# Competitive Analysis Using a Flexible Demand Specification

**[1]Hausman, J.A. and [2]G.K. Leonard**

[1]John and Jennie S. MacDonald Professor of Economics, Massachusetts Institute of Technology, Cambridge, MA 02139, [2]Vice-President, NERA Economic Consulting, Suite 2600, One Front Street, San Francisco, CA 94111.

**EXTENDED ABSTRACT**

Econometric methods can be helpful when one is trying to understand the nature of competition between products in differentiated products industries, an issue that arises in many competition policy contexts. However, misspecification of the consumer demand system can result in biased econometric results and misleading conclusions. We discuss some considerations that go into choosing a demand system specification and suggest that a ‘flexible functional form’ be used. This approach avoids placing restrictions on the demand elasticities. We identify and discuss other issues that arise in the estimation of demand systems. We conclude with an empirical example concerning the calculation of lost profits damages in a patent infringement case. This example demonstrates how using a flexible functional form leads to a substantially more reliable damages calculation than using the particular restrictive functional form that is commonly used to calculate lost profits damages in patent litigation.

## 1. INTRODUCTION

The need to understand the competitive interactions among a group of products arises in a number of litigation and regulatory settings. For example, in a patent infringement case, the extent to which the patent-owner has suffered lost sales depends upon how closely the infringer's product competes with the patent-owner's product. The closer the competition, the greater the lost sales would be expected to be. As another example, in the case of a proposed merger, the competitive effects of the merger depend in part upon how closely the products of the merging companies compete with each other.

Economists often summarize the extent of competition between products in a differentiated products industry using measures called the 'own and cross price elasticities of demand.' The own elasticity of demand measures the responsiveness of a product's demand to its own price (the own price elasticity is formally defined as the percentage change in demand for the product that would result from a 1% increase in the product's price). The cross elasticity of demand measures the responsiveness of demand for one product, say product A, with respect to the price of a second product, say product B (the cross elasticity of demand for product A with respect to product B's price is formally defined as the percentage change in the demand for product A that would result from a 1% change in product B's price). The larger the cross elasticity of demand between two products, the closer the two products are as substitutes in the eyes of consumers.

These elasticities can be calculated if the demand functions for the products in the industry are known or have been econometrically estimated. In principle, estimation of the demand functions can be accomplished using price and quantity data on the individual products. However, such data for differentiated product industries was only occasionally available to economists prior to the mid-1980's.[1] The situation changed when AC Nielsen and IRI introduced what is called 'retail scanner data.' Nielsen and IRI collected from a sample of supermarkets and other retail outlets the data that were captured by checkout scanning systems. These systems record information on every item that passes over the scanner as consumers make their purchases. Nielsen and IRI then processed the data and developed estimates of dollar sales, unit sales, and other variables by geographic area (e.g., metropolitan areas), time period (e.g., week), channel (e.g., supermarkets), and UPC code (e.g., 12 oz. package of Honey Nut Cheerios) and sold the resulting data to the product manufacturers, who then used it for market research purposes. Retail scanning data have also proved to be quite useful for economists interested in investigating the competitive interactions between products covered by these data.[2]

In this paper, we address econometric methods for analyzing competition between products using 'aggregate-level' data such as the retail scanner data that provides information on price and quantity aggregated over individual consumers within a specified set of geographic areas.

---

[1] Prior to the mid-1980's, the available aggregate-level data (i.e., data aggregated over individuals to the level of a city or some other geographic region) typically allowed the estimation of own and cross price elasticities for only broad categories of goods, e.g., food, clothing, etc., or for commodities, e.g., wheat, corn, etc. The lack of aggregate-level data on the individual products within broad categories prevented the estimation of own and cross elasticities for these products (e.g., individual brands of bread or soap). The most common exception was where data on individual decisions was available from surveys and data on prices was publicly available. Examples are demand for various modes of transportation and demand for recreational sites.

[2] See, e.g., Jerry Hausman, et al., *Competitive Analysis with Differentiated Products*, 34 ANNALES D'ECONOMIE ET DE STATISTIQUE 159 (1994); Gregory Werden and Luke Froeb, *The Effects of Mergers in Differentiated Products Industries: Logit Demand and Merger Policy*, 10 J.L. ECON. & ORG. 407 (1994); Gregory Werden, et al., *The Use of the Logit Model in Applied Industrial Organization*, 3 INTL. J. ECON. BUS. 83 (1996); Jerry Hausman, *Valuation of New Goods Under Perfect and Imperfect Competition*, in Timothy F. Bresnahan and Robert J. Gordon, THE ECONOMICS OF NEW GOODS (1997); Jerry Hausman and Gregory Leonard, *Economic Analysis of Differentiated Products Mergers Using Real World Data*, 7 GEO. MASON L. REV. 321 (1997); Aviv Nevo, *Mergers with Differentiated Products: The Case of Ready-to-Eat Cereal Industry*, 31 RAND J. ECON. 395 (2000); Aviv Nevo, *Measuring Market Power in the Ready-to-Eat Cereal Industry*, 69 ECONOMETRICA 307 (2001); Jerry Hausman and Gregory Leonard, *The Competitive Effects of a New Product Introduction: A Case Study*, 50 J. INDUS. ECON. 237 (2002).

Obtaining the necessary aggregate-level data is only the first step for the economist interested in estimating a demand system. A particular specification, or functional form, for the demand system must also be chosen. In this paper, we discuss some of the considerations that go into choosing a demand system specification. We suggest that a 'flexible functional form' be used for the demand system specification. A flexible functional form leaves the own and cross price elasticities of demand free to be estimated from the data. A non-flexible form, on the other hand, may impose restrictions on the demand elasticities, which can lead to biased results. We then describe an example of a flexible functional form that we generally favor—the Almost Ideal Demand System (AIDS)—and discuss its strengths and weaknesses in comparison to other specifications. We identify and discuss some issues that arise in the estimation of an AIDS system (and other demand systems as well).[3] Finally, we conclude with an empirical example.

## 2. CONSIDERATIONS IN CHOICE OF DEMAND SYSTEM SPECIFICATION

A reliable competitive analysis in turn requires reliable estimates of the own and cross price elasticities of demand (or, more generally, the demand functions for the set of products at issue). Reliable elasticity estimates, in turn, require an appropriate choice of demand system specification. There are two types of considerations in the choice of specification: econometric considerations and theoretical considerations.

