Nonlinearity And Hyperinflation

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Keywords*:* *Hyperinflation, nonlinearity*

EXTENDED ABSTRACT

A model of inflation and growth is presented which incorporates the standard Classical assumptions of market clearing and perfect foresight. However, a variable returns to scale production function is assumed, introducing a nonlinearity into the model. The rate of money growth is held constant in the model, ruling out accelerating money growth as an explanation of hyperinflation.

The global dynamics of the model are analysed and shown to have two, one or no stable equilibria, depending on parameter values. The standard approach to analysing this type of model is to linearise it by (perhaps implicit) appeal to Hartman's theorem. The difficulties of this procedure are discussed, in particular:

- Its reliance on jump variables.
- It only holds locally
- It is based on a topological form of equivalence (homeomorphism)

It is argued that the linearisation process obscures important economics and, accordingly, the correct modelling strategy is to consider the global dynamics of the model.

It is impossible to solve the model analytically and hence a simple Matlab program is used to provide numerical solutions. The program plots output, rate of inflation, real rate of interest and nominal rate of interest. Three cases are considered:

1. growing economy with two stable equilibria
2. growing economy with a single stable equilibrium
3. contracting economy with two stable equilibria

For the growing economy cases, inflation accelerates to approximately 3000% with a money growth rate of 5%. For the contracting economy case, inflation accelerates to approximately 25% with a money growth rate of 5%.

The paper weakens the orthodox argument that firm control of the money supply is sufficient to control the rate of inflation in the medium to long run. It argues that local linearisation is not the appropriate strategy for dealing with macroeconomic models: the global dynamics should be analysed, using simulation methods if necessary. The paper provides an example of how striking analytical (including policy) results may depend as much on the method of analysing a model as on the economic assumptions upon which it is based.

1. Introduction

Traditionally, attempts to model hyperinflationary processes have followed one of two routes. Cagan (1956) concentrated on hyperinflation within a fixed output environment. Later writers, for example Sargent (1977), Sargent and Wallace (1973) and Flood and Garber (1980) retained this assumption but extended the basic model by incorporating rational expectations. The second route involves models which consider inflation within a growing economy. Here the basic formulation is due to Sidrauski (1967), where one standard characteristic is steady-state superneutrality.

One factor common to both traditional approaches is the view that high or hyper inflation is a *purely* monetary phenomenon, and that the interesting issues relate to expectations formation. In such models accelerating inflation arises, either implicitly or explicitly, as a result of government policies which increase the rate of growth of the nominal money supply.

The purpose of this paper is to present an alternative approach to the modelling of a growing economy which is experiencing a high and stable rate of inflation. We will present a model in which the normal explanation of hyperinflation is inapplicable as the rate of growth of nominal money will be fixed throughout. However, unlike the classical dichotomy world of Cagan, Sargent and Sidrauski, the model presented will be 'coupled', the real and monetary sectors interacting through the real rate of interest.

In section 2 we outline the basic properties of a Cagan type hyperinflation model, turning in section 3 to models of growing inflationary economies. In section 4 we explain the fundamentals of our point of departure, while in section 5 our formal model is presented. Section 6 provides an analysis of the model, with section 7 devoted to an explanation of the numerical simulation results. Section 8 concludes.

2. Cagan-Type Models of Hyperinflation

Cagan's explanation of hyperinflation relies heavily on the form of the demand for real money balances:

$$\log(M_t/P_t) = \alpha p_t^e + \lambda \log Y_t + \psi + U_t \qquad \text{……………………………………………..(1)}$$

$(\alpha < 0, \lambda > 0)$

where $M_t =$ nominal money balances, $P_t =$ price level, $p_t^e =$ expected rate of inflation, $Y_t =$ real income, $U_t =$ stochastic error term and α, λ and ψ are parameters.

Cagan made several crucial assumptions which generate his conclusions. Firstly Y is assumed constant over time. This effectively dichotomises the real and monetary sectors, allowing no feedback of inflation effects on output or vice-versa. Inflation will only occur as a monetary phenomenon. Secondly the Cagan model is log-linear. This severely restricts the dynamic modelling possibilities of the inflation process, allowing only explosive or monotonically convergent solutions. Thirdly, accelerating inflation is the result of expectations of future price increases, fuelled endogenously by a demand determined, equilibrium, money growth process. Because of the assumed form of the demand for money function, the steady state inflation rate must be equated with the nominal money growth rate in order to generate a constant, steady state, level of real money balances.

