# Analysis of A Dynamic Model of Legal Process Behaviour

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**Abstract**. Legal systems are complex and difficult to understand. Up to now, much of the effort on the part of information scientists to develop improved understanding of legal systems has been directed towards understanding and interpreting the static logical structure of these systems. But, like almost all complex systems that exist and participate in the world we live in, legal systems frequently also have a complex behaviour that is not immediately evident from observations of their static, logical structure. In order to deal effectively with these systems, it is necessary to have an understanding not only of their structure, but also of their behaviour. In this study, we describe how such behavioural understanding can be obtained by presenting a dynamic behavioural model associated with the process of handling a resource consent request as it has been specified in the Resource Management Act (RMA) of New Zealand. Coloured Petri net formalism have been used in representing these models. The models are executable and can be run for different experimental situations to examine the various "what if" scenarios. In particular, the goal of this paper is to take advantage of the formal properties of Coloured Petri nets and analyse the models for some properties, such as reachability and deadlock, without the need to perform exhaustive simulation studies.

## 1. INTRODUCTION

We have constructed a legal process model of significant components of the RMA by using high-level Coloured Petri nets to specify the relationships between interacting processes specified in the Act. Even though modelling high-level, human-oriented domains, such as legal systems, is difficult, we argue that a suitable dynamic model can potentially provide a qualitative description of system behaviour that can assist in attempts to avoid undesirable outcomes [Purvis *et al.* 1994].

Once the resource consent process was constructed, it was executed for various "what if" scenarios where different model parameters were varied, and the benefits of examining the simulation results with regard to better understanding of the problem domain and fine tuning of the model itself was discussed in previous papers [Purvis 1998,99]. In the present paper, after a brief discussion of the RMA model representation, the dynamic behaviour of the model has been examined by means of formal analysis. In addition to their use for behavioural modelling by means of computer simulations, Coloured Petri net [Jensen 1992] models can also be used for formal analysis. This is a feature of Coloured Petri nets that is not offered by other simulation languages. The formal analysis properties that will be considered in this study are boundedness, reachability, liveness, and deadlock.

## 2. MODELLING RESOURCE CONSENTS

The activities associated with the resource consent process were mapped to the Coloured Petri net formalism by representing various sub-processes (activities) as Petri net transitions. Petri net tokens are used to represent various conditions, artefacts (such as resource consent applications), and resources such as government officials. Ordinarily, an applicant submits an application requesting for a resource consent. The application is evaluated by a governing authority, first to see if the application form is adequate, and then to see if the consent should be granted. The evaluation process may involve reports written by technical consultants or domain experts, and there may be a necessity of publicly notified hearings. The minimal requirements and legal time-limits for these activities are specified in the Act [New Zealand Government 1991]. A hierarchical model of these activities have been developed and published in previous papers [Purvis 1998,99]. Due to space limitations, only a portion of this model has been shown in Figure 1. The Petri net model in Figure 1 represents the process that corresponds with the initial evaluation of a resource consent application to see whether the information submitted for a proposed activity is adequate or not. If a given resource consent application is going to require public notification, then, in addition to the regular requirements, the application will need to be examined by environmental experts. These processes are depicted on the right-hand portion of Figure 1.

*Initial tokens for the Applications place*

*1`{id = 1,comp = true,notify = true,accepted = true,was_evaluated = false,*
*heard = delegation,appeal = false,s_time = 50,*
*e_req = (0,1,0,2,2),n_req = (0,1,0,10,3),d_req = (0,1,0,3,4),*
*h_req = ((0,1,0,3,2),(1,1,1,15,3),(1,2,1,15,5))}*
*+ 1`{id = 2,comp = true,notify = true,accepted = true,was_evaluated = false,*
*heard = sub_com,appeal = false,s_time = 50,*
*e_req = (0,1,0,2,2),n_req = (0,1,0,10,3),d_req = (0,1,0,3,4),*
*h_req = ((0,1,0,3,2),(1,1,1,15,3),(1,2,1,15,5))}*
*+ 1`{id = 3,comp = true,notify = false,accepted = true,was_evaluated = false,*
*heard = delegation,appeal = false,s_time = 130,*
*e_req = (0,1,0,2,2),n_req = (0,1,0,10,3),d_req = (0,1,0,3,4),*
*h_rcq = ((0,1,0,3,2),(1,1,1,15,3),(1,2,1,15,5))}*