### A. Econometric Considerations

In general, when choosing an econometric specification, a tradeoff exists between the flexibility of the specification to reflect the characteristics of the observed data and the statistical precision of the elasticity estimates. A less flexible specification generally has fewer parameters to estimate and thus may lead to more precise elasticity estimates. On the other hand, being less flexible, the specification may fail to fit the data well, which could induce bias into the elasticity estimates.[4] In other words, the specification may fail to capture important characteristics of the data. During the 1980's econometricians realized the importance of using 'flexible functional forms' that place a minimal (or no) restrictions on the estimated values of the demand elasticities.[5]

Classical statistical testing procedures may not be useful for helping to choose between alternative specifications where one alternative demand system specification is not nested within another, a situation that often arises.[6] While non-nested testing procedures could be used to choose between specifications, another approach is to use the more flexible specification as long as it produces acceptable levels of precision in the elasticity estimates.

### B. Theoretical Considerations

Under the economic theory of consumer choice, a demand system must satisfy three properties: Slutsky symmetry, homogeneity of degree zero in prices and total expenditure, and adding up.[7]

---

[3] Some of the issues we address, as well as other issues, are also discussed in Daniel Hosken, et al., *Demand Sysstem Estimation and Its Application to Hortizontal Merger Analysis*, FTC Working Paper (2002).

[4] Another consideration, particularly when estimating the price effects of a proposed merger, is the behavior of the demand system as prices move away from the point of approximation. Demand systems that yield the same elasticities at the point of approximation can predict substantially different post-merger price changes. See, e.g., Philip Crooke, et al., *Effects of Assumed Demand Form on Simulated Postmerger Equilibria*, REV. INDUS. ORG. 205 (1999).

[5] See, e.g., Erwin Diewert, *An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function*, 79 J. POL. ECON. 481 (1971) for a definition of flexible functional forms; see also Angus Deaton, *Demand Analysis*, in Zvi Griliches and Michael Intriligator, HANDBOOK OF ECONOMETRICS, VOL. 3 (1986) and Robert Pollak and Terrence Wales, DEMAND SYSTEM SPECIFICATION & ESTIMATION (1992).

[6] For example, the AIDS and log-log specifications discussed below are not nested within one another.

[7] Slutsky symmetry requires that the compensated cross price derivative of product A with respect to product B equals the compensated cross price derivative of product B with respect to product A. Homogeneity of degree zero in prices and expenditure requires that demand for all products be unchanged if the prices of the products and total expenditure all increase by the same percentage. Finally, adding up requires that the sum of expenditures on the individual products equals total expenditure.

Some demand specifications allow these properties to be easily imposed and tested, while other specifications do not. Generally, one would want to impose the restrictions implied by these properties because certain calculations of interest (e.g., consumer welfare calculations) would not be valid if the demand system did not satisfy the properties of consumer demand. On the other hand, empirical demand studies have often found that the properties of consumer demand are rejected by statistical tests.[8] Thus, the ability to both impose and test the properties of consumer demand is valuable property for a demand system specification.

A second theoretical consideration relates to whether the demand system specification can be obtained by aggregation over individual consumers.[9] A demand system and its associated properties are derived at the level of the individual utility-maximizing consumer. The question is whether the demand system and its properties transfer over to the aggregate-level data that is obtained by aggregating over individual consumers. In that case, the aggregate-level demand can be treated as the demand of a 'representative consumer' and the estimated demand system should exhibit the appropriate properties. In particular, the welfare of the representative consumer (i.e., from the estimated demand system) is equal to the true consumer welfare, i.e., the aggregation of welfare over individual consumers. If the demand system cannot be obtained by aggregating over consumers, there is no guarantee that the demand system estimated on aggregate-level data will exhibit the appropriate properties and that the consumer welfare calculated from the demand system will be equal to the true consumer welfare.

## 3. THE ALMOST IDEAL DEMAND SYSTEM

We now focus on a particular example of a flexible function form demand system. The Almost Ideal Demand System (AIDS) was proposed by A. Deaton and J. Muellbauer in 1980.[10] We describe the AIDS specification and then discuss its strengths and weaknesses in comparison to other demand system specifications.

We assume that there are N products in the industry that are to be included in the demand system and we index these products by i = 1,…,N. We note that, when choosing the products to include in the demand system, it is best to cast a wide net. After all, the purpose of estimating the demand system is to determine the extent of competition between products. Thus, it is better to be over-inclusive and let the data decide the extent to which products compete closely or not. By excluding products, one would instead be presumptively assuming that no competition exists between the excluded products and the included products. Of course, in a given real situation, data limitations may guide what products are included.

A product's 'revenue share' is equal to the revenue generated by the product divided by the total revenue generated by all products included in the demand system. We denote $s_i$ to represent the revenue share of product i. Under the AIDS, $s_i$ is specified as

$$s_i = \alpha_i + \beta_i \log(Y/P) + \sum_{j=1}^{N} \gamma_{ij} \log p_j \qquad (0.1)$$

where Y is the total revenue over all of the products in the demand system, P is an overall price index for the products, $p_j$, j=1,…, N, are the prices of the products, and $\alpha_i$, $\beta_i$, and $\gamma_{ij}$ are parameters to be estimated.

Thus, under AIDS, the revenue share of product i is the result of three terms. The first term is a constant ($\alpha_i$) that differs across products. Thus, everything else equal, some products would have higher shares than other products as a result of differing consumer preferences for products. The second term is based on the 'real' expenditure devoted to the

[8] See Deaton, supra note 5.

[9] Angus Deaton and John Muellbauer, ECONOMICS AND CONSUMER BEHAVIOR 148-59 (1980).

[10] Angus Deaton and John Meullbauer, *An Almost Ideal Demand System*, 70 AMER. ECON. REV. 312 (1980).

category (Y/P).[11] The revenue share of product i increases as the total real expenditure on the category increases if $\beta_i$, the coefficient on the log(Y/P) term, were positive. The reverse relationship would hold if $\beta_i$ were negative. The third term is based on the prices of the various products. Although not necessary to have a well-behaved demand system (and although often not observed in practice), it is helpful in understanding equation (1.1) to suppose that the 'own' price coefficient $\gamma_{ii}$ is negative and that the 'cross' price coefficients $\gamma_{ij}$ are positive. In that case, the share of product i increases when its own price decreases or when the price of another product decreases.

Under AIDS, the log price index P has the form

(0.2)

$$\log P = \alpha_0 + \sum_{i=1}^{N} \alpha_i \log p_i + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \log p_i \log p_j$$

However, it is common to employ a linear approximation to this price index, which allows for more straightforward estimation of the model parameters. In particular, Deaton and Muellbauer suggest the 'Stone price index' defined as[12]

(0.3) $$\log P = \sum_{i=1}^{N} s_i \log p_i$$

Thus, the Stone price index is a weighted average of the individual product prices, using the products' revenue shares as weights. Typically, to avoid inducing endogeneity in the log price index, we use a fixed-weight version of this index where the weights are equal to the average of the revenue shares over the entire time period covered by the data. Moreover, we typically allow the weights to differ across city to reflect differences in consumer preferences across geographies.

