A basal area model responsive to thinning for a plantation forest

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Abstract The Tarawera Valley spacing and thinning trial data were used to develop a dynamical model for basal area prediction that is responsive to both spacing and thinning. The model is a 'critically damped' second-order model, i.e. it simulates a sigmoidal response, before and after thinning. It has a scalar spacing variable which makes the model responsive to different spacing trials. It also has a vector 'reductor' term which enables simulation over different productivity sites, although this still needs to be validated with measured data. The dynamical model has the potential of being applied across a forest estate with different productivity sites, if data are available for all sites.

1. Introduction

Dynamical models are extensively used in Systems Engineering for System Identification (i.e. model development for dynamic systems) and Optimal Control (that concerns itself with the means by which to alter the future behaviour of a dynamic system to achieve an outcome at a minimal cost). They are black box models and System Identification enables the development of mathematical functions that describe the behaviour of a dynamic system rather than the process. The dynamical model has three basic parameters, i.e. input (a parameter that can be measured and directly controlled by the observer); output (an observable parameter that can be measured and indirectly controlled by the observer); and noise (the influence exerted by the external environment on the dynamic system, that includes noise that can be directly measured and that which can only be observed through its influence on the input). The most basic discrete-time dynamical model, a first-order function, has the following form:

$$y(t) = ay(t-1) + b(1-a)u(t), \qquad t = \{t \in N\},$$
$$a = \{a \in R \mid 0 < a < 1\}, b = \{b \in R \mid b > 0\} \qquad (1)$$

where

t = time;
$y(t)$ = output y at time t;
u = input vector;
a = shape parameter;
b = asymptotic limit of function;
$N = \{1, 2, 3, \ldots\}$; and
R = set of real numbers.

Using the Tarawera Valley data, a 'critically damped' second-order model (DiStefano et al. 1995) was identified for basal area prediction which has a generic form represented as follows:

$$y(t) = a_1y(t-1) + a_2y(t-2) + b(1-a_1-a_2)u(t) \qquad (2)$$

For a detailed understanding of dynamical models, a textbook by Professor Ljung (1987) is highly recommended.

2. Data

The data, from a replicated spacing and thinning trial in the Tarawera Valley, New Zealand, were used to develop the basal area prediction model. The trial was established in 1963 with the sole purpose of investigating the economic impact of a number of alternative regimes for sawlog and pulpwood production of *Pinus radiata* D. Don (James 1976). Measurements were carried out each winter from 1969 to 1989 and 1991. There were 90 plots, hexagonal in shape, laid out in a honeycomb pattern in four adjacent stands, and each established at a different planting espacement. The four planting espacements were 1.3 x 1.3 m, 1.8 x 1.8 m, 2.1 x 2.1 m and 2.7 x 2.7 m.

Each of the 90 plots had an area of *0.06* ha where trees were measured, with a surrounding buffer between plots of *0.12* ha. For each espacement, control plots were established which provided information on what would happen if the stands remained unthinned. The thinning strategies were as follows:

1. pre-commercial thinning from initial density to *1330* stems ha^{-1}, 2^{nd} thinning (selective) from *1330-375* stems ha^{-1};
2. pre-commercial thinning from initial density to *1330* stems ha^{-1}, 2^{nd} thinning (crown) from *1330-500* stems ha^{-1}; and
3. mechanical (row) thinning from initial density to *375* stems ha^{-1}.

3. Method

The method followed here was similar to the one used by Chikumbo et al. (1999) for developing a dynamical model for stand basal area prediction for both unthinned and thinned stands, using data from unthinned stands and stands 'thinned early and in advance of competition mortality'[1]. To achieve this, the basal area data were grouped for each espacement as follows: thinned plots called ET (i.e. plots thinned early and in advance of competition before measuring); unthinned plots called UT (i.e. control plots measured every winter); and thinned plots called TT (i.e. thinned plots measured before and between thinning operations).

The ETs were used for System Identification and some of the TTs used for cross validation of the models (Chikumbo et al. 1999). The UTs were not used for System Identification as in Chikumbo et al. (1999) because the initial densities were high, ranging from *1384-5157* stems ha^{-1}. Competition mortality was very high causing erratic basal area trends, although they could still be identified by a simple first-order dynamical model. This model is not discussed in this paper, because the emphasis was on a model responsive to thinning. The stand basal area trends between thinning operations, with residual densities ranging from *379-1334* stems ha^{-1} showed sigmoidal trends that were identified with a 'critically damped' second-order model. The model was complicated by designing it to respond not only to thinning but to spacing and different productivity sites. Since the Tarawera data were from one site, the model's ability to simulate different sites still require data for further validation, although its theoretical response was demonstrated.