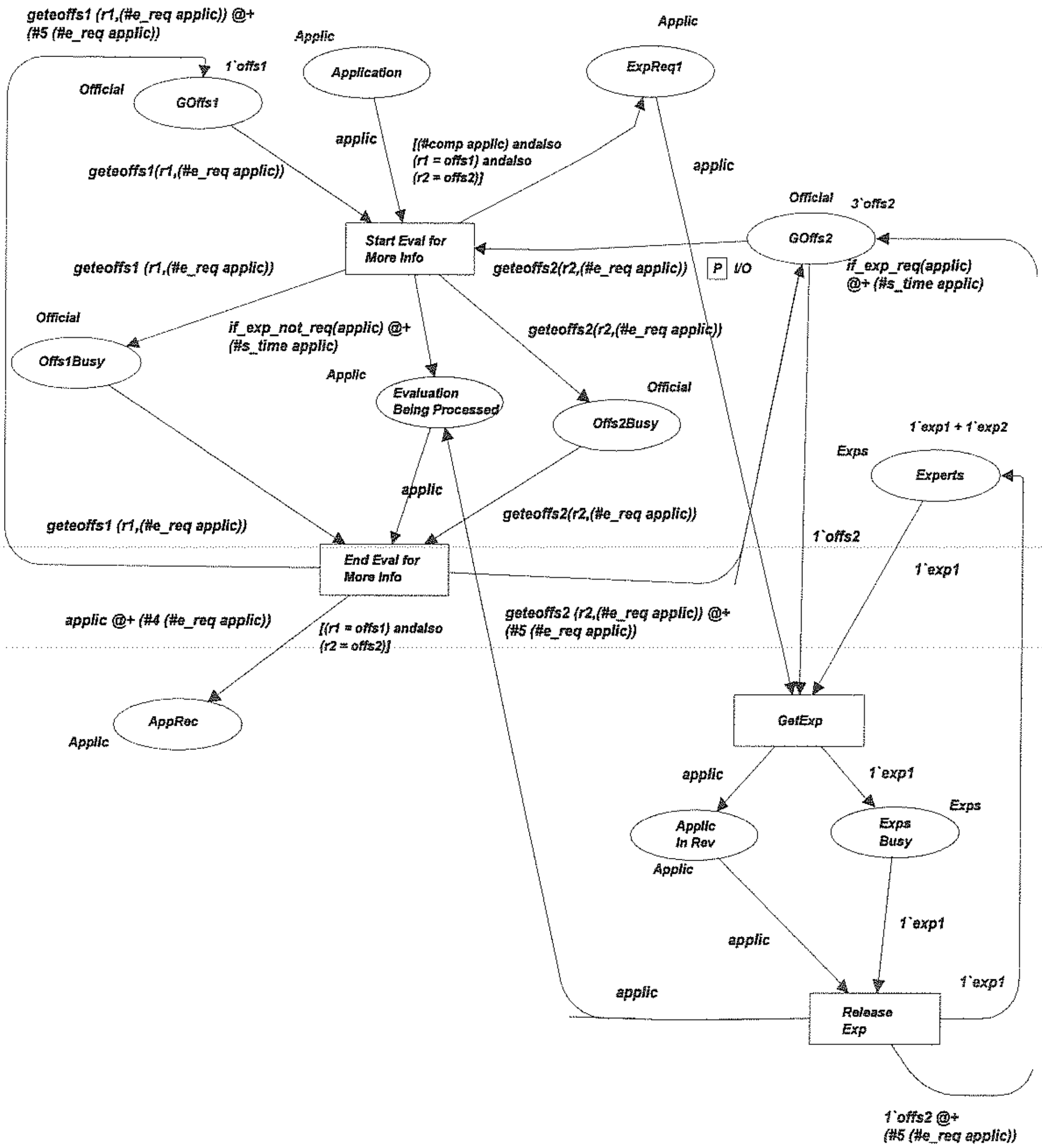


**Figure 1**. Petri net model for evaluating whether an application requires more information.

If the consent application does not require public notification, then only the processes on the left-hand side of Figure 1 are involved.

## 3. SIMULATION AND ANALYSIS

What makes the precise behaviour of such a model difficult to predict is that model performance (in terms of number of applications processed per unit time) is significantly affected by

(a) the number of government resources and administrative personnel available,
(b) by the number of applicants that are submitted in a given time,
(c) the number of applications that are already in the system at the beginning of a given time period (the "backlog"), and
(d) the percentage of submitted applications that need to be passed before a subcommittee hearing (a time-consuming component of the resource consent granting process).