AIDS has a number of desirable properties as we now discuss.

### A. Flexibility

AIDS has a high degree of flexibility in the econometric sense described above. It is derived from an expenditure function that is a second order approximation to any expenditure function. Thus, it is a flexible functional form demand system as that term has been used in the economics literature.[13] As a result, the AIDS demand specification is a first order approximation to any demand system.[14] This result implies that even if the true underlying demand system is not AIDS, AIDS will nevertheless provide a reasonably accurate approximation at any set of prices not too far from the point of approximation. For the reasons discussed above, a flexible demand system has considerable advantages over an inflexible demand system in terms of reliably estimating the cross price elasticities of demand.

The downside to flexibility is the large number of parameters that need to be estimated. Even after imposing Slutsky symmetry and homogeneity of degree zero as described below, estimation of the most parsimonious flexible function form demand system (e.g., AIDS) with N products, will generally require the estimation of at least $\frac{N^2+3N-4}{2}$ parameters.[15] For example, a system with 10 products would have at least 63 parameters.

### B. Imposing And Testing The Properties Of Consumer Demand

AIDS allows for easy imposition and testing of the properties of consumer demand.[16] Slutsky symmetry can be imposed by setting $\gamma_{ij} = \gamma_{ji}$ for i = 1,…, N and j = 1,…, N. Then, the cross price

[11] We will often refer to Y as total expenditure on the category. Note that consumers' expenditure is equivalent to producers' revenue.

[12] Deaton and Meullbauer, supra note 10, at 316.

[13] See Diewert, supra note 5.

[14] Deaton and Muellbauer, supra note 10, at 312.

[15] Pollak and Wales, supra note 5.

[16] Deaton and Muellbauer, supra note 10, at 312.

derivatives of compensated demand for products i and j will be equal as required by Slutsky symmetry. This condition is generally required to do valid consumer welfare calculations. Similarly, homogeneity of degree zero can be imposed by setting $\sum_{j=1}^{N}\gamma_{ij}=0$ for i = 1,…, N. Then, the share for each product i will not change if total expenditure Y and all prices $p_j$ are increased by the same percentage. Adding-up requires, in addition to the other restrictions, $\sum_{i=1}^{N}\alpha_i=1$ and $\sum_{i=1}^{N}\beta_i=0$ since the revenue shares must sum to one across products.

The above parameter restrictions can be imposed during estimation. Alternatively, the restrictions can be tested using standard statistical methods after estimation of the AIDS model.

### C. Aggregation

AIDS at the aggregate level can be obtained through aggregation over individual consumers.[17] Thus, AIDS estimated on aggregate-level data can be treated as the demand system for a representative consumer. The demands and welfare calculations for this representative consumer will appropriately reflect the aggregated demands and welfare of the individual consumers.

## 4. COMPARISON TO OTHER DEMAND SYSTEMS

As described above, the AIDS has a number of desirable properties. In this section, we compare the AIDS to other widely used demand systems and show that these other systems generally do not possess as many desirable properties as the AIDS.

### A. Logit

The logit model of consumer demand has been proposed for use in merger analysis and other situations under certain conditions.[18] The logit model has the advantages that it is easy to estimate, it satisfies the restrictions of consumer demand, and aggregates across individual consumers.

However, logit is not very flexible. As is well-known, logit exhibits the independence of irrelevant alternatives (IIA) property.[19] This property constrains the cross price elasticity of product i with respect to product j's price to be equal for all i. In other words, the cross elasticities of demand with respect to a particular product's price are all equal.[20] To derive this result, start with the equation for the quantity share of product i under the logit model:[21]

$$(0.4)\quad \pi_i=\frac{\exp(\alpha p_i+Z_i\gamma)}{\sum_{j=1}^{N}\exp(\alpha p_j+Z_j\gamma)}$$

The cross elasticity of product i with respect to product j's price is derived by differentiating (1.4) with respect to $p_j$ and multiplying by $p_j/\pi_i$, which yields

(0.5)

$$\frac{p_j}{\pi_i}\frac{\partial\pi_i}{\partial p_j}=-\frac{p_j}{\pi_i}\frac{\exp(\alpha p_i+Z_i\gamma)}{\left[\sum_{k=1}^{N}\exp(\alpha p_k+Z_k\gamma)\right]^2}\exp(\alpha p_j+Z_j\gamma)\alpha=-\alpha p_j\pi_j$$

[17] Deaton and Muellbauer, supra note 10, at 312.

[18] See, e.g., Werden and Froeb, supra note 2; Werden, et al., supra note 2; Gregory Werden, et al, *Economic Analysis of Lost Profits From Patent Infringement With and Without Noninfringing Substitutes*, 27 AIPLA QUARTERLY JOURNAL 305 (1997).

[19] See, e.g., Daniel McFadden, *Econometric Models of Probabilistic Choice*, in Charles Manski and Daniel McFadden, STRUCTURAL ANALYSIS OF DISCRETE DATA WITH ECONOMETRIC APPLICATIONS 222-23 (1981).

[20] Jerry Hausman, *Project Independence Report: An Appraisal of U.S. Energy Needs Up to 1985*, 6 BELL J. ECON. 517 (1975); McFadden, supra note 19, at 222; Hausman and Leonard, supra note 2, at 322.

[21] For ease of exposition, equation (1.4) assumes that each consumer inelastically purchases one unit from the category. The logit model can be generalized to allow for an "outside alternative," i.e., a choice for consumers not to purchase any product in the category. However, the conclusions regarding equal cross price elasticities continue to hold in the logit model with an outside alternative.

As equation (1.5) demonstrates, the cross elasticity for all products $i \neq j$ with respect to the price of product j are equal to the same value, $-\alpha p_j \pi_j$. Note that this cross price elasticity value is driven by $\pi_j$, the quantity share of product j. If product j has a large quantity share, its cross price elasticities will be large for all other products.

It is easy to think of examples where this property will fail to hold. Consider a case of an industry consisting of several branded 'premium' products with large industry shares and several 'economy' products with smaller shares. One would expect that the economy brands would compete more closely with each other than they do with the branded premium products, i.e., that the cross price elasticities between the economy products are larger than the cross price elasticity between the economy products and the branded premium products. Logit cannot capture this situation because it would force the cross elasticity of branded premium product A with respect to the price of economy product B to be the same as the cross elasticity of economy product C with respect to the price of economy product B.