Expectations in the original Cagan model were extrapolative, however extensions incorporating rational expectations have been considered without fundamentally affecting the qualitative properties of the model, see e.g. Sargent and Wallace, 1973.

3. Money, Growth and Inflation

Cagan assumed fixed output in his hyperinflation model, but the dynamics of inflation in a growing economy have been considered by several authors, including Sidrauski (1967), and Fischer (1979). Ambler and Cardia (1998) provide empirical evidence on the relationship between inflation and growth. Typically the models are formulated as an infinite horizon, individual utility maximisation problem which exhibits unique convergent paths to steady-state. In the Sidrauski (linearised) model the steady-state is characterised by superneutrality, whereas Fischer (1979), utilising a constant relative risk aversion utility function, demonstrates that this property does not generally hold on the transition path to steady state. On such paths capital accumulation is faster the higher the rate of money growth. When one looks at the actual models of money, growth and inflation however, again we find that the systems of equations are generally linear or linearised, as in the Sidrauski models, with the production functions exhibiting constant returns to scale throughout. The major qualitative difference of allowing growth to enter the dynamics of the inflationary process is that, on the equilibrium growth path, the expected rate of inflation will be equal to the rate of monetary expansion *minus* the economy's warranted rate of growth. In a perfect foresight world (or in steady-state where expectations are fulfilled) one can equate expected inflation with actual inflation.

4. A Point of Departure

As we have seen, hyperinflation models of the Cagan variety implicitly assume prices are rising so fast as to be able to assume output is fixed. Alternatively, models of money and growth of the Sidrauski type assume simple, constant returns to scale production functions. The model developed here has the following properties, many of which are absent from traditional models. Firstly the real and monetary sectors interact, the main linkage operating via the rate of interest. The demand for money depends on income and the nominal rate of interest, which in turn is equal to the real rate of interest (marginal product of capital) plus the fully anticipated rate of inflation. This is in effect an asset market equilibrium condition under perfect foresight. The real rate of interest is determined on the real side of the economy, as the marginal product capital, derived from a variable returns to scale production function. For the sake of tractability, output depends only on the capital input.

Taken in isolation some of these properties appear standard; for example see the demand for money function in Sargent and Walllace, (1973) or the identification of the real rate of interest with the marginal product of capital in Begg (1982). However both models feature a *constant* marginal product of capital, and in the Sargent and Wallace model the whole system is assumed to be linear. The model developed here, by contrast, incorporates a production function with variable returns to scale. This generates a dynamical system which is inherently nonlinear. Similar nonlinearities could arise from a technical progress function of Kaldorian type.

The second important assumption of the model is a constant growth rate of the nominal money stock, which has the effect of ruling out the Cagan explanation of accelerating inflation. One conclusion of the paper is that, even with a constant growth rate of nominal money, the economy can experience accelerating inflation.

Finally, we will assume perfect foresight rather than rational expectations. It is now well known (see, for example, George and Oxley, (1985) that the qualitative behaviour of perfect foresight models is identical to that of certainty equivalence rational expectations models. More importantly though, the nonlinearities in our model considerably complicate the standard rational expectations solution techniques without affecting the qualitative properties of the model.

5. A Simple Alternative Model

The production function is chosen to exhibit variable returns to scale, this being the source of non-linearity in the model. A constant labour input is assumed for the sake of simplicity, so output depends only on the capital input:

$$Y = F(K) = A[\tan^{-1}(K-\alpha) + \tan^{-1}\alpha] \qquad \text{……………………………………………..(2)}$$

where Y = output, K = capital, A and α are non-negative constants. With this specification the marginal product of capital increases with K for $K < \alpha$, decreases for $K > \alpha$ and is momentarily constant for $K = \alpha$. Note that the production function (2) has the property that $F(0) = 0$: in this case, $K = \alpha$ is a *Frisch point* at which returns to scale switch from increasing to decreasing.

A proportional consumption function is assumed:

$$C = cY \qquad (3)$$

where $0 < c < 1$, giving macroeconomic equilibrium condition:

$$Y = cY + \dot{K} + \delta K \qquad (4)$$

where $\delta =$ depreciation rate of capital ($0 < \delta < 1$) and $\dot{K} =$ net investment.[1]

A constant exogenous growth rate (θ) is assumed for the nominal money supply (M):

$$\dot{M} = \theta M \qquad (5)$$

A standard demand for money function of the following form is assumed:

$$\frac{M}{P} = b_1 Y - b_2 r \qquad (6)$$

[1]The 'dot' notation represents time derivatives throughout.

