A Combination of Linear and Nonlinear Optimisation Algorithms to Maximise Net Return in Feedlots

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ABSTRACT

Linear programming (LP) is widely used to formulate diets for farm animals. In most LP models, the objective function is the minimum cost of the diet dry matter (MCDM), subject to the animals' nutritional requirements, usually stated as concentrations of nutrients in the diet dry matter. LP solutions apply to a single expected animal performance assumed by the user. However, there are multiple least cost diets at different nutrient concentrations and corresponding animal performances. Thus, traditional LP will not converge to the diet that minimises the cost of production. With the objective of overcoming LP limitations, a static and deterministic model of beef cattle growth response to diet composition was programmed in a Microsoft Excel® spreadsheet. This model predicts feed intake and animal performance from energy concentration of the diet and then calculates the requirements for the remaining nutrients. A series of linear optimisations (TDNS) and one (OPOP) or two (TPOP) phase optimisation procedures were compared in finding the minimum cost of carcass weight gain (MCCWG). TDNS generates a series of solutions for the MCDM problem for several levels of a constraint for total digestible nutrients (TDN). TDNS results are compared for MCCWG. OPOP uses a Generalised Reduced Gradient Algorithm (XLGRG) assigning a set of predefined starting values. TPOP uses a LP algorithm to solve a MCDM problem with a user-defined diet concentration constraint and assigns the solution as starting values to the MCCWG problem using XLGRG. TPOP procedure had better performance than OPOP and TDNS in all experiments. OPOP presented unsatisfactory results both by failing to solve or converging to sub-optimal solutions. TDN was inferior to TPOP due to both the safety factors used to approximate nonlinear constraints and precision in the discrete search. TPOP appeared to have high potential to be incorporated into commercial diet optimisation software.

INTRODUCTION

Diet composition is the major driving factor of animal performance, and accounts for more than 70% of the total costs in beef cattle feedlots. Feedlots usually operate with low gross margins. Therefore, diet formulation is the process that mostly affects net returns in such enterprise.

Linear programming (LP) is widely used to formulate diets for farm animals (Scott [1972]). In most LP models the objective function is to minimise cost per kg of dry matter of the diet (MCDM), subject to the animals' nutritional requirements, usually stated as nutrient concentrations in the diet dry matter (Hertzler [1988]). Animal's nutritional requirements hold constant only within a single performance level. Changes in diet composition cause variations in feed intake, animal's nutritional requirements and animal performance. The traditional LP formulation ignores animal performance and feed intake. There are multiple least cost diets at different levels of nutrient concentrations and corresponding animal performances. Thus, a single LP formulation will not necessarily converge to the minimum cost of production or maximum profit, both nonlinear functions of diet composition.

Either minimisation of production costs or maximisation of profit requires a model that enables prediction of dry matter intake (DMI) and animal performance. The CNCPS (*Cornell Net Carbohydrate and Protein System*; Fox et al. [1992]) and the *National Research Council* (NRC [1996]) models have been widely used to predict animal performance given a diet composition. Both models were the basis for the implementation of the RLM (Lanna et al. [1999a]), a commercial computer program for formulation of diets for beef cattle.

With a previous version of RLM, Lanna et al. [1999b] obtained a 6.2% reduction of feeding costs per unit of carcass weight using the Excel Solver® Generalised Reduced Gradient Algorithm (XLGRG; Ladson et al. [1978]) compared to the MCDM formulated through LP. However, those authors warned about the problems of local optima'solutions and starting values when using XLGRG, an usual problem well documented in the literature (Goldberg [1989]).

Previous works have demonstrated advantages of utilizing robust optimisation procedures to generate initial values for "hill climbing" algorithms in attenuating that problem (Mayer et al. [1995]; Hart et al. [1998]). In this paper, solutions of linear programming optimisations were used to set the initial values to XLGRG. This procedure was compared with the straight use of the nonlinear algorithm and series of linear optimisations.

MATERIAL AND METHODS

Model

A commercial computer program for diet formulation and evaluation, RLM, slightly modified from that reported by Lanna et al. [1999b], was used to test the optimisation procedures. RLM is a static model implemented in Microsoft Excel®. The model requires the following inputs to describe the animal: shrunk body weight (SBW), previous 60 days average daily gain, breed, body frame size score, sex, ionophores and hormone implants. The variables required for feed description are price, dry matter, total digestible nutrients (TDN), crude protein, ether extract, acid detergent fibre insoluble protein, rumen degradable protein, fibre, and minerals.

Feed energy is based on TDN content. One kilogram of TDN is assumed equal to 4.4 Mcal (Megacalorie) of digestible energy (NRC [1996]).

