Fuzzy Graph Model for Sociometry and its Analysis Methodology

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ABSTRACT The fuzzy graph will make it possible to approximately represent fuzzy information such as human relations and mental processes. Fuzzy information can be modelled by using a fuzzy graph. In order to clarify the main feature of a fuzzy graph for a group of human relations, we would represent it as an approximate graph and extract its characteristics, such as relational and similar structures. But we must proceed to process many kinds of information concerning the structure of a fuzzy graph, and also draw quickly and display comprehensively fuzzy graphs and the related graphs in the analysis process. Fuzzy graphs with a few nodes are quickly drawn, easily and comprehensively analysed by hand. However, it is practically impossible to process a fuzzy graph with more than ten nodes by the same method. Consequently, a new method for approximate analysis of fuzzy graphs is desired. To solve these needs, we have proposed a computer-aided system for analyzing fuzzy graphs through human interaction.

This system can quickly and comprehensively draw a graph arranged on a circular, a partition tree , cluster representative graph corresponding to a cluster and a specified shape of approximate n-valued fuzzy graph. However, it can not draw automatically a graph with nodes on the lattice intersection. So, we would rearrange the nodes on the lattice intersection for any fuzzy graph through man-machine interface. This method is good for traditionally analyzing fuzzy graphs.

We propose an analysis methodology using the lattice type of fuzzy graph display. Here, it plays an important role in this system. In this paper, we describe the analysis methodology, the functions, the man-machine interface of our system and present a case study.

1. INTRODUCTION

Inexact information such as human cognition and evaluation can be modelled by utilizing the concept of a fuzzy graph. By applying the fuzzy graph, it will be possible to quantitatively process fuzzy information. To clarify the global features of a fuzzy graph, we represent it as an approximate graph and extract the characteristics of fuzzy graph, such as relational and similar structures. The information processing requires rapid drawing and comprehensive display of fuzzy graphs and related graphs. A computer-aided analysis method could be effective, but such an effective one have not yet been developed.

To fill these needs, we have proposed a method for analysis of a fuzzy graph and a computer-aided methodology that analyses fuzzy graphs through man-machine interaction. It was developed on a workstation. It is easy to draw a partition tree for a cluster analysis and to draw an approximate fuzzy graph. We could transform display of an fuzzy graph into a shape of a fuzzy graph in which all the nodes are arranged on the intersection of the lattices through man-machine interaction. We call it the lattice type of fuzzy graph display in this paper. It could be more effective to analyse fuzzy graphs. This display plays an important role on our methodology of fuzzy graph analysis. Here, we would not only discuss an analysis methodology of the fuzzy sociogram, but also present its practical effectiveness with the case study.

This paper describes fuzzy graph modelling and analysis methodology in chapter 2, man-machine interface in chapter 3, case study in chapter 4 and concluding remarks in chapter 5.

2. FUZZY GRAPH MODELLING AND ANALYSIS METHODOLOGY

Sociometry which was proposed by Moreno[1960] is one of the measurement and evaluation methods of a group structure by using tables and diagrams.

But it could be more effective to represent human relations by applying a fuzzy graph. With the data from simple questionnaires, we could analyse the connective structure among the members of a group and have the approximate sociogram.

For our analysis, we would build a fuzzy model from the answers of simple questionnaires, apply the fuzzy clusterings and orderings, and reasonably clarify the human relation in a group.

2.1 Fuzzy Graph Modelling

If we ask M questionnaires to L students; "Write your friendly members in order with whom you would like to do activities A_p ($1 \leq p \leq M$) ", and we have the response matrix $K = (k_{ij}^{(p)})$, where $k_{ij}^{(p)}$ is the order number which a student S_i has selected another student S_j. Eexample of questionnares is as follows;

Please write your friends in order with whom;

(A1) you would like to study in group.

(A2) you would like to take lunch together.

Table 1. Questionnars

	1	2	3	. . .
A_1				
A_2				
A_3				

(A3) you would like to do club activity.

Let $n^{(p)}$ be Σ Max{ $k_{ij}^{(p)}$}, and let N be [$\Sigma\, n^{(p)}$ / LM + 0.5], where N means the mean value of the friendly members of whole students in the class room. And, we have evaluation matrix $R = (r_{ij})$, where $r_{ij} = n^{(p)} N - \Sigma\, k_{ij}^{(p)} + n$ with $r_{ij} \geq 0$.