where $b_1, b_2 > 0$, P = price level and r = nominal interest rate. The money market is assumed permanently in equilibrium. Equation (6) yields:

$$r = \left[\frac{b_1}{b_2}\right]Y - \frac{M}{b_2 P} \qquad (7)$$

The nominal interest rate is taken to be the sum of the real interest rate and the perfectly foreseen rate of inflation, and the real interest rate is assumed equal to the marginal product of capital (d). Thus:

$$r = d + \frac{\dot{P}}{P} \qquad (8)$$

This condition ensures that the sum of total profit and total (perfectly foreseen) capital gain ($PF(K) - rPK + \dot{P}K$) is continuously maximised. From (2) we have:

$$d = F'(K) = \frac{A}{1 + (K - \alpha)^2} \qquad (9)$$

Equation (4) gives:

$$\dot{K} = (1 - c)Y - \delta K \qquad (10)$$

Equation (8) gives:

$$\dot{P} = P(r - d) \qquad (11)$$

To permit exposition in two dimensions we combine the nominal money supply and the price level into a single variable, the real money supply (m):

$$m = \frac{M}{P} \qquad (12)$$

From (12), (5) and (11) it is readily seen that:

$$\dot{m} = m(\theta - r + d) \qquad (13)$$

which, from (9) and (7) yields:

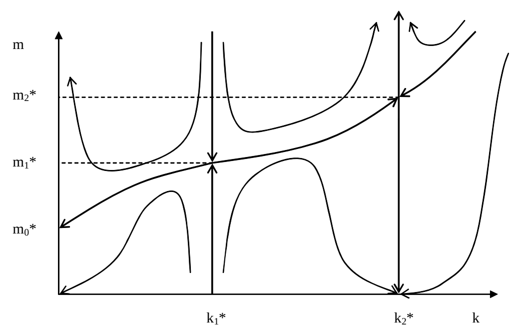
$$\dot{m} = m(\theta - \frac{b_1 F(K)}{b_2} + \frac{m}{b_2} + F'(K)) \qquad (14)$$

Equations (10) and (2) together yield:

$$\dot{K} = (1 - c)F(K) - \delta K \qquad (15)$$

Equations (14) and (15) together constitute a dynamical system in m and K, though note that equation (15) is independent of m. This dynamical system may have two, one or no stable equilibria, depending on parameter values. The phase portrait of this dynamical system (in its 'two stable equilibria version') is depicted in figure 1. Note that only positive values of m and K are considered since negative ones would be economically meaningless. In addition to the two stable equilibria, the dynamical system of figure 1 has three saddlepoints, $(0, m_0^*)$, (k_1^*, m_1^*) and (k_2^*, m_2^*), and one unstable equilibrium, $(k_1^*, 0)$. The 'boundary line' of figure 1 divides the positive quadrant into two segments. Paths lying above this line exhibit m tending to infinity, paths lying below it exhibit m tending to zero and paths actually lying in the boundary exhibit m tending to a positive, finite value (either m_0^* or m_2^*).

Figure 1. Global dynamics of the model



6. Analysing the Model

The standard approach to analysing this type of model would be to linearise it about an equilibrium by (sometimes implicit) appeal to Hartman's Theorem. This theorem ensures that the phase portrait of the linearisation is (in most cases) locally homeomorphic to the phase portrait of the original system. The use of this theorem in the context of dynamic economic models involves considerable difficulties (see George and Oxley, 1985 for a fuller discussion), some of which are associated with the *local* nature of the equivalence. For example, the standard analysis of the model of section 4 would involve linearising it about an equilibrium such as (k_2^*, m_2^*) or $(0, m_0^*)$. These are clearly saddlepoints and the corresponding linearisations have the appearance of figure 2. Note that no equilibria with $m = 0$ appear in the linearisation. This is because Hartman's Theorem applies only locally, in this case in a neighborhood of the equilibrium (k_2^*, m_2^*).

Figure 2. Local dynamics of the model

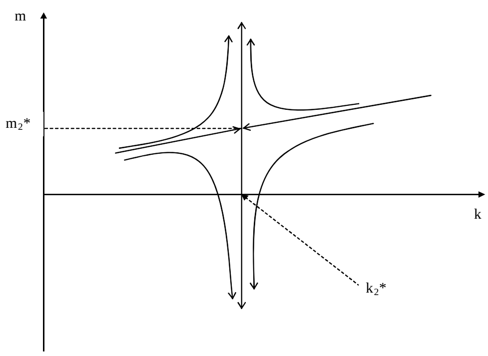


Figure 2 depicts a linear saddlepoint in which there is a single "stable branch". Any paths with initial conditions lying on this stable branch converge to the equilibrium; all other paths exhibit diverging m. In particular paths with initial conditions lying below the stable branch exhibit m diverging to $-\infty$. Since it is economically meaningless for the real money supply to be negative, these paths would be ruled out of consideration. The analysis might be augmented by noting that a divergent path will not in general satisfy the transversality condition arising from a standard intertemporal utility maximising problem (see George and Oxley, 1985 George/Oxley (1985) for a further discussion of this argument).

