A Study of Multi-stage Flowshop Scheduling Problem with Alternative Operation Assignments

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ABSTRACT In small-item large lot production, one work person had only to do single processing. A recent tendency of the production system is the multi-item small lot production. In this type of production system, one work person should have a lot of skills adaptable to multi-item articles. Because of the fact that one work person has to have more than one skill, one processing operation which composes a job has come to be handled by several work persons. From the viewpoint of a job, one elementary operation has become to be processed by several alternative operation assignments.

The scheduling problem mentioned below is assumed to be operated under the above-mentioned situation. The n jobs processed by three work persons (or machines) are given, and either of n jobs is composed of 5 elementary operations. As for the 1st, 3rd and 5th operations, they are assigned decisively to certain work persons. As for the 2nd operation, it can be processed by either the work person who processes the 1st or the work person who processes the 3rd. As for the 4th operation, it can be processed with either the work person who processes the 3rd or the work person who processes the 5th.

Under these conditions we discussed a problem how to minimize the total elapsed time from the start time up to the completion time of a job. In order to solve this problem, we analyzed it theoretically in the first place under a simpler condition. Then we proposed an approximation algorithm based on the theoretical analysis applicable to the more generalized problem. We had a number of numerical simulation runs to make a comparison of advantages and disadvantages of various approximation methods.

1. INTRODUCTION

This study aims at discussing the problem where the assignment of processes to work persons are flexible and the operation time is different from an assignment to another. We are to obtain the optimal schedule which gives the minimum total elapsed time in the case where the above-stated operations are processed in the flowshop lines. For example, we consider producing the electric parts of plural kinds which consist of five elementary works of soldering, operation confirmation, painting, externals inspection, and wrapping. These electric parts is produced by three work persons (Fig.1). It is assumed that work person M1, work person M2, and work person M3 take charge of soldering, painting and wrapping, respectively.

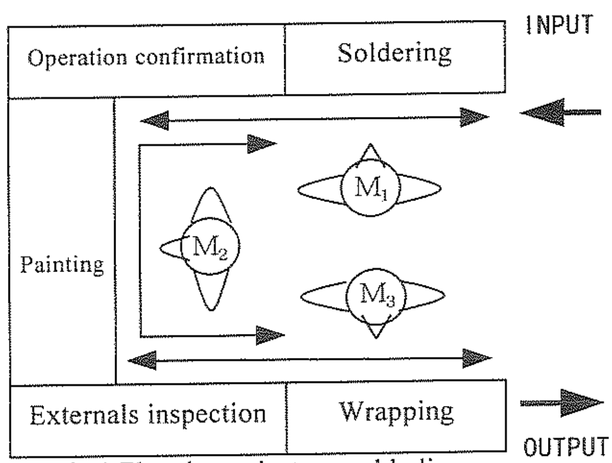

Fig.1 Electric product assembly line

Work person M1 and work person M2 can also process the operation confirmation, and work person M2 and work person M3 can also process externals inspection. For this case, we aim at setting up the production schedule during a day in which the time from the beginning of work to the completion of the entire work is minimized.

Z.Nakamura and I.Watanabe[1] proposed an optimal solution for two-stage flowshop scheduling problem in which one of the three operations have flexible assignment to work persons. As for the three-stage flowshop scheduling problem, I.Watanabe and T.Nakanishi[2] proposed eight kinds of approximation methods, and made a comparative study. The optimal method which Z.Nakamura and I.Watanabe[1] proposed were applied to these approximation methods. Y.Futatsuishi and I.Watanabe[3][4] proposed a new approximation method through an approach different from the past one. In addition, a new approximation method and the past approximation method were compared with each other.

In short, this problem is the three-stage flowshop scheduling problem in which two of five operations have flexible assignments to work persons (or machines) (Fig.2). We analyzed theoretically a problem where all the processed items are of the same property. Then we proposed approximation method based on theoretical analysis.

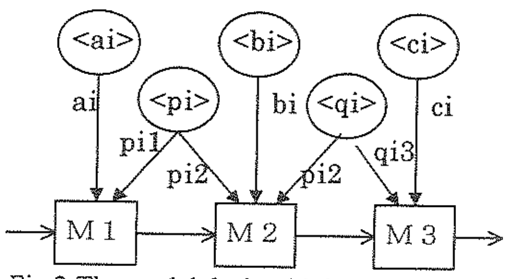

Fig.2 The model dealt with in the paper

In addition, the advantages and disadvantages of our proposed approximation method and the past approximation method were compared through numerical simulation in this research study.