Analyzing the matrix $R = (r_{ij})$, we could define the fuzzy friendly degrees and prefering degrees among the members in the class. Let $f_{ij} = r_{ij}/N$, and we consider f_{ij} as the degree of fuzziness of which a member S_i prefer the member S_j. We have a fuzzy matrix $F = (f_{ij})$, $0 \leq f_{ij} \leq 1$, where $f_{ij} = 1$ if $i = j$.

On the other hand, a graph was defined mathematically as $G = (V,F)$, where $V = \{v_i\}$ is the set of nodes and $F = \{f_{ij}\}$ is relation matrix, here f_{ij} is the value of arc from v_i to v_j. A graph is called a fuzzy graph if its value is between 0 and 1. The fuzzy matrix F is equivalent to a fuzzy graph.

Here, we could build a fuzzy model representing to the connective structure among the members in a group.

2.2 Cluster Analysis of Fuzzy Graph

In this section, we would illustrate methods of cluster analysis for a fuzzy graph. Let $2/s_{ij} = 1/f_{ij} + 1/f_{ji}$, and we consider s_{ij} as the degree of the fuzziness of which two members S_i and S_j are mutually friendly and we have a fuzzy matrix $S = (s_{ij})$, $0 \leq s_{ij} \leq 1$, and $s_{ij} = 0$, if $f_{ij}f_{ji} = 0$. Here, $s_{ij} \sim 1$ means that S_i and S_j are very friendly, and $s_{ij} \sim 0$ does that they are scarcely friendly.

In order to analyse the similarity structure of nodes for a fuzzy graph, we use the symmetric relational matrix $S = (s_{ij})$ and compute it transitive closure $\hat{S} = (\hat{s}_{ij})$. After that, we define the c-cut matrix S^c as follows:

$$S^c = (s^c_{ij}) \ , \ s^c_{ij} = \begin{cases} 1 & (\hat{s}_{ij} \geqq c) \\ 0 & (\hat{s}_{ij} < c) \end{cases}$$

The relation R_c of the matrix S^c gives a similarity relation among nodes. Thence, we have the partition tree by changing the level c of the c-cut matrix which represents the clustering situation of nodes in a fuzzy graph. By analysing S, we could have a partition tree (dendrogram) P which shows the clustering situation and the branching process among members.

2.3 Analysis Method of Fuzzy Graph

In this section, we would illustrate methods of approximate expression for a fuzzy graph. In order to analyse the global structure of a fuzzy graph $F = (f_{ij})$, we define the p-graph F^p of F for $p \in [0, 0.5)$.

Thence, the maximal decision p* of the fuzzy decision

$f_G(p) = f_D(p) \wedge f_E(p)$ [Yamashita 1995]

should be an optimal value of p, and we could optimally have the approximate ternary graph F* of F by using p*.

Finally, summaring the graph F* and the tree P, we have the approximate sociogram U*, where z is the friendly level of each minor groups.

2.4 Analysis Process

Our goal is to develop computer-aided analysis methodology for analyzing fuzzy graph through man-machine interaction, which can efficiently analyse its features. This is composed of three parts: input, processing and man-machine interaction.

(1) Input part

A response matrix $K=(k_{ij}^{(P)})$ is input by keyboarding. If the input data is in error, an user can correct it by using keyboard and mouse. The process executes either similar-structure or relational structure analysis according to each user direction.

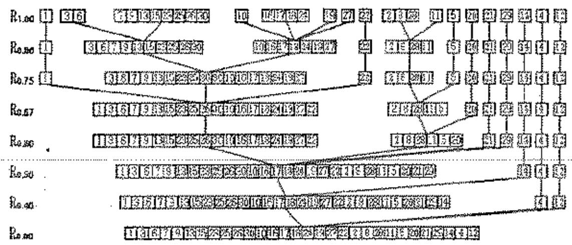


Figure 1. Partion tree

(2) Processing part

The system executes either similar-structure or relational structure analysis. The algorithm of similar-structure analysis processing is as follows:

(a) Compute a transitive closure matrix

(b) Determine the degree of the rank R_z of the cluster analysis

(c) Determine the order of the node in the first rank

(d) Cluster the nodes according to previously determined R_z

(e) Classify the R_z and draw and display a partition tree (Fig.1)

The algorithm of relational structure analysis is as follows:

(f) Compute the p-graph for the fuzzy graph F

(g) Compute the distance function d(p) between F and F_p

(h) Compute the vagueness function e(p) of F

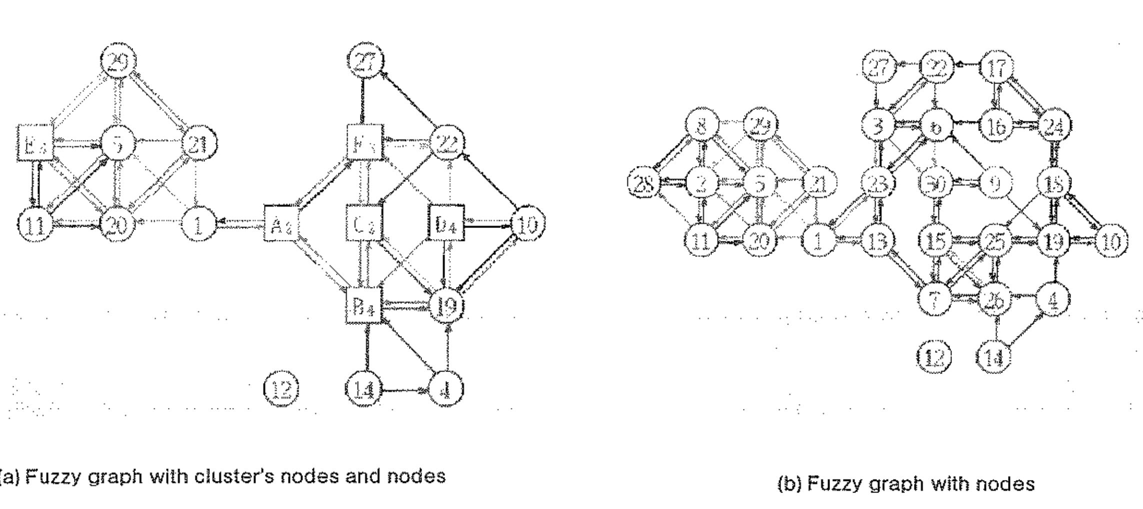
(i) Compute the maximal decision p* based on d(p) and e(p)

(j) Compute an approximate N-valued (ternary) graph and draw and display it

(k) Freely change the shape of the graph for easy analysis

(3) Man-machine interaction

A partition tree is firstly displayed in similar-structure analysis and a fundamental fuzzy graph is firstly displayed in relational structure analysis. These shapes are rearranged and/or changed in or-



(a) Fuzzy graph with cluster's nodes and nodes

(b) Fuzzy graph with nodes

Figure 2. Example of the lattices type of fuzzy graph display

der to easily grasp the global structure of the fuzzy graph and to comprehensively understand it. Finally we would rearrange nodes on the lattice intersection for any fuzzy graph through man-machine interaction. This is start of our analysis in new methodology. This display is good for traditionally analysing fuzzy graphs.

3. MAN-MACHINE INTERFACE

Fuzzy graphs have been traditionally analyzed by the trial-and-error method, because the algorithm has not yet been established. Hence we have been developing a methodology to help easy and comprehensive analysis for a fuzzy graph through human interaction. Man-machine interaction are carried out through the graphic user interface.

We would firstly transform a display of an fuzzy graph into one in which all the nodes are rearranged on the intersection of the lattice through man-machine interface as shown in Fig.2. It could be more effective to analyze an approximate fuzzy graph.

3.1 Concept of Man-machine Interaction

We have improved the methodology to analyse fuzzy graph so as to do on the lattices type of fuzzy graph display in which all the nodes are rearranged on the intersection of the lattices. Its methodology could be more effective to analyze an approximate fuzzy graph.

We have improved man-machine interface from a practical viewpoint as follows;

(1) Speedy computation ; to process speedily such computations without man-machine interaction as matrix computation.