But how is convergence to be guaranteed? In the standard analysis, the situation is saved by the intervention of so-called 'jump variables', usually prices, which adjust infinitely fast to ensure initial conditions which do lie on the stable branch, thus guaranteeing convergence. In this model the price level could play this role, allowing the real money supply to adjust appropriately. The capital stock would be treated as a "pre-determined" or "backward looking" variable.

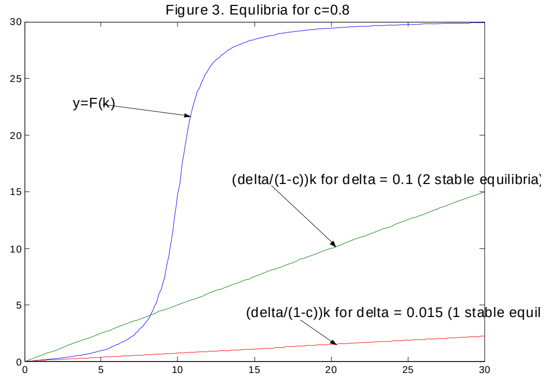
In the linearisation, convergent paths entail the real money supply tending to a positive limiting value as $t \to \infty$, which in turn, implies a steady state rate of inflation equal to the (exogenous) rate of nominal money supply growth. In such a case the steady state rate of inflation is constrained to equal the rate of nominal money growth in true monetarist fashion. But when the model is considered globally, another possibility emerges. There exists a whole additional class of convergent paths, all of which converge to an equilibrium such as $(k_2^*, 0)$ in figure 1, at which $m = 0$. In contrast to the linearisation, the original phase portrait has *no* paths which start in the positive quadrant but eventually leave it. Along paths converging to equilibria such as D or F, the real money supply is *always strictly positive*, but tending to zero. In fact along such paths, the price level cannot accelerate indefinitely. The rate of inflation $(\frac{\dot{P}}{P})$ tends to a limiting (or "steady state") value which, in general, is many times greater than θ, the rate of growth of the money supply.

This situation will not arise of course if the price level jumps so as to place the system initially on the boundary line in figure 1. But why should such a jump occur? The rate of change of prices is determined by equation (9) which is a kind of arbitrage condition in the asset market. The nominal rate of interest is fixed at each instant via equilibrium in the money market and the marginal product of capital is fixed at each instant by the amount of capital in existence. Thus $\dot{P}$ is always finite: *the price level can never jump*. Suppose at time zero the price level is P_0, such that the initial conditions (K_0, m_0) lie below the boundary line in figure 1. Then the economy will follow a path tending to $m = 0$: it may grow or contract (see figure 1 for examples of both types of path). In such cases the steady state inflation rate is many times the growth of the nominal money supply. Note that *all* paths in figure 1 are perfect foresight paths. We have thus demonstrated a simple model with perfect foresight, in which all markets clear continuously and the growth rate of the nominal money supply is exogenous, but which nevertheless has a steady state inflation rate many times greater than the rate of growth of the nominal money supply.

7. Numerical Solutions of the Model

The dynamical system composed of equations (14) and (15) cannot be solved analytically. A simple Matlab program has therefore been written to provide numerical solutions. Parameter values are set at c = 0.8, A = 10, α = 10.0, θ = 0.05, b_1 = 1.028, b_2 = 1.125 and two values of the depreciation rate (δ) are considered. The first ($\delta = 0.1$) generates two stable equilibria (one at the origin) while the second ($\delta = 0.015$) generates a single stable equilibrium. Both cases are illustrated in figure 3. The program iterates for 20 periods. Figures 4, 5 and 6 show time paths for output (Y), inflation rate $(\frac{\dot{P}}{P})$, nominal interest rate (r) and real interest rate (d) for three different sets of parameter values and initial conditions. Figure 4 shows time paths of these variables for an economy with two stable equilibria $(c = 0.8, \delta = 0.1)$. With the initial condition k(1) = 9, the economy grows rapidly between years 2 and 5. The real rate of interest (marginal product of capital) grows at first, but as the Frisch point is crossed, it starts to decline. At this stage, hyperinflation sets in: the rate of inflation increases dramatically, tending to a value of approximately 2757% (many times the rate of monetary growth of 5%).