This property of logit is highly undesirable when the goal of a given analysis is to determine how closely two or more products compete with each other. A demand specification that severely limits the values that the cross price elasticities can take could result in badly biased cross price elasticity estimates and, therefore, incorrect conclusions concerning the extent of competition between products.

### B. Nested Logit

Nested logit models improve upon the basic logit model by grouping products into 'nests.'[22] Products within a nest are allowed to compete more closely with each other than they do with products outside the nest, thus reducing the problem of equal cross price elasticities. The problem is not entirely eliminated, however, since the cross price elasticities within a nest are still constrained to be equal.

[22] See, e.g., Steve Berry, *Estimating Discrete Choice Models of Product Differentiation*, 25 RAND J. ECON. 242 (1994).

In addition, the nested logit is somewhat more difficult to estimate than the basic logit. Moreover, the econometrician must decide how to group products into nests. While external information (e.g., market research) and statistical testing procedures can aid in these decisions, an element of judgment is still involved.

### C. Random Effects Logit

A relatively new extension to the logit model is the 'random effects' logit or 'mixed' logit.[23] This model can be thought of as assuming that each consumer has logit demand, but that consumers differ in the value weights they place on price and other product attributes. As a result, aggregate demand does not exhibit the equal cross price elasticity property although the property continues to hold for each individual. For example, people who bought a Toyota station wagon and place a good deal of weight on having a station wagon would be more likely to switch to a Honda station wagon than to a sports car if the Toyota station wagon price were to increase. In aggregating over individuals, the people who choose station wagons largely determine the cross price elasticities among station wagons, while the people who choose sports cars largely determine the cross price elasticity of sports cars with respect to station wagons. Therefore, in the aggregate, the cross price elasticities among station wagons are 'large' and the cross price elasticities of sports cars with respect to station wagons are 'small.'

The random effects logit has the advantage that it requires that substantially fewer parameters be estimated than a typical flexible functional form such as AIDS. However, this benefit comes at the cost that the random effects logit is substantially more difficult to estimate than AIDS in a typical application. In addition, although it is less restrictive than the basic logit model, the random effects logit may not have the flexibility to perform as well as AIDS in many situations. In the one direct comparison of which

[23] Steve Berry, et al., *Automobile Prices In Market Equilibrium*, 63 ECONOMETRICA 841 (1995). A similar model was first proposed and estimated in Nicholas S. Cardell, *Extensions of Multinomial Logit: The Hedonic Demand Model, the Non-Independent Logit Model, and the Ranked Logit Model,* Harvard University Ph.D. Thesis (1989).

we are aware, the results for AIDS and the random effects logit were similar in some respects, but different in others.[24] A topic for future research is determining the conditions under which the random effects logit or, alternatively, a flexible functional form would be preferred.

### D. Log-Log Demand

A log-log demand system takes its name from the fact that the log of a product's quantity is related to the logs of the prices of all the products as well as the log of category expenditure. Specifically, under the log-log specification, the demand equation for product i is

$$\log Q_i = \alpha + \beta \log Y + \sum_{j=1}^{N} \gamma_{ij} \log p_j \tag{0.6}$$

where $Q_i$ is the quantity of product i, Y is category expenditure, $p_j$ is the price of product j, and $\alpha$, $\beta$, and the $\gamma_{ij}$'s are parameters to be estimated.

The log-log demand system is flexible in that it can approximate any demand system at a given set of prices. It is also relatively easy to estimate in an unrestricted fashion. However, imposing the restrictions of consumer theory is not straightforward.[25] In addition, the log-log system as applied to aggregate-level data cannot be obtained through aggregation over individuals. Finally, the log-log system has the undesirable attribute that the elasticities of demand are constant for all prices. Thus, although the log-log system might approximate a general demand system at the point of approximation, it may fail to approximate it well as one moves away from the point of approximation.

[24] Nevo, supra note 2.

[25] Indeed, imposition of the adding-up restriction is problematic, see Deaton and Muellbauer, supra note 9, at 17.

### E. Other Flexible Demand Systems

A wide variety of other flexible demand systems exist, e.g., the various translog forms.[26] These systems share many of the properties of the AIDS. However, in general, they are not as easy to estimate as the AIDS because of non-linearities in the share equations. A topic for future research is the comparison between the AIDS and translog forms in terms of how well they perform moving away from the point of approximation.

## 5. EMPIRICAL IMPLEMENTATION OF THE AIDS

We discuss empirical implementation of the AIDS in the typical context where retail scanning data is available. We assume that data is available on N brands (indexed by i) for M cities (indexed by j) and T time periods (indexed by t). For example, we might have data on 10 brands in 25 cities for 106 weeks.

### A. AIDS Revenue Share Equation

The AIDS revenue share equation (1.1) needs to be modified to account for the fact that a product's revenue share might be expected to differ across time and cities for reasons other than differences in prices and expenditure. For example, consumer preferences for the product might grow over time or might be seasonal. As another example, consumers in one geographic area might have a greater preference for the product than consumers in other geographic areas. To account for time-invariant differences in demographics or preferences across cities, we include city-brand fixed effects, or separate constants for each city and brand, in the specification. These city-brand fixed effects are often quite important. To account for changes in demographics or preferences over time, we include time trend variables and seasonal variables in the specification.

Accordingly, a typical specification is

[26] See, e.g., Pollak and Wales, supra note 5, at 53-59; Deaton, supra note 8, at 1788-93.

(0.7)

$$s_{ijt} = \alpha_{ij} + Z_t\phi_i + \beta_i \log(Y_{jt} / P_{jt}) + \sum_{k=1}^{N} \gamma_{ik} \log p_{kjt} + \varepsilon_{ijt}$$

where are $\alpha_{ij}$ are the city-brand-specific fixed effects, the vector $Z_t$ includes the time trend and seasonal variables, $\varepsilon_{ijt}$ is an error term, and the other variables are defined as in (1.1).

### B. Top Level Equation

The AIDS model as described above is a 'conditional' model. It describes consumer demand for a product in the category of interest conditional on category expenditure (Y in equation (1.1)). However, category expenditure is itself determined as part of the consumer's overall decision as to how to allocate his or her total expenditure across the full range of product categories. In other words, the consumer's unconditional demand for a product can be broken into two parts, or stages, from a conceptual point of view.[27] In the first stage, the consumer decides how to allocate total expenditure among the various product categories. In the second stage, the consumer decides how to allocate the expenditure for a given category across the products within the category. This approach is called two-stage budgeting and was developed by W. Gorman.[28]

Under two-stage budgeting, to determine the unconditional demand for a given product, one must combine the demand for the product conditional on category expenditure with the demand for the category as a whole. Thus, one needs to estimate the demand for the category as a whole. We refer to the demand equation for the category as the 'top-level' demand equation.