A factor of 0.82 is used to convert digestible energy (DE; Mcal.kg^{-1}) to metabolisable energy (ME; Mcal.kg^{-1}). Net energy for maintenance (NEm; Mcal.kg^{-1}) and net energy for gain (NEg; Mcal.kg^{-1}) are predicted according to equations 1 and 2, respectively (NRC [1996]).

$$NEm = 1.37 \cdot ME - 0.138 \cdot ME^2 + 0.0105 \cdot ME^3 - 1.12 \qquad (1)$$

$$NEg = 1.42 \cdot ME - 0.174 \cdot ME^2 + 0.0122 \cdot ME^3 - 1.65 \qquad (2)$$

Feed intake is calculated according to the equation proposed by NRC [1996] and adjusted for *Bos indicus*:

$$NEmI = ASBW^{0.75} \cdot \left(a \cdot NEm - b \cdot NEm^2 - c\right) \cdot IA \qquad (3)$$

Where NEmI is the intake of net energy for maintenance (Mcal.day^{-1}), ASBW is the shrunk body weight adjusted for animal frame size (kg) and sex in order to account for the nutritional requirements of animals. These adjustments are based on the values proposed by Fox et al. (1988) and implemented in the CNCPS model, a = 0.2435 or 0.2438; b = 0.0466 or 0.049; c = -0.1128 or -0.1068 for *Bos taurus* (European) and *Bos indicus* (Zebu) cattle, respectively, and IA is an intake adjustment for the use of hormone implants. Parameters a, b, c and IA are dimensionless.

Energy available for gain (NEAg; Mcal.day^{-1}) is calculated by subtracting NEm requirements for maintenance (NEmM; Mcal.day^{-1}) from total NEmI (equation 4).

$$NEAg = \frac{(NEmI - NEmM) \cdot NEg}{NEm} \qquad (4)$$

The liveweight gain ADG (kg.day^{-1}) is calculated according to equation 5 (NRC [1996]).

$$ADG = 13.91 \cdot NEAg^{0.9116} \cdot ASBW^{-0.6837} \qquad (5)$$

Carcass weight gain (CWG, kg.day^{-1}) is calculated by equation 6.

$$CWG = ADG \cdot CD \qquad (6)$$

CD is the carcass dressing, a user-defined dimensionless factor (0 to 1) that converts ADG to carcass weight.

Protein requirements are calculated based on animal SBW, ADG and rumen microbial protein yield. (Lanna et al. [1996b]) The optimal range of rumen degradable protein is calculated from diet TDN concentration according to NRC [1996].

The lower and upper limits to RDP (RDP_{min} and RDP_{max}) were set as stated in equations 7 and 8:

$$RDP_{\min} = 0.083 \cdot MG \cdot TDN \quad (7)$$

$$RDP_{\max} = 1.100 \cdot MG \cdot TDN \quad (8)$$

In this model, animal performance and feed intake are driven by diet TDN concentration only. Other nutrient requirements, except microminerals, are functions of ADG. Therefore, after the inputs are defined, TDN is the only variable driving the cost of carcass weight gain. An important propriety of the diet optimisation problem is the optimum solution for maximum profit or MCCWG is the same as the MCDM solution at the optimum TDN level for MCCWG. This propriety holds only when constraints are the same for MCCWG and MCDM problems. MCCWG optimisations, however, do not hold the TDN concentration constant, but modify it in order to find a TDN concentration that generates the optimum solution.

Model equations and further detail of the program operation may be found in Lanna et al. [1999a,b].

Optimisation

The objective function was to minimise feed costs per unit of carcass produced, shown in equation 9.

$$Min\left(\frac{DMI \cdot \sum_{i=1}^{n}(DMPF_i \cdot PF_i)}{ADG \cdot CD}\right) \quad (9)$$

Where $DMPF_i$ is the proportion of the i^{th} feed in the diet dry matter which make the vector of natural variables for the optimisation algorithm (Ladson et al.[1978]), PF_i is the price of the i^{th} feed, and n is the number of feeds in the problem.

Only constants were allowed on the right hand side of the constraint inequalities following the XLGRG algorithm requirement. The optimisation was subjected to the following constraints:

$$\sum_{i=1}^{n} DMPF_i = 1 \quad (10)$$

$$DCP - RCP \geq 0 \quad (11)$$

$$EE \leq 50 \quad (12)$$

$$DRDP - RDP_{\max} \geq 0 \quad (13)$$

$$DRDP - RDP_{\min} \geq 0 \quad (14)$$

$$NPN - 500DRDP \geq 0 \quad (15)$$

$$FI \geq 150 \quad (16)$$

Where, DCP is the crude protein concentration in the diet dry matter ($g.kg^{-1}$), RCP is the requirement of crude protein concentration in the diet ($g.kg^{-1}$), EE is the concentration of fat (ether extract) in the diet ($g.kg^{-1}$), DRDP is the diet concentration of rumen-degradable protein ($g.kg^{-1}$), RDP_{min} and RDP_{max} are $g.kg^{-1}$calculated by equations 7 and 8, NPN is the concentration of non-protein nitrogen ($g.kg^{-1}$) in the DRDP and FI is a fibre concentration index ($g.kg^{-1}$).