In order to predict the performance for a particular government authority, it is necessary to supply the model with appropriate figures for these parameters and then run experimental simulations. During this study a number of simulation experiments were designed and the model was calibrated according to the simulation results that were obtained. The result of these experiments have been previously published [Purvis 1998, 1999].

It should be noted that even with this limited number of model parameters, there are many possible configurations of the model for the purpose of simulation. Moreover for each specific configuration of the model (the initial marking of the Petri net model), there may be many possible paths (or 'trajectories') which start with the initial state (the initial marking) and end with some terminal state (terminal marking). This is due to the non-determinism that is inherent in most concurrent, dynamic models of this sort. During a simulation run, normally one of these paths is randomly selected and evaluated. Given the complexity of large models, the number of such paths through the system, and hence the number of possible different simulation runs, may be very large. In the analysis approach, however, all possible paths through the state-space are considered. Therefore when a statement is made about some basic property of the model, such as *boundedness*, that statement holds for all possible paths (all possible states) that the Petri net can take starting from a particular initial state. Due to the large number of possible paths, it is normally not practical to cover all these paths during simulation. Consequently it is often highly desirable to carry out a formal analysis and explore the entire state space in this fashion.

The focus of this paper is to analyse the dynamic model of the resource consent process with respect to the formal analysis capability of Coloured Petri nets, such as boundedness, reachability, liveness, and deadlock. The examination of these properties, provide additional information concerning the dynamic behaviour of the model over what is obtained from simulation experiments by themselves [Keller et al. 1994, Rasmussen and Singh 1996]. Other (non-Petri-net) modelling representations do not ordinarily yield this type of additional information.

### 3.1 Occurrence Graph Analysis

There are several different approaches for formal analysis of Petri net models, including occurrence graphs, coverability trees, and place invariant analysis. Occurrence graphs, or reachability graphs, are graphs whose nodes indicate different states that the system can be in and whose arcs indicate a transformation from the current state to possibly another state. An occurrence graph indicates all the possible sequences of transitions that can take place from a particular initial state. The use of occurrence graphs can be of practical value, since the process of constructing occurrence graphs can be more easily automated than some of the other analytical techniques. Once a graph is constructed, the verification of all the dynamic properties noted earlier can be automated as well. The disadvantage of this method is the rapid explosion of the state space as the size of the model increases. The size of an occurrence graph grows rapidly for large colour sets (large number of tokens). However if a correct behavior is obtained for a small colour set, one can gain confidence that the basic logic of the model would be satisfactory for a larger colour set as well [Jensen 1994, pp. 20]. As will be discussed in the next Section, it was possible to construct a full occurrence graph for the resource consent process model, but with a limited number of tokens. Once an occurrence graph is constructed, the desired behavioural properties noted earlier such as reachability and boundedness can be identified.

### 3.2 The boundedness property

A number of full occurrence graphs were generated for a limited number of tokens (applications and government officials). Table 1 shows the values of

Table 1. Occurrence graph analyser with resource consent model.

| Experiment | Gov. Offs. (#A, #P, #C) | Applications | Occurrence Graph (Nodes/Arcs) |
|---|---|---|---|
| 1 | 1,3,1 | 3: (E,N,Del,D) + (E,-,Del,D) + (E,N,Pub,D) | 14371/52450 |
| 2 | 1,2,1 | 3: (E,N,Del,D) + (E,N,Sub,D) + (E,-,Del,D) | 11582/38976 |
| 3 | 1,3,1 | 3: (E,N,Del,D) + (E,N,Sub,D) + (E,-,Del,D) @(5,5,13) | 15029/55962 |

Table 2. The boundedness property for selected nodes in the resource consent model.