The basal area prediction had the following form:

$$y(t) = a_1 y(t-1) + a_2 y(t-2) + \alpha\beta\, u(t),$$
$$y(t_1) = \lambda y(t_0) \qquad (3)$$

where

y = stand basal area, $m^2\ ha^{-1}$;

α = spacing variable;

$\beta = b(1-a_1-a_2)$;

λ = parameter that is a function of residual density following thinning;

$u(t)$ = reductor term (adapts model to different sites); and

t_0 = initial time at start of model simulation.

A second-order model is initialised by two values, $y(t_0)$ and $y(t_1)$. $y(t_1)$ is a function of the previous value, $\lambda y(t_0)$ and λ will influence the response of the model, depending on the residual density.

4. Results and discussion

Using a single ET plot from the 1.3 x 1.3 m espacement a second-order model (as in equation (3)) was identified using a System Identification toolbox running under MATLAB (MathWorks Inc. 1992). Constrained optimisation was then used to fit the rest of the ET data to this model structure. Since the a_1, a_2 and b are not in any way unique, it is important to use constrained optimisation such that changes in these parameters can be tracked with response to changes in the residual densities (d) from the ET plots. The following set of residual densities were available for all the espacements, $\{d \in N \mid d = \{379, 395, 741, 1334\}\}$. The changes in the parameters were only accepted on the basis of satisfying a low mean squared error (MSE), high R-squared value, homoscedasticity, the autocorrelation function of residuals and the cross correlation between the residuals and the input(s).

Because the dynamical models are identified by regressing lagged variables, the statistical properties are investigated by looking at correlations in residuals: the correlations between the residuals, s steps apart, should always be the same (Chatfield 1984) and this

[1] This treatment of thinning early and in advance of competition mortality provides a means of establishing nominal plots with varying initial stand densities (Chikumbo et al. 1999).

was satisfied in all the models developed. The residuals of the dynamical model are shown as a correlogram as in Figure 1. A correlogram is an aid for interpreting a set of autocorrelation coefficients (defined as the ratio of the autocovariance at lag k to the autocovariance at lag zero – i.e. a scaled autocovariance). The rule is that if the correlation coefficients, which approximate a normal distribution of mean zero and variance $1/n$, go significantly outside the $\pm 2/\sqrt{n}$ confidence band (where n is the size of the time series), the corresponding model should not be accepted (Chikumbo et al. 1998).

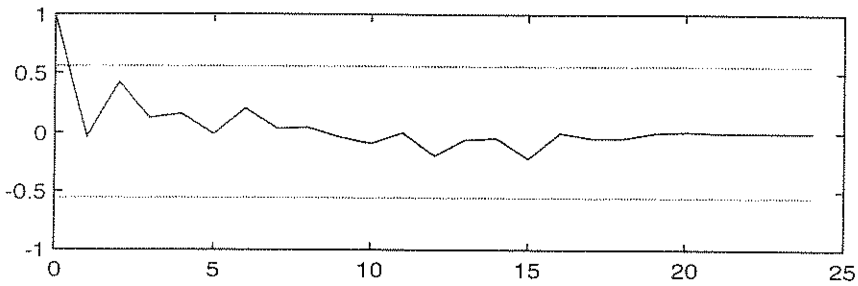

Figure 1. (a) the autocorrelation function of residuals; and (b) the cross correlation between the residuals and the b parameter, for one of the ET plots modelled with equation (3).

The parameters a_1 (*1.1764*) and a_2 (*-0.2336*) remained static for all thinning responses, and β varied monotonically with increasing residual densities, i.e β = {*4, 4, 5.13* and *5.4*}, for residual densities, {*379, 395, 741* and *1334* stems ha^{-1}} respectively. λ switched between 2 values, *1.09* (for *379-741* stems ha^{-1}) and *1.18* (for *1334* stems ha^{-1}). With greater variation in the residual densities, a linear function for β, dependent on the residual densities, can be developed and the same done for λ. The spacing variable, α, varied as follows; {*1, 1.15, 1.2* and *1.12*} for {1.3 x 1.3 m, 1.8 x 1.8 m, 2.1 x 2.1 m, and 2.7 x 2.7 m} respectively.

