difficult to estimate evaporation with any reasonable certainty. The Penman Method is universally acknowledged as one of the best physically based evaporation estimation methods to date (Penman, 1948, 1956). Use of this method on a continual basis is a difficult task due to limited data availability of a fairly large number of variables. These variables are a crude representation of the thermodynamic and aerodynamic properties of an open water surface.

Focusing on the thermodynamic component of the Penman Method, the study sought to reduce errors associated with the estimation of solar energy flux into a water surface, particularly in Sydney. In most applications of the Penman Method, a series of semi-physical and semi-empirical equations are used to derive net energy flux based on a solar radiation constant, air and water temperature readings, and a cloudiness coefficient, as the input variables.

This paper presents modifications to the Penman Method which make it suitable for use with high quality solar radiation measurements. These modifications involved exclusion of some empirical equations reported to have errors of up to ±20%, and resulted in a statistical improvement of general evaporation estimation.

The study also produces a set of solar radiation constants calibrated based on the results of the best evaporation estimates. These constants are useful substitutes for the frequent gaps in measured solar radiation data, and allow consistency for long periods of estimation.

Surprisingly, it was discovered that the semi-empirical approximations of solar radiation terms cause an average annual error of only 23mm, compared to the estimates derived from real measured solar radiation data.

This level of improvement may be useful where accuracy of evaporation estimates is an absolute priority, or where vast water surfaces mean that evaporation rate equates to the loss of a very large volume of water. The findings can also benefit cases where the resolution of evaporation estimates is critical, by improving accuracy for shorter time steps.

Two conclusions were drawn from this study which can benefit the use and understanding of the Penman Method in general. The first was that the semi-empirical equations suggested by some texts such as Thompson (1999), which are commonly used to approximate many of the inputs into the Penman Method, though seemingly far fetched and based on very limited data, do not result in any acute errors in the annual total evaporation estimate.

The second conclusion drawn from this study was that the Pan Conversion Method of estimating evaporation has a strong tendency to over estimate evaporation by as much as 7% if compared to the best Penman Method estimates made in this study.

1. INTRODUCTION

This paper primarily focuses on the Penman Method for estimating evaporation (Penman, 1948; 1956). It has undergone widespread use worldwide proving it to be a robust and comprehensive method of estimating evaporation losses from open water surfaces.

While there are many methods of estimating open water evaporation losses based on various theories, the Penman Method encompasses several of the most fundamental physical evaporation principles. It can be generalised as a combination of two key principles, aerodynamics and thermodynamics. Thermodynamics cover the requirement of energy as a driving force of vaporisation. While this may be a commonly known scientific fact, the distinction between vaporisation and evaporation is less commonly known. In laboratory conditions (no wind agitation), vaporisation and condensation would occur in equilibrium, with the volume of vapour at any given time being dependant upon the temperature of the system. Hence for evaporation to take place the second essential component is introduced, aerodynamics.

Aerodynamics encompasses the mechanisms of vapour transport from a water surface. Reducing vapour pressure above a water surface maintains a vapour pressure gradient in favour of vaporisation. It is the net vaporisation - the difference between the rate of vaporisation and condensation - which defines the true rate of evaporation.

According to Penman Method, evaporation fails to cease when wind speed is zero. However, given that the method does not account for convection currents by any direct means, this oversight is not entirely incorrect. In fact, the Penman Method's combination of aerodynamics and thermodynamics has been proven to match the best estimates of evaporation with remarkable accuracy.

The individual components of the Penman Method are by no means original with regards to the physical principles of evaporation. Aerodynamic evaporation theory precedes the Penman Method, appearing in earlier stand-alone aerodynamic estimation methods. Thermodynamics too is a firmly established evaporation science in its own right. Evaporation estimates derived from thermodynamic studies of large water bodies are collectively known as "Energy Budget" evaporation estimates. To date this is the most accurate method of estimating evaporation losses from open water bodies, a fact openly acknowledged by leading academic publications on the subject. This standing places the "Energy Budget" evaporation estimate as the closest thing to a true "measure" of evaporation loss rates. The results of such studies are very useful as benchmarks for the calibration of relatively simpler evaporation estimation methods, such as the Penman Method.

2. BACKGROUND

This paper is part of a much wider study addressing the many ways of improving evaporation estimates. The study was commissioned by the Sydney Catchment Authority (SCA) in 2004, in response to independent recommendations for the improvement of evaporation estimates being used in SCA models.

Given the severe drought conditions currently afflicting Sydney's water supply, the importance of accurate water balance modeling has increased by orders of magnitude. The results of such models have much further reaching influence. Forecasting Sydney's water supply is necessary to give appropriate notice of shortage, and support for political approval of alternative measures of water supply, such as desalination.