2. FORMULATION OF THE PROBLEM

①The n processed items are given and each processed item goes through the processes of M1, M2 and M3 in this order.

②Each processed item i(i =1,2,⋯,n) has five production elements <ai>, <bi>, <ci>, <pi> and <qi>. <ai>, <bi> and <ci> are processed by M1, M2 and M3 respectively. <pi> can be processed by either M1 or M2. <qi> can be processed by either M2 or M3.

③Processed item i is called (Ⅰ,Ⅱ)-type job when <pi> is assigned to M1 and <qi> is assigned to M2, (Ⅰ,Ⅲ)-type job when <pi> is assigned to M1 and <qi> is assigned to M3, (Ⅱ,Ⅱ)-type job when <pi> is assigned to M2 and <qi> is assigned to M2, and (Ⅱ,Ⅲ)-type job when <pi> is assigned to M2 and <qi> is assigned to M3.

④Processing time of production elements <ai>, <bi>, <ci>, <pi> and <qi> for each processed item i is given by ai, bi, ci, pij and qij respectively.
pij designates the processing time of <pi> on Mj (j=1,2). qij designates the processing time of <qi> on Mk (k=2,3).

⑤Only a job can be in process on one operation at a time. Once an operation starts on Mj (j=1,2,3), another operation has to wait until the preceding operation is over.

⑥Mj (j=1,2,3) can handle at most one operation at a time.

⑦Each processed item is available at time zero.

Under the above-mentioned condition, we discuss the scheduling problem for minimizing total elapsed time. The schedule is given according to the decision of the job type for each processed item and of the processing sequence for all the processed items.

3. THEORETICAL ANALYSIS UNDER THE SIMPLEST CONDITION

Here we add the following two conditions⑧and ⑨to the conditions①through ⑦in section 2.

⑧The processing time with <ai>, <bi>, <ci> for every i is X and definite.

⑨The processing time of production element <pi> even if it was processed with any of the M1 and M2, is Y1 and definite. The processing time of production element <qi> even if it was processed with any of the M2 and M3, is Y2 and definite.

For the discussion hereafter, schedule S is shown by S=(1,2,⋯,n) or S=(α , β , γ). Here α , β and γ are the subsequences of processed items.

We clarified the characteristics of the conditions which restrict the range of the existence of optimal schedule. They are as follows.

Property 1. There exists an optimal solution where job 1 is (Ⅱ,Ⅲ)-type job and job n is (Ⅰ , Ⅱ)-type job.

Some properties are enumerated in consideration of the condition of Y1 and Y2. Hereafter, only the range of Y1≦Y2 will be chosen because there is a symmetry between Y1≦Y2 and Y1≧Y2.

i) An optimal schedule under the condition Y1 ≦Y2

Property 2. There exists an optimal schedule among the schedules where S=(α , β). Here α is composed of only (Ⅱ,Ⅲ)-type jobs and β is composed of (Ⅰ ,Ⅲ)-type jobs or (Ⅰ , Ⅱ)-type jobs.

ii) An optimal schedule under the condition Y1=Y2

Property 3. If n=3m, there exists an optimal schedule S* where (Ⅱ , Ⅲ)-type jobs are composed of job 1 to job m, (Ⅰ ,Ⅲ)-type jobs are composed of job m+1 to job 2m, and (Ⅰ , Ⅱ)-type jobs are composed of job 2m+1 to job 3m. The total elapsed time is T(S*)=(n+2)X +(2m+1)Y.

Property 4. If n=3m+1 or n=3m+2, there exists an optimal schedule S* where (Ⅱ,Ⅲ)-type jobs are composed of job 1 to job m+1, (Ⅰ ,Ⅲ)-type jobs are composed of job m+2 to job 2m+1 and (Ⅰ , Ⅱ)-type jobs are composed of job 2m+2 to job n. The total elapsed time is T(S*)=(n+2)X +(2m+2)Y

iii) An optimal schedule under the condition 2Y1≦Y2

Property 5. If n=2m, there exists an optimal schedule S* where (Ⅱ , Ⅲ)-type job is composed of job 1, (Ⅰ , Ⅱ)-type jobs and (Ⅰ , Ⅲ)-type jobs of job 2 to job n-1 are alternately arranged m-1 times, and (Ⅰ , Ⅱ)-type job is composed of job n. The total elapsed time is T(S*)=(n+2)X+Y1+mY2.