| Experiment | Upper/Lower Bounds | Gov. Offs. (#A,#P,#C) | Applications in sub-processes (Eval, Notify, Hearing, Decision) |
|---|---|---|---|
| 1 | Upper: | 1,3,1 | (3,2,2,3) |
| | Lower: | 0,0,0 | (0,0,0,0) |
| 2 | Upper: | 1,2,1 | (2,2,2,2) |
| | Lower: | 0,0,0 | (0,0,0,0) |
| 3 | Upper: | 1,3,1 | (1,1,1,1) |
| | Lower: | 0,0,0 | (0,0,0,0) |

some of the input parameters to the model and the size of the constructed occurrence graphs in terms of the number of arcs and nodes that were generated. The column in Table 1 labelled "Applications" shows the number of applications that were analysed and the configuration of each one. Here "E" indicates that the application must be evaluated for more information, "N" indicates that a public notification will be required, "-" indicates that a public notification will not be required, "Del" indicates that a "hearing by delegation" is required, "Sub" indicates that a "hearing by subcommittee" is required, "Pub" indicates that a public hearing is required, and "D" indicates that a decision must be made for it. Thus the first application of experiment #1 had to be evaluated, notified, heard by delegation, and a decision had to be made for it. This information is indicated by (E,N,Del,D) in the first row of Table 1.

Table 2 shows the maximum and minimum number of tokens that can be present in various places in the model. For example in experiment 1 where we had 1 administrative person, 3 planners, and 1 consultant, the lower bound associated with the number of applications that could be processed at various phases of evaluation was zero, where as the upper bound associated with these places were (3, 2, 2, 3) which corresponds to the maximum number of applications that could be evaluated for *more information, notification, hearing, and decision making* activities. In experiment 3, the applications had time stamps. The submission time is shown in the table by means of the @-sign. It can be observed that the number of nodes and arcs generated for the timed models are considerably less than the untimed model. This arises from the fact that, in the timed case, fewer tokens are available at a given point in the execution due to the different time stamps that they have.

Note that the upper bounds associated with the number of applications that were processed at the same time are generally higher for experiment #1 than they are for experiment #2 (due to higher number of officials available in experiment #1). Also the upper bound for applications that are being processed at the same time is higher for experiment #1 than for experiment #3, in which the applications were timed and consequently not available at the same time.

### 3.3 The reachability property

To investigate how the occurrence graph could be used to identify whether a particular marking could be reached, the following experiment was performed. It should be noted that in the overall model of the resource consent process, various sub-processes are competing for the limited number of resources such as the government officials and environmental experts (consultants), since both the process of evaluating the application for "more information" and the "hearing" process may require the use of environmental experts. This experiment sought to answer the question: whether, in the current model with the configurations used for occurrence graph analysis, it was possible to encounter a marking in which the "hearing" process was delayed due to the fact that there were no available environmental experts (*i.e.* the environmental experts were busy with the evaluation process for "more information"). It was found in experiment #3 that there were a number of markings in which this situation did, in fact, occur (see Figure 1). In experiment #3 there are two applications that require public notification, and the situation occurred that when one of the notified applications was about to enter the process of being heard, another application requiring notification was being evaluated for "more information" and had already secured the use of the environmental experts. Thus the "hearing" had to be held up until the needed officials were released.

Once it is known that such a state is reachable, it may be advantageous to change the process being modelled so that this kind of situation is avoided. For example, it may be useful to increase the number of available environmental experts. Or it may be decided to impose a condition such that it is only permissible to use environmental experts in the evaluation process for "more information" when there is an adequate supply of environmental experts available such that the public hearing process will not be put on hold.

### 3.4 The liveness property

The three occurrence graph experiments listed previously were all checked for dead markings. In all the experiments other than experiment #2, the only dead marks found were the terminal nodes that indicate that the applications have completed processing. In experiment #2, however, there was an additional, non-terminal node dead marking.

For experiment #2, the number of available officials of type 2 (planners) is 2. The dead marking here corresponds to the case when two applications that are to require public notification concurrently enable the transition *Start Eval for More Info*(see Figure 1). Since both of these applications require notification, they both require an environmental expert in connection with another official of type 2. When the *Start Eval for More Info* transition fires concurrently for the two applications, it will consume two officials of type 2 (one for each of the applications) and deposit two applications in the *ExpReq1* place (in the upper right-hand corner of Figure 1). The applications are now waiting for the *GetExp* transition to be enabled, but this transition also requires the availability of officials of type 2 (none of which are, at the moment, available). Thus there is a deadlock.

During the time when the RMA model was under development, checking for dead markings provided assistance in the process of debugging the model by identifying possible problems in the model that may have been overlooked by visual inspection or simulation. (Note that it is also possible to encounter dead transitions. For example if no applications in the model are to require public notification, then the transitions corresponding to the notification process will be identified as dead transitions.)