An old adage rightfully states that a model can only ever be as accurate as the information put into it. Hence, the notorious margins of error associated with evaporation estimates used in any water balance study can translate into and even procreate larger errors within water resources modeling results.

In addressing evaporation estimation, this paper addresses only a small part of the problem in the hope of contributing to more accurate water balance modeling capabilities in the future. Further still it focuses on only one of the many possible avenues for improving evaporation estimates. This is firstly by suggesting the Penman Method as a standard practice for estimation of evaporation in Sydney, but more specifically, improving the Thermodynamic component of the Penman Method by adopting real solar radiation measurements.

3. SITE AND DATA

This study required solar radiation measurements at the earth's surface on a daily basis, which was obtained from the Australian Bureau of Meteorology, measured at Observatory Hill, Sydney.

The study period covered January 1996 to May 2001 as this was the only common period of the available data across each of the input variables in the Penman Method, with priority given to solar radiation data. Gaps in data were filled with appropriate statistical methods based on recent trends, as well as seasonal or yearly trends (which ever was deemed most appropriate to match the nature of variability for each type of data).

A critical component of this study was the standard measure of "Class A" Pan evaporation. For comparison purposes, Class-A pan estimates and Penman estimates at the same location would be ideal.

Class-A Pan evaporation data was available in best record at Prospect Reservoir, in the center of the Sydney metropolitan area. For consistency all hydro-meteorological variables were collected for this location, as best as possible. Hydrometric variables however are not available at Prospect such as is required for this study. To substitute, hydrometric variables (e.g. water temperature etc.) were assessed for Warragamba and Woronora Dams, two large water bodies with sufficient spatial, altitudinal, and volumetric differences to represent a variety of open water bodies in Sydney. Comparisons suggested very little variation between the hydrometric variables of the two dams. Hence the differences were deemed insignificant and hydrometric variables for Warragamba were used in conjunction with the Hydro-meteorological variables for Prospect to create the study data set.

Further attempts were made to assess the spatial variability of all the hydro-meteorological variables used in this study. This would have been useful if it could have proved that all variables were suitably applicable to Warragamba Dam, Sydney's primary water supply reservoir. However, no useful patterns of spatial variability could be determined for any of the variables between any of the 3 sites (Warragamba, Woronora, and Prospect). Particularly, and somewhat unfortunately, no conclusions could be drawn regarding the applicability of Prospect atmospheric variables to the Warragamba site, other than to say that both typically follow Sydney's weather patterns with minor random spatial variability. The study area could not simply be moved to Warragamba (where both hydrometric and meteorological data are available), since the "Class A" evaporation pan was only available at Prospect, and the Penman estimates needed to be comparable to this as much as possible. This was accepted as a forced limitation to the study, imposed by a lack of all necessary data at any single location.

Data was obtained from two sources, the Sydney Catchment Authority, and the Australian Bureau of Meteorology. The SCA provided data that covered hydrometric variables for Warragamba and Woronora, as well as "Class A" Pan evaporation at Prospect.

4. METHODOLOGY

The Penman Method itself can be modularised based on the "intended" purpose of each of the equations in the method. Each "module" can be individually assessed on how well it achieves its purpose, and improved if possible.

As mentioned earlier, the Penman Method is considered to be a physically based method. This is true in that it is a relationship between evaporation and hydro-meteorological variables, based on physical principles. However this assumes that these variables, such as long and short wave solar radiation etc, can be measured accurately. In most cases, it is either not possible, or more often, not practical to measure these variables. In their places semi-empirical formulas are used, and this is where most of the weakness in the Penman Method currently lies.

This study maintained the physical theory shell, and addressed each internal component individually. In this way the Penman Method maintains its general structure, remaining true to its base theory. Changes to each component are verified by the effect on the overall method's performance. This provides some independence between the improvement and verification processes. For example, a change in one component might seem beneficial, however the performance of the overall method will suffer if it is a poor change, or if it somehow clashes with the fundamental theory. Examples of evidently poor changes are negative estimates, inability to maintain the Priestly Taylor relationship (Priestly and Taylor, 1972), and irregular or disproportionate behaviour with regards to short term and seasonal trends.

These semi-empirical additions are not truly part of the Penman Method. They are simply a means of estimating some of the inputs into the method.