[27] It is not necessary that the consumer actually go through this thought process for the two-stage budgeting methodology to be an appropriate way of modeling the consumer's decision problem.

[28] See, e.g., Charles Blackorby and Anthony Shorrocks, COLLECTED WORKS OF W. M. GORMAN, VOLUME 1: SEPARABILITY AND AGGREGATION (1995).

The two-stage budgeting approach can be extended to three or more stages. This approach can be useful if the category contains a large number of products, rendering estimation of a single AIDS specification including all of the products unwieldy. In that case, one can divide the category's products into segments, perhaps according to product characteristics or the views of company market research. For example, the products in the beer category might be segmented into light beers, premium beers, and low-priced beers. Then, a separate AIDS model can be estimated for each segment, conditional on segment expenditure. A segment demand model can be estimated, conditional on beer expenditure. Finally, a top-level beer demand model can be estimated, conditional on total expenditure. The three models can be combined to derive the unconditional demand for any given product.[29]

Returning to the two-stage case, we typically use a log-log specification for the top-level demand equation:[30]

(0.8)

$$\log Q_{jt} = \delta_{0j} + Z_t\theta + \delta_1 \log P_{jt} + \lambda \log X_{jt} + \eta_{jt}$$

where $Q_{jt}$ is overall category quantity in city j in time t, $\delta_{0j}$ is a fixed effect for city j (again representing time-invariant demographics and preferences), $X_{jt}$ is total expenditure for city j and time t, $P_{jt}$ is the category price index, $Z_t$ is the vector of seasonal and time trend variables, and $\eta_{jt}$ is an error term. Prices and total expenditure should be deflated using the CPI.[31]

### C. Instruments

In estimating a demand system consisting of N-1 share equations like (1.7) and a top level

[29] Use of multi-stage budgeting does impose restrictions on the demand system. However, these restrictions can be tested, see, e.g., Hausman, et al., surpa note 2.

[30] Many of the disadvantages of the log-log-specification discussed above are not present when only a single product (or in this case, category) is addressed.

[31] Often total expenditure is not available by city. In that case, another income measure can be used instead, such as personal disposable income.

equation like (1.8), a potential ‘simultaneity problem’ arises. Specifically, there may exist some factors unobserved to the econometrician that affect both consumer demand and the price-setting of firms. In that case, the prices appearing on the right-hand-sides of equations (1.7) and (1.8) would be correlated with the error terms of these equations. Ordinary least squares or its variants (e.g., seemingly unrelated regressions, with cross-equation restrictions) would be biased and inconsistent.

In general, the solution to the simultaneity problem is to employ an ‘instrumental variables’ technique. An instrumental variables technique involves finding variables (the instruments) that are correlated with the endogenous variables (in this case, prices), but not correlated with the error terms. Loosely speaking, one replaces the endogenous variables with the instruments and the simultaneity problem disappears (since the instruments are not correlated with the error terms, as were the endogenous variables). The question in implementing an instrumental variables technique is where to obtain the necessary instruments.

One possibility for developing instruments is to obtain data on cost variables that do not appear in the demand equations. However, to be useful instruments, such variables would have to be measured with a great degree of frequency and specificity (i.e., separately for the individual manufacturers). Cost variables measured monthly and at a national level (e.g., Bureau of Labor Statistics input price indices) would not ultimately be very helpful in estimating a demand equation based on the prices of N individual brands, measured weekly in a large number of cities. While plant-specific variable cost data for each manufacturer would be more helpful, having access to such data is rare.

As an alternative solution, we have proposed using the panel structure of the underlying data. After allowing for the brand-city fixed effects, we use the prices from one city as instruments for other cities.[32] The intuition is that prices in each city reflect both underlying product costs and city-specific factors that vary over time as supermarkets run promotions on a particular product. To the extent that the stochastic city-specific factors are independent of each other, prices from one city can serve as instruments for another city.

We now discuss the conditions under which this approach would be valid. Consider the case of two cities, indexed by j =1 or 2, and the estimation of the share equation (1.7) for city 1. The reduced form equations for the prices of brand i in the two cities are

$$\text{(9)} \qquad \begin{aligned} \log p_{i2t} &= \Pi_1 \log c_{it} + \mu_{i2} + Z_{2t}\Pi_2 + v_{i2t} \\ \log p_{i1t} &= \Pi_1 \log c_{it} + \mu_{i1} + Z_{1t}\Pi_2 + v_{i1t} \end{aligned}$$

A common determinant of the prices in the two cities is $c_{it}$, a non-city-specific cost element that arises because of the regional or national manufacture and shipping of the products. Also appearing in the reduced form equation are the demand shifter variables ($Z_{jt}$), a city-specific brand differential due to transportation costs or local wages ($\mu_{ij}$), and an error term ($v_{ijt}$). In general, the error term $\varepsilon_{i1t}$ from share equation (1.7) for city 1 will be correlated with $v_{i1t}$. If so, then OLS would yield inconsistent estimates of the parameters in equation (1.7).

However, as long as $v_{i2t}$ is uncorrelated with $v_{i1t}$, city 2’s price satisfies the first requirement to be a valid instrument for city 1’s price, i.e., it is uncorrelated with the error term in equation (1.7). Moreover, since city 2’s price, after elimination of city- and brand-specific effects and the demand shifter variables, is driven by the same underlying costs, log $c_{it}$, as city 1’s price, city 2’s price also satisfies the second requirement to be a valid instrument for city 1’s price.

Next, we address the conditions under which $v_{i2t}$ would be uncorrelated with $v_{i1t}$. For that purpose, it is useful to consider the error term from the share equation (1.7), $\varepsilon_{i1t}$. This error term will contain demand-shifting factors not accounted for by $Z_{1t}$. These demand-shifting factors can be divided into three categories. First, since supermarket shelf prices are generally set and posted in advance of the realization of demand, some factors in $\varepsilon_{i1t}$ are not observed when prices are set. Such factors would not appear in the reduced form equations (1.9) and thus would not cause correlation

[32] Jerry Hausman and William Taylor, *Panel Data and Unobservable Individual Effects*, 49 ECONOMETRICA 1377 (1981).

between $v_{i1t}$ and $v_{i2t}$. Second, some factors in $\varepsilon_{i1t}$ are purely city-specific, e.g., the effects of local advertising and promotion. These factors also would not cause correlation between $v_{i1t}$ and $v_{i2t}$. Third, some part of $\varepsilon_{i1t}$ may arise from a factor that is both present across cities and not already picked up by $Z_{1t}$. Only this third category of factors could cause a correlation between $v_{i1t}$ and $v_{i2t}$.