(2) Comprehensive display ; to display clearly and comprehensively such information concerning to man-machine interaction as a shape of a fuzzy graph in which all the nodes are rearranged on the intersection of the lattices.

(3) Display of supporting man-machine interaction ; to display such information to support man-machine interaction as the numbers of cross-arcs of fuzzy graph at real time

3.2 Fundamental Functions

For a fuzzy graph on a workstation, the maximum permissible number of nodes is about 50 nodes. The fundamental functions for man-machine interface are described as follows.

(1) Fuzzy Graph Displays Functions; Color display of arc, Zoom in and zoom out functions, Display of partition tree P and Display of N-valued graph.

(2) Fuzzy Graph Transformations Functions; Display based on a specified node, Rearrangement of nodes for reducing cross-arcs, Rearrangement of nodes by hand, Making and breaking clusters for a fuzzy graph and Arrangement of nodes on the lattice intersections.

(3) Additional functions for man-machine interaction; Comprehensive display, Display of supporting man-machine and Multi-evaluations.

4. CASE STUDY OF MAN-MACHINE INTERACTION

As a case study, we would show a practical analysis of a fuzzy sociogram in primary school. We asked the questionnares to 30 students. As the results, we have obtained the response matrix K=(k_{ij}). From the response matrix K, we had the evaluation matrix R=(r_{ij}). By analysing the evaluation matrix R, we had the fuzzy sociogram F. and also its approximate graph F*. By analysing the evaluation matrix R, we can also had the partition tree P as shown in Fig. 1. By summarizing F* and partition tree P, we had the approximate sociogram U_z with z=1.00 and z=0.67, which is shown in Fig 3. This display is represented by a graph which all nodes

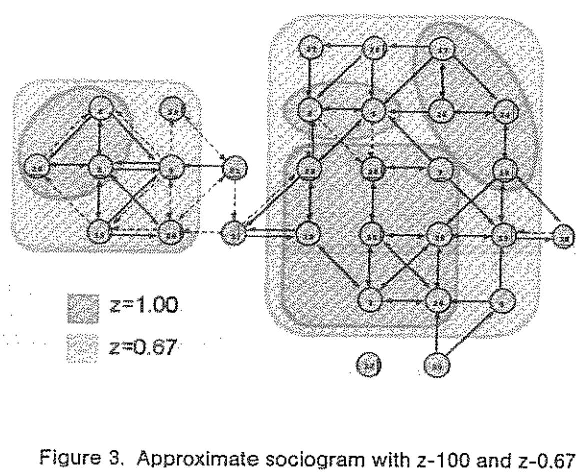


Figure 3. Approximate sociogram with z-100 and z-0.67

are arranged on the intersections of the lattice.

Shape of fuzzy graphs, which all nodes are arranged on the intersections of the lattice, can be easily transformed from any shapes of fuzzy graphs through man-machine interaction .

An analysis of new method starts from the lattice type of fuzzy graph display.

4.1 Analysis of Subgroups

The approximate sociogram U_z with z=1.00 and z=0.67 is shown in Fig.4. Here, two kinds of subclusters approximated with z=1.00 and z=0.67 is shown in it. We can understand the relations of members in this classroom through the clusters. This classroom is composed of two subgroups with z=0.67, and four subgroups with z=1.00. We can more clearly analyse relations of subgroups from the viewpoints of whole organization with every z (the fuzzy friendly degrees).

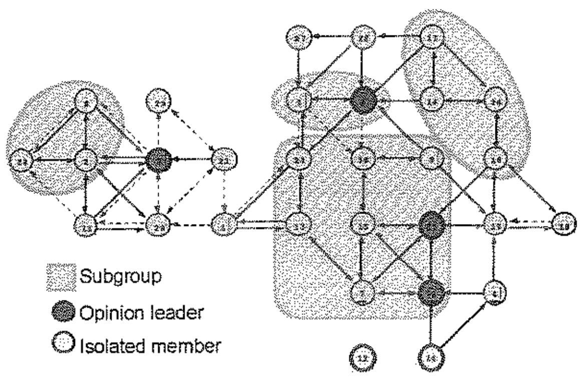


Figure 4. Example of fuzzy graph analysis(1)