Property 6. If n=2m+1, there exists an optimal schedule S* where (Ⅱ , Ⅲ)-type job is composed of job 1, (Ⅰ ,Ⅲ)-type jobs and (Ⅰ , Ⅱ)-type jobs composed of job 2 to job n are alternately arranged m times. The total elapsed time is T(S*)=(n+2)X+Y1+(m+1)Y2.

4. APPROXIMATION METHOD APPLIED TO MORE GENERALIZED PROBLEM

In this chapter, we aim at finding a good approximation method. Here, there are two kinds of compared approximation methods.

Approximation method no.1 (it is called AP1 in the following) is a method that has already been shown by S.Nakayama and I.Watanabe[5]. AP1 generates n! processing sequences every time the type of one job is changed (Refer to step2 of AP1 mentioned below). Then, approximation method no.2 (called AP2 later) proposed by this research study decides the processing sequence at the end after having changed and decided the type of job. Therefore, the number of generated schedules decreases more greatly than those of AP1.

The procedure of each approximation method is shown as follows.

[AP1]

step1. Change in the type of job

< the first time >

All the jobs to be processed are (Ⅱ, Ⅱ)-type jobs.

< after the second time >

Job i where pi1/pi2 or qi3/qi2 is minimized is chosen, and the type of job is changed according to Fig.3.

step2. Decision of the processing sequence

The processing sequence is decided through all searches methodology.

Step3. Comparison of the total elapsed times

The present schedule is considered to be the best solution when the total elapsed time is smaller than the that of past one. Go to step 1. Otherwise, the total elapsed time of the past schedule is assumed to be the best solution. Go to step 4.

step4. Decision of the best solution

The present best solutions is compared with the schedule in which the first job is changed to (Ⅱ, Ⅲ) -type job and the n-th job to (Ⅰ, Ⅱ) -type job in terms of total elapsed time. The schedule with smaller total elapsed time is assumed to be the best solution. The end.

The object of change	Change in the of job
About I where pi1/pi2 is minimized	(Ⅱ, Ⅱ) → (Ⅰ, Ⅱ) -type job
	(Ⅱ, Ⅲ) → (Ⅰ, Ⅲ) -type job
About I where qi3/qi2 is minimized	(Ⅱ, Ⅱ) → (Ⅱ, Ⅲ) -type job
	(Ⅰ, Ⅱ) → (Ⅰ, Ⅲ) -type job

Fig 3 METHOD OF CHANGE IN TYPE OF JOB

[AP2]

step1. Change in the type of job

< the first time > All the jobs to be processed are (Ⅱ, Ⅱ)-type jobs.

< after the second time >

The total processing time of M1 is compared with that of M3. If the former is smaller than the latter, the job with smaller pi1/pi2 is chosen, and the job type is changed according to the upper row of Fig.3. In the opposite case, the job with smaller qi3/qi2 is chosen, and the job type is changed according to the lower row of Fig.3.

step2. Comparison of processing time

To step1 when the total of the processing time of M2 is larger than that of M1 or M3. Otherwise, go to step3.

step3. Decision of the processing sequence

The processing sequence is decided as follows. First arrange the jobs in the order of (Ⅱ, Ⅲ)-type jobs, (Ⅱ, Ⅱ)-type jobs, (Ⅰ, Ⅲ)-type jobs, (Ⅰ, Ⅱ) -type jobs. But when there are plural (Ⅱ, Ⅲ)-type jobs, process the job with smaller processing time in advance. When there are plural (Ⅰ, Ⅱ)-type jobs, process the job with smaller processing time later. The total elapsed time is calculated.

step4. Decision of the best solution

(Ⅰ, Ⅲ) -type job and (Ⅰ, Ⅱ) -type job are alternately arranged based on the schedule of step3, and the total elapsed time is calculated. And consider here the order exchange of (Ⅰ, Ⅲ) -type job and (Ⅰ, Ⅱ) -type job which satisfies property 2, and the total elapsed times of both are calculated. The smaller one with the total elapsed time is assumed to be the best solution. The end.

(However, the selected schedule here has to satisfy property1)

Step4 of AP2 takes into consideration property 1, 2, 3, and 4. Step5 of AP2 takes into consideration property 5 and 6 in addition to the above-mentioned one.