An example of such a factor might be a national advertising campaign, which might both affect demand in all cities and be taken into account when retail prices are set. The variables we included in $Z_{1t}$ may well capture the effects of national advertising. However, it is possible to test whether national advertising is causing a simultaneity problem by allowing for a more flexible effect of time on demand in the specification in order to pick up more of any nationwide factors. In particular, one can include separate indicator variables for each month-year period in the data. Since manufacturers' national advertising plans are often broken into monthly segments, this specification has the potential to capture the effects on demand of national advertising, eliminating the correlation that might exist among the $v_{int}$. In one case where we implemented this test, we found that the results of this more flexible specification were quite similar to our original specification, indicating that no serious simultaneity problem existed in the original specification.[33] We conclude that the instrumental variable approach discussed above is likely to be useful in many circumstances.

### D. Elasticity Estimates

Having estimated the parameters of equations (1.7) and (1.8), one can estimate the own and cross price elasticities of demand for the products in the category. With the AIDS model and the Stone price index, the equation for the elasticity of product i with respect to product j's price is

(10)

$$e_{ij} = \frac{1}{s_i}\left[\gamma_{ij} - \beta_i w_j\right] - 1\left[i = j\right] + \left(1 + \frac{\beta_i}{s_i}\right)(1+\delta) w_j$$

where $w_j$ is the average revenue share of product j and the other variables and parameters are defined in equations (1.7) and (1.8).

## 6. ISSUES IN ESTIMATION

When estimating an AIDS system in practice, a number of econometric issues may arise. We discuss some of these issues and potential solutions.

### A. Negative Cross Elasticity Estimates

With an AIDS system, the estimated cross elasticities are not guaranteed to be positive. It is not unusual in our experience to find some of the estimated cross elasticities to be negative, particularly when the number of products is large. Negative cross elasticities can be a cause for concern both because they are counter-intuitive and also because they can lead to odd results for consumer welfare calculations or merger simulations.

The first question to ask is whether the two products in question might, in fact, be complements rather than substitutes in which case the true cross elasticities would be negative. If so, the negative cross elasticity estimates do not pose a problem.

If the products should be substitutes, the next question to ask is whether the estimates are statistically significantly different from zero. If not, the negative estimated cross elasticities should be of no particular concern unless they unduly affect subsequent calculations of interest, e.g., merger simulations. In that case, the cross elasticity in question can be constrained to be zero, although one must be proceed carefully if the Slutsky symmetry and homogeneity of degree zero restrictions have been imposed since these properties link the elasticities together.

If one or more cross elasticities are estimated to be negative and statistically significantly different from zero, the appropriate response

[33] Hausman and Leonard, supra note 2, at 250.

depends upon the number of negative estimated cross elasticities relative to the total number of estimated cross elasticities. If the number of products is large, so that many cross elasticities have been estimated, it would not be surprising to find some negative and statistically significant cross elasticities. However, a relatively large number of negative cross elasticity estimates would suggest a problem with the data or the model specification. The appropriate response would be to examine the data for errors and try different model specifications, e.g., add other variables to the specification or implement a different flexible functional form.[34]

### B. Rejection of the Properties of Consumer Demand

A common finding in studies of consumer demand is the rejection of Slutsky symmetry and homogeneity of degree zero. As discussed above, one would generally want to impose these properties, particularly if consumer welfare calculations are to be performed using the estimated demand system. The consumer welfare calculations are not valid if the properties do not hold. If, on the other hand, one is performing other types of calculations, e.g., merger simulations, the properties are less important. In that case, if the properties are rejected, one might want to proceed without imposing them.

Generally, one should examine the reason for the rejection of the properties. If the difference between the unrestricted model and the restricted model is small from an economic point of view, we suggest imposing the restrictions even if they have been rejected by the statistical test. If the difference between the models is economically important, then one needs to reconsider the econometric specification.

### C. Aggregation of Products

For a given category, e.g., facial tissue, the number of individual products can be quite large because each brand (e.g., Kleenex) might have many different package sizes or types (e.g., stand-up versus flat) and many varieties (e.g., different colors). In general, specifying a demand system to account for all of the individual products is not realistic. Instead, as a practical matter, the products must be aggregated to some degree and the demand system specified for the aggregates. The question then arises as to the proper degree of aggregation and the appropriate aggregation method to use.

Sometimes the degree of aggregation (and the method) will be predetermined. For example, the econometrician may not have access to disaggregated data without substantial additional cost. In this situation, the econometrician will have little control over the degree of aggregation.

When disaggregated data are available, the degree of aggregation that should be undertaken is the outcome of practical considerations and the desire not to distort the econometric estimates. A good way to proceed is to test the effect of using different levels of aggregation within a range dictated by the practical considerations given the number of products in the category. Our experience has been that the degree of aggregation does not significantly affect the results. However, others have found otherwise.[35] Therefore, it would appear to be a situation-specific issue.

With regard to the method of aggregation, economic theory would dictate that an appropriate price index with a corresponding quantity index be used to aggregate products. In many circumstances, this approach is feasible. However, in a situation where one or more new varieties (i.e., package sizes or flavors) have been introduced during the period covered by the data, formation of appropriate price and quantity indices is more problematic. Incorporating a new product into a price index is a complex undertaking in and of itself and correctly addressing this issue may not be desirable when it is the not the primary focus of the exercise. As an alternative solution that is straightforward though less than completely satisfying, a new variety's revenue and quantity can be aggregated with those of some other products and then this aggregate can be further aggregated with other

[34] One must be careful not to choose an alternative demand system specification that forces all cross price elasticities to be positive, e.g. the logit demand system. These restrictions may well be inconsistent with the data.

[35] Hosken, et al., supra note 3, at 7.

products using economically correct price and quantity indices.

### D. Incorporation of Promotional Variables

Retail scanning data may include information on the extent of in-store promotional and advertising activity. Consumer demand for a product would be expected to be affected by such activity. Therefore, it might be useful to incorporate this information into the AIDS demand system.

Advertising and promotion should work in the same fashion as prices. In other words, an increase in the advertising and promotion for one product would both increase the demand for that product (the 'own' effect) and decrease the demand for competing products (the 'cross' effects). The natural way to incorporate this information is to make the city-brand specific effects in equation (1.7) a function of the extent of advertising and promotion of each of the products. Similarly, in the top level, an 'index' that combines the advertising and promotional activities of all the brands could be entered as an additional variable in equation (1.8). This approach would add a significant number of parameters to be estimated (N*(N-1) additional parameters since there are no restrictions from economic theory to reduce the number of parameters). Therefore, a modified approach might be useful in some situations to reduce the number of parameters. For example, an index that combines the advertising and promotion variables of the other products into a single variable might be used in place of the individual advertising and promotion variables for each product.