The Penman Method can be generally represented by the following equation (Thompson, 1999):

$$E_o = \left[\frac{\Delta}{\Delta + \gamma}\right] E_n + \left[\frac{\gamma}{\Delta + \gamma}\right] E_a \qquad (1)$$

In this equation, E_o is the evaporation rate in cm/day; Δ is the slope of saturated vapour pressure per temperature; γ is the psychometric constant; E_n is the evaporation rate due to net all wave radiation (cm/day); and E_a is the evaporation due to aerodynamic forces (cm/day). It is easy to see how the method is a direct combination of two parts: thermodynamics (term with E_n part of Equation 1), and aerodynamics (term with E_a part of Equation 1). Each term expands to create a large hierarchy of equations. It should be noted here that the Penman Method adopted in this study was taken from Thompson (1999) in that Dalton's wind function (Dalton 1802, as cited in Thompson 1999) is recommended, and is used throughout this study to be consistent with Thompson (1999).

This paper addresses the term E_n (evaporation resulting from net allwave solar radiation). This term is derived simply from net allwave solar energy flux (Q_n) (calories/cm^2/day), the density of water at a given temperature (ρ) (g/cm3), and the latent heat of vaporisation of water L_v (calories / g):

$$E_n = \frac{Q_n}{\rho L_v} \qquad (2)$$

Further expansion of the net allwave solar energy term (Q_n) brings the thermodynamic component to the point where data input is required:

$$Q_n = Q_s - Q_{rs} - Q_{lw} \qquad (3)$$

In Equation 3, Q_s is the gross solar radiation, Q_{rs} is the solar radiation reflected off a water surface, and Q_{lw} represents the net longwave energy, which does not play a significant role in the process of evaporation, and hence is subtracted from the gross energy flux. These three terms are actually a simplification of the Energy Budget theory. The three terms considered in Equation 3 are those which are most influential on evaporation. To this point, the thermodynamics part of the Penman Method appears completely physical. It is beyond this point that the method is affected by the use of empirical relationships.

Rather than measuring each of the variables in Equation 3 on a daily basis, most applications of the Penman Method instead use equations which can estimate each of the variables based on empirical relationships. The temptation lies in that obtaining such specific radiation measurements requires expensive and very sensitive equipment. Empirical relationships on the other-hand can approximate them with reasonable accuracy based on much more rudimentary atmospheric variables, which are easily obtained in long record.

Some of the most recent texts (Thompson, 1999), quote empirical equations to estimate the variables in Equation 3. Some of these equations have purported confidence intervals of ±20%. While this is commendable for an empirical equation, it must be remembered that being empirical, these equations are not necessarily applicable to different climates and locations. Furthermore, accumulation of errors throughout the Penman Method greatly magnifies any uncertainties. For instance, if all 7 of the standard Penman input variables have a 20% error, elementary statistics suggest a joint probable error as high as 79%.

This paper presents a method that could identify and reduce the errors associated with the empirical means of approximating the solar radiation variables of Equation 3.

The key term in this equation is Q_s (gross solar radiation). One of the best empirical methods of estimating Q_s is presented in Thompson (1999) (Equation 4). This method uses solar radiation constants quoted by Dunne & Leopold (1978), and reduces them for atmospheric and cloud related losses based on an empirical formula. These atmospheric constants represent the average daily solar radiation energy projected onto the earth by the Sun for given latitudes and given month (average daily radiation for each month).

$$Q_s = I_o\left(0.803 - 0.340C - 0.458C^2\right) \qquad (4)$$

In Equation 4, solar radiation arriving at the earth (I_o) is reduced by 19.7% (on account of atmospheric deflection and absorption), and then reduced further as a polynomial function of cloudiness coefficient C. Cloudiness coefficient is available from the Australian Bureau of Meteorology, and is a measure of relative cloudiness on a scale of 0 to 8. Cloudiness coefficient is generally acknowledged as a ratio of measured daylight hours to maximum daylight hours for a given day in the year. Such a measure is clearly subjective, since clouds vary in height, consistency and density.

In this study, the entire Q_s term was replaced by actual daily measurements of solar radiation measured at Observatory Hill, Sydney. Radiation measured at the earth's surface inherently accounts for atmospheric and cloud losses. Hence, Q_s can be defined as radiation arriving at a water surface.

The term Q_{rs} benefits equally from this change, since it is simply Q_s multiplied by a constant fraction albedo (α), which represents radiation reflected off an open water surface at zenith angle. Albedo is a physical characteristic of water and only varies with angle and possible surface agitation. These variations were assessed in the wider study, but are outside the immediate scope of this paper.

Finally the term Q_{lw} remains. This is addressed by Thompson (1999), again using a complex semi-physical equation. While no alternatives could be rationally proposed to better this equation, some important observations were made.