## 4. CONCLUSION

Overall, the simulation and analysis process lead to an increased understanding of the model. The analysis helps one explore all possible states that the system can be in. In particular the user can examine if it is possible to reach certain undesirable states and try to examine the execution paths that lead to those nodes (states) and perhaps avoid them if possible.

It is true that for large nets, the state space explodes rapidly. For large nets where it is not possible to generate a full occurrence graph, one may try to analyse the critical parts of the model. At times some of the problems may be revealed when smaller components of a bigger model are analysed. However there is still the problem of behaviour that involves interacting elements of the model that lie outside the boundaries of these smaller components. Recent research in this area by Christensen and Petrucci [1995] has attempted to address this difficulty. Their approach seeks, as above, to decompose a model into smaller modules and then to analyse the individual modules separately. Then they attempt to construct a state space of the interactions between modules in order to attempt to analyse the whole model. This is a manual activity, and, as far as the authors know, there have been no tools developed up to now that provide assistance with this process.

## REFERENCES


Askar-Tehrany, M. and Purvis, M. K. (1997). "Modelling and Simulation of Legal Processes". *The Third Annual Conference of Computer Society of Iran (CSICC 97)*, (ed.) Fathi, M. Tehran, Iran, Computer Engineering Department, Iran University of Science and Technology: 7-12.

Christensen, S. and Petrucci, L. (1995). "Modular State Space Analysis of Coloured Petri Nets". *16th International Conference on Application and Theory of Petri Nets*, (eds). De Michelis, G. and Diaz, M. Turin, Italy, Springer Verlag: 201-217.

Jensen, K. (1992). *Coloured Petri Nets - Basic Concepts, Analysis Methods and Practical Use, Vol. 1: Basic Concepts*. Berlin, Springer-Verlag.

Jensen, K. (1994). *Coloured Petri Nets - Basic Concepts, Analysis Methods and Practical Use, Vol. 2: Analysis Methods*. Berlin, Springer-Verlag.

Jensen, K. (1997). *Coloured Petri Nets - Basic Concepts, Analysis Methods and Practical Use, Vol. 3: Practical Use*. Berlin, Springer-Verlag.

Keller, R., Ozkan, M. and Shen, X. (1994). "Towards Comprehensive Support for the Dynamic Analysis of Petri Net based Models". *Application and Theory of Petri Nets 1994, Lecture Notes in Computer Science*, (ed.) Valette, R. Zaragoza, Spain, Springer-Verlag: 298-317.

Meldman, J. A. and Holt, A. W. (1971). "Petri Nets and Legal Systems." *Jurimetrics Journal*, **12**(1): 65-75.

Meldman, J. A. (1978). "A Petri-Net Representation of Civil Procedure." *IDEA -- The Journal of Law and Technology*, **19**(2): 123-148.

*New Zealand Government (1994). Resource Management Act, 1991*. Wellington, New Zealand, *New Zealand Government*.

Purvis, M. A. (1998) *Dynamic Modelling of Legal Processes with Petri Nets*, PhD Thesis conducted at Information Science Department, University of Otago. Dunedin, New Zealand, 1-261.

M. A. Purvis, and M. K. Purvis, (1999) A Dynamic Model of Legal Process Behaviour, Proceeding of the IASTED International Conference – Applied Modelling and Simulation, (ed.) M.H. Hamza, Cairns, Queensland, Australia, 124-129.

Purvis, M. K., Benwell, G. L. and Purvis, M. A. (1994a). "Dynamic Modelling of the Resource Management Act." *New Zealand Journal of Computing*, **5**(1): 45-56.

Purvis, M. K., Benwell, G. L. and Purvis, M. A. (1994b). "Dynamic Modelling of the Resource Consent Process in the Resource Management Act". *Environmental and Resource Management in New Zealand*, (eds). Selsky, J., Morgan, R. and Memon, A. Dunedin, New Zealand, Environmental Policy and Resource Management Centre at University of Otago: 381-394

Purvis, M. K., Purvis, M. A. and Benwell, G. L. (1995). "Modelling and Simulation of the New Zealand Resource Management Act." *Journal of Law and Information Science*, **6**(2): 181-192.

Rasmussen, J. L. and Singh, M. (1996). "Designing a Security System by means of Coloured Petri Nets". *17th International Conference on Application and Theory of Petri Net*, (eds). Billington, J. and Reisig, W. Osaka, Japan: 400-419.