Figure 3. Equlibria for c=0.8



The nominal rate of interest trends upwards with the inflation rate and output growth peters out as the inflation rate levels off. Thus the economy has experienced an approximately three year period of rapid growth, with inflation many times the rate of monetary growth (which is held constant at 5% at all times).

Figure 4. Growing economy with hyperinflation

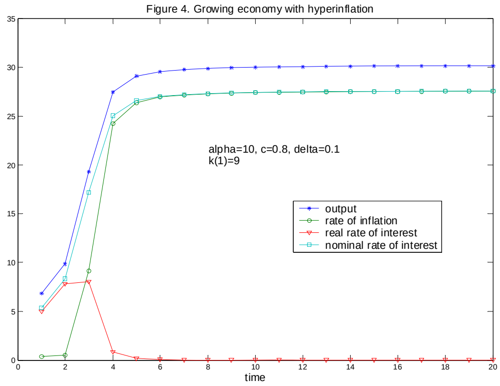


Figure 5 shows time paths of the same variables, with the same parameter values, but with the initial condition k(1) = 8, leading to a contracting economy. In this case output falls steadily along with the real rate of interest (marginal product of capital). The nominal rate of interest and the rate of inflation rise at first but then fall. However, the rate of inflation tends towards 25%, (with monetary growth again held constant at 5%) showing that, even in a contracitng economy there can be high rates of inflation. (We refer to this as 'high', not 'hyper' inflation.)

Figure 5. Contracting economy with high inflation

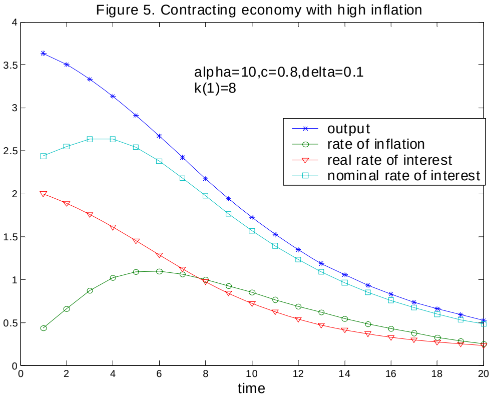
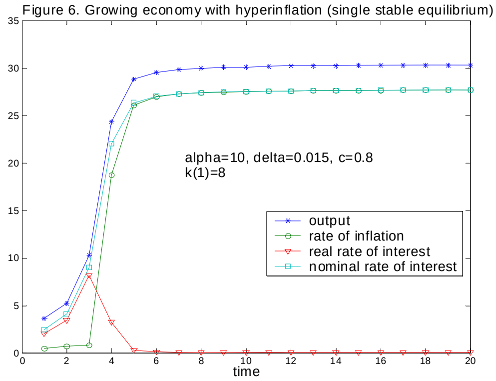


Figure 6 shows timepaths for the same variables, with the same parameter values, except that the depreciation rate (δ) is set at 0.015, generating a model with a single stable equilibrium. The initial value of k is set at k(1) = 8, leading to growth with hyperinflation. Growth is gradual until period 3, when the real rate of interest falls sharply and inflation takes off, reaching a limiting value of 2769%. Again the rate of monetary growth is set at 5%.

Figure 6. Growing economy with hyperinflation (single stable equilibrium)



8. Conclusions

In a perfect foresight, market clearing model with an exogenously growing nominal money supply it is possible for the steady state rate of inflation to be many times greater than the rate of money growth. This type of inflation is of the bootstrap variety and cannot accelerate indefinitely. Nevertheless plausible parameter values indicate, for a growing economy, possible inflation rates of 2000% - 3000%, enough to worry most policymakers. Even for a contracting economy, an inflation rate several times the rate of money growth is a possibility.

These conclusions weaken the orthodox argument that firm control of the money supply is sufficient to control the inflation rate in the medium to long run. They are derived from a more or less orthodox model by the addition of a simple non-linearity and the simultaneous abandoning of local linearisation methods and rejection of the 'jump variable' notion. A full global analysis of the model's dynamics is presented and no *ad hoc* jump mechanism is invoked. Price dynamics are explained simply by the structural equation sof the model. The paper therefore provides an example of how striking analytical (including policy) results may depend as much on the method of analysing a model as on the economic assumptions upon which it is based.

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