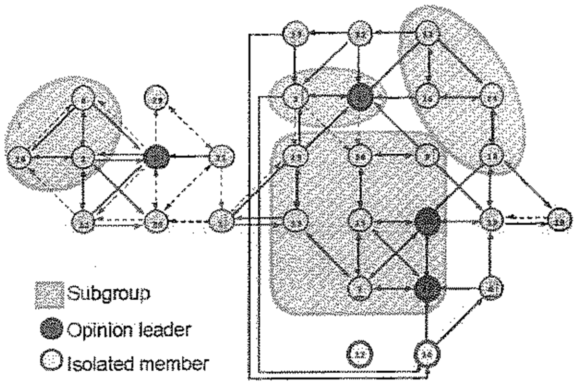


Figure 5. Example of fuzzy graph analysis(2)

4.2 Opinion Leaders, Isolated Members

Four subgroups, four opinion leaders and two isolated members are shown in Fig. 4. The relations of opinion leaders among subgroups and isolated members in the classroom are clarified in Fig. 5. We understand clearly the relations of opinion leaders among subgroups and isolated members from the fuzzy graph, in which all nodes are arranged on the intersections of the lattices.

We found the following reasons for the relations among students in the class room.

(i) Members 5,6,25 and 26 are opinion leaders in the classroom.

(ii) Members 12 and 14 are isolated ones in the classroom.

In addition to the above information, we may observe the relations of between isolated members and the other members as shown in Fig. 5. So we can see the practical relation of the isolated members and the others.

4.3 The Other Information

(i) Supporters for opinion leaders and secondary suporters for opinion leaders(Fig. 6)

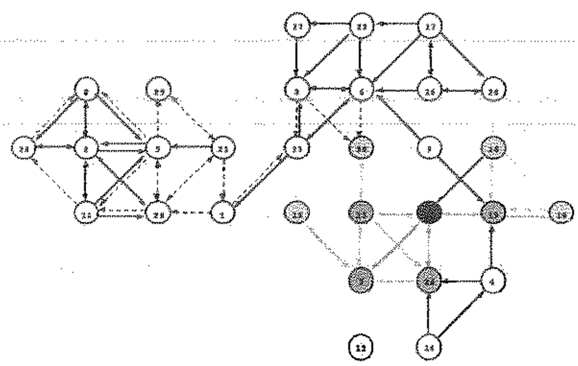


Figure 6. Example of fuzzy graph analysis(3)

(ii) A group of friends(Fig. 7)

With these analysis and information, we could effectively investigate the connective structure among students and manage human relations in the classroom.

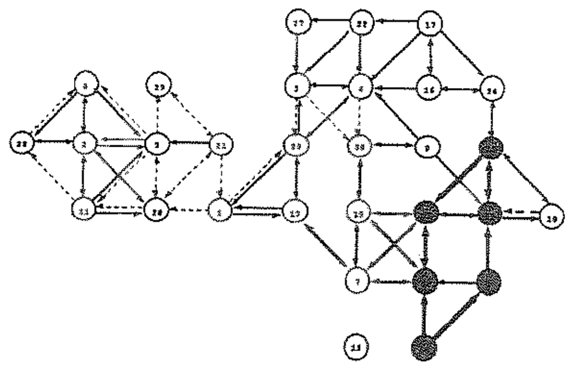


Figure 7. Example of fuzzy graph analysio(4)

5. CONCLUDING REMARKS

We have proposed an analysis methodology for a fuzzy graph through man-machine interaction based on the shape of a fuzzy graph in which all the nodes are rearranged on the intersection of the lattices. This can effectively analyse the fuzzy information and make a reasonable decision on fuzzy phenomena. Using this methodology, it is possible to analyze speedily, comprehensively and easily similar-structure and relational structure of fuzzy graphs.

Two main problems remain to be solved in the near future, to enhance this analysis methodology. One is the improvement of the man-machine interface in the rearranging nodes to reduce the numbers of the cross-arcs in a fuzzy graph with many nodes and in the arranging nodes on the intersections of the lattice. The other is the addition of other functions for our purpose. In the future, it is desired to draw automatically the shape of fuzzy graph in which all the nodes are rearranged on the intersection of the lattices.

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