### E. Inventorying Behavior

When weekly data is used, a danger exists that the elasticity estimates obtained from a demand system represent short-run behavior rather than long-run behavior. Specifically, if consumers stock up on products when they go on sale, their short-run responsiveness to price changes (i.e., sales) might exceed their long-run responsiveness to price changes (i.e., permanent price changes). Since we in general would be interested in long-run elasticities, consumer inventorying behavior could lead to incorrect conclusions.

One implication of consumer inventorying behavior is that we would expect to see that a week with larger than normal demand (e.g., due to a sale) would be followed by weeks with smaller than normal demand (as consumers depleted their inventories rather than purchasing at the full price). In other words, demand should be negatively correlated over time. In our experience, we have observed the opposite: larger than normal demand one week is followed by larger than normal demand the next week as well. This result is inconsistent with substantial consumer inventorying behavior. However, other recent studies have reported finding evidence of inventorying behavior.[36]

Of course, the situation likely differs across product categories and thus the extent of inventorying behavior should be investigated in a given situation. If it appears to be an issue, there are two ways to account for it. First, the dynamic behavior can be explicitly modeled and the long-run elasticities can be solved for as a function of the short-run elasticities and the parameters describing the dynamic behavior. Second, the data can be aggregated over time (say to month) and the model re-estimated on the time-aggregated data. The results should represent longer-run elasticities than the weekly data.[37]

### F. Standard Errors for Predicted Post-Merger Price Changes and Welfare Calculations

The estimated demand system is typically used for some other purpose, e.g., to estimate the likely effects of a merger on prices, to calculate the welfare changes induced by changes in prices or qualities of products, or to determine the lost profits damages resulting from patent infringement. Since the demand system has been estimated, any calculations based on the demand system will reflect the statistical variation inherent in the estimated demand system

[36] See, e.g., Igal Hendel and Aviv Nevo, *Sales and Consumer Inventory*, Unpublished Paper (2002).

[37] Time-aggregation provides another test of inventorying behavior. If the estimated elasticities are significantly lower in the time-aggregated model, inventorying behavior might be present. Hosken, et al., supra note 3, report that they have found significant differences in elasticities after aggregating over time.

parameters. Thus, it is typically desirable to calculate standard errors for any results derived from the estimated demand system.

Often, the results derived from the demand system must be obtained via an iterative equation solving procedure and thus do not have a closed form solution. However, calculating standard errors in such a situation is straightforward from a computational point of view using the 'delta method.' The delta method involves linearizing the result in question around the demand system parameters. Having linearized the problem, calculation of the standard errors is straightforward. Alternatively, bootstrap methods can also be used, although these are quite time-intensive.

## 7. EMPIRICAL EXAMPLE

We now present an example of estimating an AIDS and using the results in a litigation setting. The product category is a health and beauty aid.[38] The retail scanner data available to us covered five cities and 108 weeks per city. There are four products, which we will label Products 1 through 4. Products 1, 2, and 3 are offered by different companies. Product 4 is an aggregate of products in the category other than Products 1 through 3.

An AIDS share equation of the form (1.7) was specified for each product (due to the adding-up constraint, one of the four equations can be dropped). Included in the specification were city fixed effects, a time trend, and month indicator variables. A top-level equation of the form (1.8) was also specified. Included in this specification were city fixed effects, a time trend, month indicator variables, and log personal disposable income. The instrumental variables technique described above was used to estimate the AIDS model. Finally, standard errors were estimated using the Newey-West procedure to account for serially correlated error terms. The resulting own and cross price elasticity estimates, evaluated at mean prices and shares, are presented below.

**Estimated Demand Elasticities**

| Elasticity of Demand For | With Respect to the Price of: Product 1 | Product 2 | Product 3 | Product 4 |
|---|---|---|---|---|
| Product 1 | -2.109 | 0.318 | 0.751 | -0.078 |
| | (0.128) | (0.114) | (0.124) | (0.072) |
| Product 2 | 0.280 | -1.854 | 0.226 | 0.119 |
| | (0.116) | (0.125) | (0.116) | (0.071) |
| Product 3 | 0.560 | 0.170 | -2.175 | 0.154 |
| | (0.108) | (0.098) | (0.140) | (0.069) |
| Product 4 | -0.164 | 0.213 | 0.353 | -1.627 |
| | (0.126) | (0.120) | (0.138) | (0.113) |

Note: Standard errors in parentheses.

The table is read as follows. The number in the first row, first column (-2.109) represents the elasticity of the demand for Product 1 with respect to its own price; the number in the first row, second column (0.318) represents the elasticity of the demand for Product 1 with respect to the price of Product 2; and so on.

The elasticity table contains several results worth noting. First, Products 1 and 3 appear to be closer substitutes for each other than either is for Product 2. This result is somewhat surprising because Product 1 uses a different technology than Products 2 and 3, i.e., the attributes of Product 1 would appear to be different from those of Products 2 and 3. Second, the cross price elasticities between Products 1 and 4 are negative. While this result is counter-intuitive, note that the standard errors are sufficiently large that neither cross price elasticity is statistically significantly different from zero.

We now illustrate how these results might be used in a litigation setting.[39] Suppose that Product 3 was found to infringe a patent held by the manufacturer of Product 2.[40] We will estimate the lost profits damages sustained by the manufacturer of Product 2 under several different

[38] Due to confidentiality reasons, we cannot disclose the exact product category.

[39] The hypothetical example we are about to use bears no relationship whatsoever to any actual events.

[40] One might think that patent infringement litigation rarely involves products for which scanner data would be available. However, we are aware of patent infringement cases involving products in a number of supermarket, drug store, and mass merchandiser categories including lipstick, diapers, and contact lens cleaners. In addition, IMS provides data similar to scannder data that are available for pharmaceuticals and medical devices, which are frequently the subject of patent infringement litigation.

approaches.[41] Lost profits damages consist of the incremental profits on the sales lost to the infringing product, plus the profits lost due to 'price erosion,' i.e., the reduction in prices caused by competition from the infringing product.

We assume that the appropriate 'but for' world is one in which the infringing Product 3 is completely removed from the market, leaving only Products 1, 2, and 4.[42] We calculate damages under three approaches. First, we calculate damages under the 'market share rule,' whereby the infringing sales are divided among the remaining firms in the market according to those firms' market shares. The market share rule has been endorsed by federal courts.[43] However, economists would expect that this rule may fail to accurately measure lost sales in many cases. First, the market share rule is implicitly based on the logit model of demand. Because of the logit model's assumption of equal cross price elasticities, it may provide a poor description of many industries, leading to either an understatement or overstatement of lost sales. Second, the market share rule implicitly assumes that the infringing product led to no expansion of total category sales. In a differentiated product industry, a new product often leads to an expansion in sales because it provides additional variety to consumers that did not previously exist.