Firstly, Q_s was replaced by the measured solar radiation data (I_m) and run through the Penman Method. It became immediately obvious that the Q_{lw} equation began to act disproportionately to the other components of the radiation balance (Equation 3). For cloudier days in the study period, Equation 3 was resulting in a negative net solar radiation as a result of the disproportion. In-depth analyses of the measured solar radiation (I_m) revealed that the inherent cloud losses of the radiation data did not match those implied by the cloudiness coefficients measured by the Bureau of Meteorology. In simple terms, the measure of cloud in the Q_s term and Q_{lw} term were not consistent. To reconcile this difference a new measure of cloudiness was "extracted" from the solar radiation data.

This new measure of relative cloudiness was termed "Radiation Deficit Cloudiness" throughout the study, and denoted as C_{rd}. It was called this because, rather than being derived from missing sunlight for a given day, as is traditionally the case, it was derived from the missing radiation, or "radiation deficit".

Radiation arriving at the earth's atmosphere is defined with acceptable accuracy by Dunne & Leopold's (1978) constants. These are a function of radiation travel through space and the earth's position relative to the sun. These can be assumed sufficiently constant and accurate for the purposes of this study. Dunne & Leopold's daily averages for each month were applied at mid month and interpolated for days in between to create a smoother daily pattern.

At this stage, there were two daily radiation time series, one arriving at the earth's atmosphere free of losses (Dunne & Leopold's 1978 radiation constants), and one measured at the earth's surface after atmospheric and cloud related losses. It was then simply a case of identifying the losses which are cloud related, and presenting them as a fraction comparable against the conventional cloudiness coefficient. It is known at this point that atmospheric losses account for approximately 20% of gross radiation. Therefore the gross radiation time series is multiplied at every ordinate by a factor of 0.8. This was then superimposed over a plot of measured solar radiation on "clear days" (no cloud):

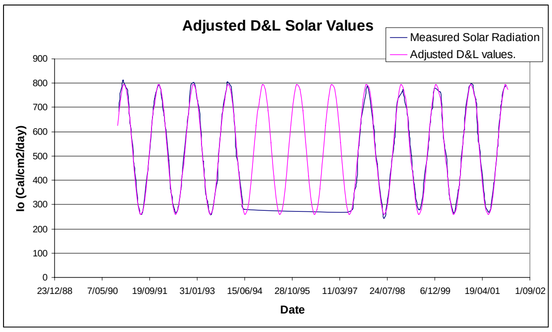

Figure 1. 80% of gross daily radiation, plotted against solar radiation measurements for cloudless days.

Figure 1 shows the two time series are almost identical (except for the flat-line period of missing data). This is the definition of "clear day" radiation used throughout the study. This time series was then statistically "ironed out" to reduce the influence of short term irregularities, creating an average annual clear day radiation pattern for Sydney.

Establishing the "clear day" radiation pattern was an important step in deducing radiation deficit. For it was then very evident that for a given day of the year, the difference between the predicted "clear-day" radiation for that day, and the actual measure of solar radiation on that day, could safely be attributed to cloud losses. This solar radiation deficit, expressed as a fraction was plotted against conventional cloudiness coefficient in Figure 2.

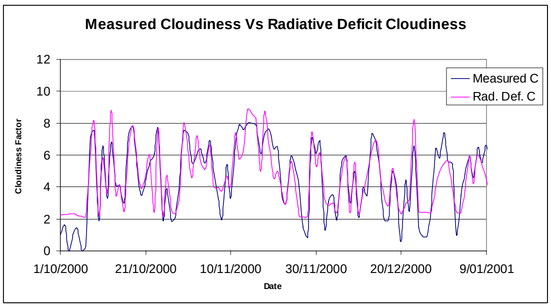

Figure 2. Conventional Cloudiness Coefficient vs. Radiation Deficit Cloudiness Coefficient.

The similarities are compelling, proving that the radiation difference was successfully reconciled. Despite the similarities, and assuming that radiation deficit (being a direct measurement) had become the more accurate representation of cloud activity, it was clear that the conventional cloudiness coefficient does fail to represent the occasional cloud "event".

Evaporation was estimated once again using the amended Penman Method on a daily basis for the study period. Amendments include using measured daily solar radiation in place of Q_s, and "radiation deficit" cloudiness coefficient in place of conventional cloudiness coefficient in the Q_{lw} equation.

The evaporation estimates made post amendment could then be considered as the better estimate for the study period, since they were based on real solar radiation inputs, with less empirical approximations. The results are presented in Section 5.

It is not always practical in routine application of the Penman Method to obtain measured solar radiation data. This study encountered difficulty in obtaining such data as a homogenous time series. It would be even more difficult if the data was required for a specific time period. Therefore it would be very useful if the findings of this study could be instilled into the standard Penman Method, so the method itself can improve even in the absence of measured solar radiation.