Minerals were not restricted and assumed to be adequately supplied by a mineral supplement, which was set to enter in a proportion of 10 $g.kg^{-1}$ of the diet dry matter.

Animals were 343 kg SBW Zebu x Holstein crossbreed castrated males without hormone implants and not supplemented with ionophores. Body frame size score was 8. Previous ADG was 0.5 $kg.day^{-1}$. Carcass dressing was defined as 0.52.

The optimisation procedures tested were one (OPOP), two (TPOP) phase optimisation procedures and a series of linear programming optimisation solutions with a series of diet TDN concentration constraint (TDNS). OPOP was tested with two sets of starting values. In the first (OPOP1), the proportion of all feeds in the diet was assigned to naught. In the second (OPOP2), the proportion of all feeds, except urea and mineral supplements were assigned to (1-Cm-Cu)/n, where Cm is the concentration of urea in the diet and Cu is the proportion of urea in the diet dry matter. Cu and Cm were assigned to 0.01 as it is close to the expected concentration of those components in the optimum solution. Both, OPOP1 and OPOP2, call the XLGRG algorithm straight after setting the starting values.

TPOP optimisation procedure calls a Simplex LP algorithm to solve the MCDM problem. The LP problem is solved for a user-defined level for the TDN constraint (TDNc; equation 18). As there is a small range of TDN levels that can be used pragmatically in a feedlot diet, usually from 650 to 800 $g.kg^{-1}$, the TDN constraint for the linear optimisation can be easily defined by the user. In the experiments reported here, TDNc was set to 700 $g.kg^{-1}$. The vector of natural variables ($DMPF_1$ to $DMPF_n$) in the solution of the MCDM problem is then assigned to XLGRG to solve the MCCWG problem.

The MCCWG solution is the same as the MCDM solution when the latter is restricted to the same level of TDN the first converged. Thus, if a series of MCDM diets are generated by exploiting the possible levels of diet TDN, allowed by the composition of the available feeds, the minimum of those should be close to the optimum for the nonlinear problem. Thus the best solution of a minimisation of series of optimisations of the MCDM problem should be close to the optimum for the MCCWG problem. The TDN series starts by setting the constraint for minimum TDN to 500 (equation 18). This level was chosen based on a practical knowledge. The diet was then formulated for that level of TDN. TDN constraint was successively increased by 10 $g.kg^{-1}$ and a new diet formulated. This procedure was repeated until the problem was infeasible. All solutions were then compared for minimum carcass cost as described in the objective function of equation (9).

$$Min\left(\sum_{i=1}^{n}(DMPF_i \cdot PF_i)\right) \qquad (17)$$

Linear programming optimisations (TDNS and the linear phase of TPOP) were subject to the same constraints of the nonlinear procedures except for a constraint of minimum level of TDN (equation 18)

$$TDN \geq TDNc \qquad (18)$$

Where TDNc is the constraint for minimum TDN level for the LP problem. Also, constraints shown in equations 13 to 15 are not suitable to linear programming problems. These constraints were approximated by the constraints in equations 19 to 21. Note that constraints 19 and 20 are the same as constraints 13 and 14 when TDN converge to TDNc.

Since the RDP value is unknown before the optimisation, the NPN constraint was calculated using RDPmin to ensure that no diet would have excessive levels of NPN as high levels of NPN may be very toxic. The NPN constraint was then approximated as in equation 21.

$$RDP_{\min} = 0.083 \cdot MG \cdot TDNc \qquad (19)$$

$$RDP_{\max} = 1.100 \cdot MG \cdot TDNc \qquad (20)$$

$$NNPMS - 0.083 \cdot MG \cdot TDNc \leq 0 \qquad (21)$$

Where MG is microbial growth rate ($g.gTDN^{-1}$).

The procedures were tested with 5, 10 and 15 feeds with three different combinations of feeds, resulting in 9 optimisations for each procedure. The experiments were coined 5a, 5b, 5c, 10a, 10b, 10c, 15a, 15b, 15c where the number refers to the number of feeds in the problem and the letter represent a particular combination of feeds tested. Feeds made available to the optimisations and their prices (based on the Brazilian Market in July 1999; Table 1). Most nutritional composition inputs for those feeds are based on NRC [1996] feed library.