To demonstrate the shortcomings of the market share rule in our particular example, we calculate the equilibrium prices and sales that would have resulted if the infringing Product 3 had been removed from the market (as it would have been in the 'but for' world). The removal of Product 3 from the market is achieved by raising its price to the point where its demand is zero. This approach has a long history in economics, as we discuss elsewhere in more detail.[44] Given the absence of Product 3, the equilibrium prices and sales of the remaining three products are then determined using the estimated demand system and an assumption about the form of competition.[45]

We start with the case where the infringing product is assumed to have caused no price erosion. In that case, prices for the products of the other three products are the same in the 'but for' world as they were in the actual world. Part of Product 3's quantity sales are distributed among the other three products depending on their cross elasticities of demand with Product 3. The remaining part of Product 3's sales leave the category in the 'but for' world, i.e., these sales were due to expansion of the market by Product 3.

We next take into account the possibility of price erosion. In that case, the other three manufacturers may charge higher prices in the 'but for' world due to the absence of competition from Product 2. As a result of the higher category prices and the category elasticity of demand, category sales would be expected to be lower.

A comparison of quantities and prices between the actual world and the three 'but for' worlds is given below.

---

[41] Werden, et al., supra note 18, discuss a similar approach to estimating lost profits damages in patent infringement cases. The plaintiff may be entitled to reasonable royalty damages in addition to lost profits damages. We focus on lost profits damages only in this example.

[42] A defendant might have available to it a non-infringing alternative that it could have sold in the 'but for' world. If it would have been profitable for the defendant to do so, the appropriate 'but for' world would be one where the defendant would have offered that non-infringing alternative product. Damages would depend in part upon how much lower consumer demand for the non-infringing product would have been. For our example, we assume that there was no such product available to the defendant.

[43] *State Industries, Inc. v. Mor-Flo Industries, Inc.,* 883 F.2d 1573 (Fed. Cir. 1989)

[44] Hausman, supra note 2, and Hausman and Leonard, supra note 2.

[45] Specifically, we use the Nash-Bertrand model of price competition. This model is widely used in competitive analyses of differentiated products industries, see the citations in note 2 supra.

**Quantity Sales**

| | | But For World | | |
|---|---|---|---|---|
| | Actual | Market Share Rule | Demand System - No Price Erosion | Demand System - Price Erosion |
| Product 1 | 64,322 | 99,245 | 105,245 | 74,973 |
| Product 2 | 123,290 | 190,229 | 146,685 | 126,472 |
| Product 3 | 135,608 | 0 | 0 | 0 |
| Product 4 | 62,153 | 95,898 | 80,671 | 63,044 |
| Total | 385,373 | 385,373 | 332,601 | 264,489 |

**Prices**

| | | But For World | | |
|---|---|---|---|---|
| | Actual | Market Share Rule | Demand System - No Price Erosion | Demand System - Price Erosion |
| Product 1 | $0.64 | $0.64 | $0.64 | $0.83 |
| Product 2 | $0.34 | $0.34 | $0.34 | $0.40 |
| Product 3 | $0.38 | --- | $0.94 | $1.16 |
| Product 4 | $0.38 | $0.38 | $0.38 | $0.46 |

Under the market share rule, total category quantity remains the same in the 'but for' world because Product 3 is not assumed to have expanded the market at all. When the possibility of market expansion is taken into account (the column entitled 'Demand System – No Price Erosion'), total category quantity in the 'but for' world is lower than in the actual world. The sales that represent market expansion by Product 3 would not have been in the category in the 'but for' world where Product 3 would not have been on the market. The market expansion effect accounts for about 39% of Product 3's actual world sales.

The market share rule also overstates the percentage of Product 3's sales that would have gone to Product 2 in the 'but for' world. The market share rule assigns 49% of Product 3's sales to Product 2. Using the demand system, however, Product 2 would receive only 28% of the (non-market expansion) sales of Product 3. The reason for this outcome can be seen in the elasticity table. The cross price elasticity between Product 3 and Product 1 is much larger than the cross price elasticity between Product 3 and Product 2. The demand system approach, therefore, gives relatively fewer of the sales to Product 2 and relatively more of the sales to Product 1 as compared to the market share rule, which assumes equal cross price elasticities.

When the possibility of price erosion is taken into account (the column entitled 'Demand System – Price Erosion'), prices in the 'but for' world are predicted to be substantially higher than in the actual world. Specifically, the price of Product 1 is predicted to have been 30% higher in the 'but for' world than in the actual world, while the price of Product 2 is predicted to have been 18% higher. Again, because Product 1 and Product 3 are closer substitutes than Product 2 and Product 3, the effect of price erosion was larger for Product 1 than for Product 2. At the higher 'but for' world prices, total category sales are substantially reduced, by 31% as compared to the actual level of sales.

Company 2's damages under each of the three approaches is given in the table below (damages are expressed as the number of dollars per week for the average city).

**Company 2's Damages**

| | Damages |
|---|---|
| Market Share Rule | $ 12,333 |
| | ---- |
| Demand System - No Price Erosion | $ 4,310 |
| | $ (2,577) |
| Demand System - Price Erosion | $ 7,577 |
| | $ (4,684) |

Note: Standard error in parentheses.

Under the market share rule, damages are calculated to be $12,333. Using the demand system and assuming no price erosion, damages are calculated to be $4,310. As discussed above, the difference between the two is due to two factors that the market share rule does not properly take into account: market expansion and the less competition between Products 2 and 3 than assumed under the logit model. Using the demand system and accounting for price erosion, damages increase to $7,577. Even though Company 2 has only slightly higher sales in the 'but for' world than in the actual world under this approach, its prices are 18% higher on all of these sales. As a result, damages taking into account price erosion are substantially higher than if price erosion is not taken into account.

## 8. SUMMARY

We have discussed the use of the AIDS in analyzing competition between products in differentiated product industries. AIDS has a number of desirable properties and, indeed, would be preferred over other demand systems in many situations involving aggregate-level data such as retail scanning data. We described how the AIDS works and what types of issues can arise in practical applications.

Finally, we illustrated how AIDS and, more generally, flexible demand systems, can be used to analyze competition in a litigation setting. We showed how use of the ‘market share rule’ can substantially overstate damages in a patent infringement case. We also showed how price erosion damages can be sizable.


## ACKNOWLEDGMENTS

We thank Luke Froeb for helpful comments.