In the wider study, many options of achieving this were tested, the most obvious of which would be to adjust the empirical constants in the equations to better re-create the results of this study. However, the option which maintained the best short term accuracy was instead to adjust Dunne & Leopold's solar radiation constants.

5. RESULTS

Since there is no real "answer" to the actual rate of evaporation loss, the benefits of the changes made to the Penman Method can only be expressed and evaluated in relative terms.

Evaporation estimates were calculated using the standard Penman Method as presented by Thompson (1999) as a control. The same process was repeated using the Penman Method amended according to this study. Figure 1 showed that a large gap of about 3 years existed in the measured radiation data. For this time period, the standard Penman Method was used with amended solar radiation constants.

Estimates were calculated on a daily time step, and the results were summarised into monthly total evaporation. Figure 3 compares the results for the study period before and after the amendments, plotted against the raw Class-A Pan readings.

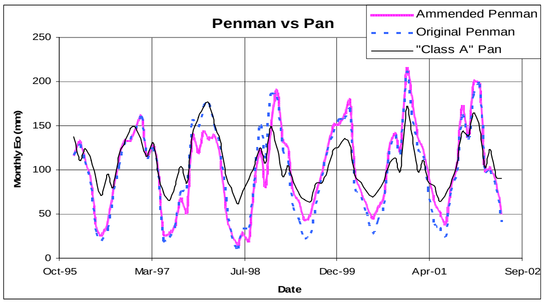

Figure 3. Monthly evaporation estimates for standard Penman Method, amended Penman Method, and raw Class-A Pan readings.

The coincidence between the Penman Method evaporation pattern for the study period, and the raw Class-A pan readings is remarkable, as shown in Figure 3. While raw Class-A pan readings are by no means the most reliable evaporation estimates, it must be remembered that they and the Penman Method are two completely different methods of estimating evaporation. Other studies such as that by Mosner & Aulenbach (2003) covering a variety of evaporation methods show that, even amongst the most popular evaporation estimation methods, this level of correlation is unusual.

Table 1. Result summary of comparisons between amended Penman Method, standard Penman Method, and raw Class-A Pan readings.

	Total Depth *mm*
Standard Penman	7,591
Amended Penman	7,739
Class A Pan	8,276

The standard and amended Penman Methods in Figure 3 seem to be very similar. Table 1 illustrates that the total difference between the two methods over the study period was 148mm. This is an annual average of 23mm, which is surprisingly low, considering how the standard Penman Method approximates net solar radiation with empirical equations. Unfortunately there are no evaporation estimates of this level of accuracy against which these results can be validated.

The substitute solar constants created for use in the absence of solar radiation data is able to emulate the results gained using real solar radiation data to within 3.4mm per annum.

Dunne & Leopold (1978) solar radiation constants are denoted as I_o. The substitute solar radiation constants created in this study are shown in Table 2 as I_{sub}. The units for these constants are calories/cm^2/day, and represent average daily radiation for each month of the year.

Table 2. Solar radiation constants designed to emulate the use of measured solar radiation when used in the standard Penman Method.

Month	I sub
1	1031
2	922
3	735
4	554
5	416
6	337
7	363
8	478
9	661
10	836
11	940
12	1054

It must be pointed out that while these values are based on Dunne & Leopold's (1978) values, they do not maintain the same meaning. Dunne & Leopold's values actually represent solar radiation reaching the earth's atmosphere at given latitude and given time of year. The substitutes presented in Table 2 however are notional, and were developed only for the purposes on maintaining some consistency in Penman estimates across a time period where gaps are likely to occur in measured solar radiation data.

6. CONCLUSIONS

The error incurred by the use of semi-empirical equations to approximate the radiation terms in the Penman Method seems not to be as dramatic as claimed by publications in the past. According to the findings of this study, use of these semi-empirical radiation equations will cause the standard Penman Method to underestimate evaporation by little over 2% compared to estimates using real radiation data.

For most applications, such a level of improvement is not required. Hence the standard Penman Method would suffice. For studies of water bodies with large water surfaces, or where higher levels of accuracy are sought in a water budget, the findings of this study may prove to be valuable.

A second conclusion to be drawn from this study is that the Pan Conversion Method of estimating evaporation currently used by water resources authorities has a tendency to over estimate evaporation by as much as 7% if compared to the post amendment Penman Method estimates made in this study.

Overall, the Penman Method appears to be a sound method for estimating evaporation. Even with the use of empirical approximations instead of real data, the method gives very reasonable results.

7. ACKNOWLEDGMENTS

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