5. NUMERICAL SIMULATION AND ITS RESULTS

5. 1 NUMERICAL SIMULATION

The content of the numerical experiment is as follows.

(i)THE PUROPOSE OF EXPERIMENTAL

We aimed at comparing AP2 proposed in this paper with conventional AP1, and confirming the validity of theoretical analysis. Thus, we payed attention to whether the operations $<p_i>$ and $<q_i>$ have flexible assignment. And we set up a problem as follows to consider what influence is affected upon the solution by the change of processing time.

(ii) PROBLEM FORMULATION

The number of processed items is fixed to eight. Processing time a_1, b_i, and c_i were given by the uniform integer random numbers of 30～40. Processing time pi and qi were given by five kinds of uniform integer random number (10～20, 20～30, 30～40, 40～50, and 50～60) as a range of the processing time. However, (pi1, pi2) and (qi2, qi3) are given by random numbers in the same range. Thus, the total patterns of processing time pi and qi are of 25 patterns(=5×5). For instance, pi is given within the range of 30～40, and qi is given within the range of 10～20. Next, consider pi is given within the range of 10～20, and qi is given within the range of 30～40. These cases have the symmetric property. Therefore our experiment were done with about 15 patterns (Fig.4). The 200 problems were made with each pattern, thus the total number of numerical simulation reached 3000(=15×200).

(iii)EVALUATION INDICES

The solutions of AP1 and AP2 were compared. Our evaluation indices are as follows, ① DIFFERENCE: Average value of difference between the total elapsed time of AP1 and the total elapsed time of AP2, ②RATIO1: Average value of ratio(%) of the total elapsed time of AP2 to the total elapsed time of AP1, ③RATIO2: Ratio (%) of the number of problems in which a solution with good total elapsed time by AP2 was used.

5. 2 COMPUTATION RESULTS

The CPU time of calculation to obtain one solution was 0.5 seconds for AP2 and 2.6 seconds for AP1 on the average. It has turned out that there is not too much difference between the total elapsed time of AP1 and that of AP2, although the calculation time has been improved to one fifth.

As computation results, the evaluation indices by the pattern of processing time pi, qi are shown in Fig.4. The result of the analyses are enumerated as follows.

pi \ qi	Evaluation	10～20	20～30	30～40	40～50	50～60
10	Difference	7	12	21	23	23
～	Ratio1(%)	105	107	108	108	107
20	Ratio2(%)	7	3	0.5	0	0
20	Difference		5	9	25	32
～	Ratio1(%)		104	109	109	109
30	Ratio2(%)		20	9	1	0.5
30	Difference			3	6	21
～	Ratio1(%)			103	104	108
40	Ratio2(%)			25	16	2
40	Difference				2	4
～	Ratio1(%)				102	102
50	Ratio2(%)				29	22
50	Difference					2
～	Ratio1(%)					102
60	Ratio2(%)					30

Fig.4 Comparison between patterns of processing time pi and qi

1) When processing time pi and qi are almost equal (column on the diagonal part in Fig.4), an acceptable result is obtained even by AP2. This is because, the situations conforms nearly to the condition of property 3 and property 4. The evaluation indices worsens as the processing times pi and qi go away from each other.

2) Even if the evaluation indices of ① or ③ are bad, ratio1 exists within 109%.

6. CONCLUSIONS AND FUTURE PROBLEMS

In this study, the following results have been obtained.

1) We conducted theoretical studies with regard to three-stage scheduling problems where two elementary operations out of five have flexible assignment to work persons, under the special added conditions as to processing time of each operation, and have obtained the sufficient conditions for optimal schedule.

2) AP2 could decrease the search times as a result of application of the theory obtained in the chapter3.

3) Numerical experiments have been conducted in order to compare and evaluate the two approximation methods. It has turned out that average value of ratio of the total elapsed time of AP2 to the total elapsed time of AP1 was within 109% though the search number of AP2 was less than that of AP1. It has proved that the good solution has been obtained when processing time pi and qi are nearly equal.

4) As for the schedule obtained by AP1, the types of the job mix irregularly. On the other hand, the schedule obtained by AP2 divides into (Ⅱ, Ⅲ) -type job, (Ⅱ, Ⅱ) -type job, and (Ⅰ, Ⅲ) -type job and (Ⅰ, Ⅱ) -type job, and line up in this order.

How to cope with the situations where processing times ai, bi and ci are different, and the differences of processing times pi and qi are large, is the future problem to be solved.

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