RESULTS AND DISCUSSION

Optimisation results are shown in table 2. XLGRG did not find a viable solution for OPOP1 in any experiment. In OPOP1, the proportion of all feeds in the dry matter was assigned to naught; therefore the starting value for TDN was zero. The model was not robust to those unrealistic initial values, therefore it is possible that the response surfaces do not show a good behaviour. Since TDNS used the simplex LP algorithm, it may be regarded as the most robust optimisation procedure applied to the experiments reported. However, LP was restrictive in including all the constraints in their original form. The approximation of the constraint in equation 15 to fit it to LP (as in constraint 21) made it more restrictive in order to ensure the original constraints would be satisfied. It caused TDNS to converge to higher production costs.

TPOP was superior to the other optimisation procedures in all experiments. OPOP2 converged to the same results of TPOP in six out of nine experiments. In three it converged to a higher production costs. Surprisingly, the convergence to sub-optimal results with OPOP2 occurred in the experiments with 5 and 10 feeds, which have lower number of dimensions. Preliminary sensitivity analysis indicated that TPOP was not very sensitive to the TDN constraint defined for the linear phase.

CONCLUSIONS AND RECOMENDATIONS

The results showed that XLGRG was sensitive to initial values in the diet optimisation problems tested. Initial values had high influence on this algorithms performance. Assigning zero to the initial concentration of each feed in the diet resulted in the worst performance. Equal proportions of every feed in the diet (OPOP2) were also not reliable.

TDN series was a viable optimisation procedure to optimise diets. However, it is restrictive in terms of problem definition. As models for diet formulation becomes increasingly complex, including interaction between diet characteristics and nonlinear relationships are introduced, the approximations made by LP formulations will result in increased costs.

Table 1. Feeds in each optimisation experiment and their prices.

Feed	5^a	5b	5c	10a	10b	10c	15a	15b	15c	Price $/ton
Bahiagrass hay						●	●	●		150.00
Citrus Pulp dehydrated		●		●	●	●	●	●	●	70.00
Corn Grain Cracked	●			●		●	●		●	162.00
Cottonseed				●		●	●	●	●	175.00
Cottonseed meal (41% CP^1)		●		●	●	●	●	●	●	216.00
Elephantgrass silage (60 days of growth)			●		●	●	●	●	●	16.00
Maize Silage (35% grain)	●			●	●		●	●	●	25.00
Mineral Supplement	●	●	●	●	●	●	●	●	●	420.00
Poultry manure			●				●	●	●	46.00
Corn Gluten Feed					●		●	●	●	152.00
Rice Meal									●	160.00
Sorghum Grain Cracked			●		●		●	●	●	130.00
Soybean Meal (49% CP^2)	●			●	●	●	●	●	●	265.00
Sugarcane		●		●		●	●	●	●	16.00
Sugarcane Molasses								●		78.00
Urea	●	●	●	●	●	●	●	●	●	300.00
Soybean whole roasted				●	●		●	●	●	235.00

Table 2. Cost ($/15 kg of Carcass Weight) and TDN values in the solutions of the optimisation experiments.

Experiment	TPOP		TDNS		OPOP1		OPOP2	
	Cost	TDN[1]	Cost	TDN	Cost	TDN	Cost	TDN
5a	24.42	68.67	24.59	71.00	Nc[2]	-	24.42	68.67
5b	18.91	74.48	19.21	74.00	Nc	-	21.05	72.03
5c	24.46	67.96	24.47	68.00	Nc	-	24.46	67.96
10a	17.28	76.00	17.60	75.00	Nc	-	17.98	76.82
10b	14.59	76.82	16.10	76.00	Nc	-	14.61	76.78
10c	17.30	74.33	17.66	74.00	Nc	-	18.16	77.77
15a	16.69	71.00	17.30	72.00	Nc	-	16.69	70.94
15b	15.65	71.79	17.30	72.00	Nc	-	15.67	71.57
15c	16.69	71.00	17.30	72.00	Nc	-	16.72	70.68

[1] Total digestible nutrients level to which the optimisation converged.

[2] Optimisation did not found a viable solution.

A linear programming optimisation to 70% TDN followed by the Microsoft Excel Solver GRG algorithm was a suitable procedure. As the gradient methods improve or maintain the initial solution, TPOP solutions have the theoretical advantage of being equal or better than the solution of LP formulation for MCCWG. As an easy to implement procedure, particularly when working with spreadsheets, the TPOP appeared to have high potential to be incorporated into diet optimisation programs.

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