This model was validated with the TTs and Table 1 shows the summary statistics of the validation for the some of the plots. Since there were no TT plots for the 2.7 x 2.7 m espacement, some of the ET plots were used for the validation.

Table 1. Statistical validation of model (3). The plots are given as follows: plot group/plot number/spacing.

PLOT	AGE RANGE (years)	RESIDUAL DENSITY (trees ha^{-1})	MSE	R-squared	Residuals
TT/14/1.3x1.3m	9-28	379	0.8	0.99	increasing trend
TT/18/1.3x1.3m	9-28	379	0.3	0.99	horizontal band
TT/19/1.3x1.3m	11-28	379	0.3	0.99	horizontal band
TT/02/1.8x1.8m	11-28	379	0.35	0.99	horizontal band
TT/04/1.8x1.8m	6-9	1334	0.04	0.99	N/A
TT/18/1.8x1.8m	11-28	379	1.06	0.99	horizontal band
TT/01/2.1x2.1m	11-28	379	0.8	0.99	horizontal band
TT/04/2.1x2.1m	11-28	379	0.15	0.99	horizontal band
TT/13/2.1x2.1m	6-9	1334	0.9	0.99	N/A
ET/07/2.7x2.7m	10-28	379	0.02	0.99	horizontal band
ET/09/2.7x2.7m	6-28	741	1.1	0.99	horizontal band

Since data were not available for other productivity sites, only predictive simulations are shown in Figure 2, in order to illustrate the effect of the reductor term.

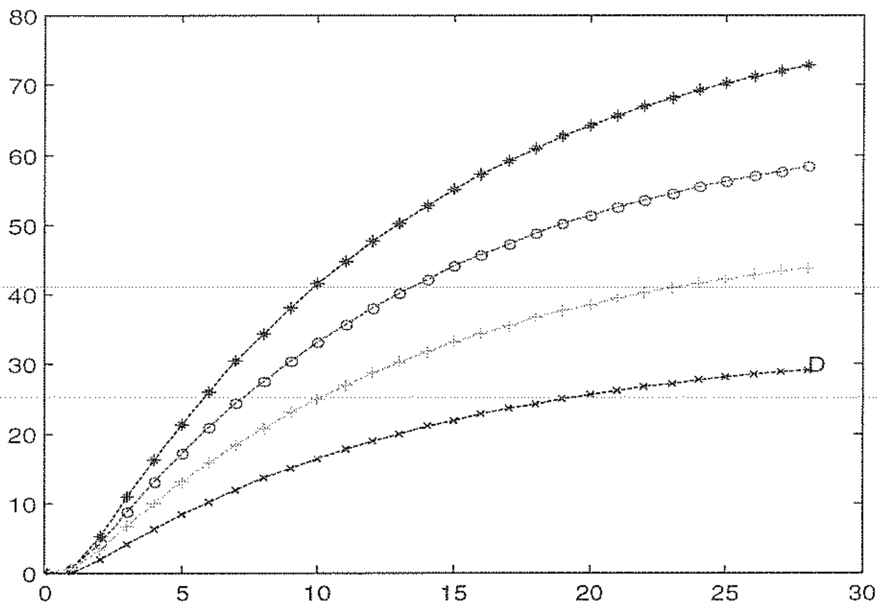

Figure 2. Trends from fictitious productivity sites (*A*, *B*, *C* and *D*) generated using equation (3) for a 1.3 x 1.3 m spacing with the vector u is as follows: $u_A = [1, 1,\ldots, 1]$; $u_B=[0.8, 0.8,\ldots, 0.8]$; $u_C=[0.6, 0.6,\ldots, 0.6]$; and $u_D=[0.4, 0.4,\ldots, 0.4]$, $\forall\, t$, $t \in N \mid t = \{1, 2, 3 \ldots, 28\}$.

5. Conclusion

A dynamical model can be developed for plantation forestry with a flexibility that accurately reflects initial spacing and thinning. The model also carries a potential of being extended to other productivity sites although this still has to be validated with measured data.

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