Estimating Long Run Pricing Models for Copper Futures Contracts

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Abstract: The London Metal Exchange (LME) is a centre for spot and futures trading in the main industrially-used non-ferrous metals. In this paper, the market for 3-month LME copper futures contracts is analysed. The risk premium hypothesis and the cost-of-carry model are the standard theoretical models for pricing futures contracts, but these two models have rarely been estimated within a unified framework. Single equation versions of the risk premium hypothesis and the cost-of-carry model are nested within a general model. If the spot price, futures price, interest rate and stock level variables contain stochastic trends, long run versions of the general model can be estimated within the cointegration framework. The long run pricing models are estimated using daily LME copper price data over the period 3 January 1989 to 30 September 1998. Likelihood ratio tests are used to test restrictions on the general model.

1 INTRODUCTION

The London Metal Exchange (LME) is the major international market for the main industrially-used non-ferrous metals, namely aluminium, aluminium alloy, copper, lead, nickel, tin, zinc and silver. It is used worldwide by producers and consumers of non-ferrous metals as a centre for spot, futures and options trading in these metals.

Three primary functions are performed by the non-ferrous metals markets on the LME. First, the exchange provides a market where non-ferrous metal industry participants can hedge against risks arising from price fluctuations in world metals markets. Second, settlement prices determined on the LME are used internationally as reference prices for the valuation of activities relating to non-ferrous metals. Third, the LME also provides appropriately located storage facilities to enable market participants to take or make physical delivery of approved brands of non-ferrous metals.

Approximately 95% of the total world trade in copper futures occurs though the LME, with the bulk of the remaining 5% in the copper market on the Commodity Exchange of New York (COMEX). The copper settlement price determined on the LME is effectively the world copper price (Gilbert, 1996).

The plan of the paper is as follows. Section 2 briefly examines previous empirical work on pricing and efficiency aspects of copper futures markets. Futures pricing models are examined in section 3. The data used are described in section 4. Tests for non-stationarity are discussed in section 5. Section 6 presents tests for cointegration and estimates of the long run pricing models. Some concluding comments are given in section 7.

2 PREVIOUS STUDIES OF COPPER SPOT AND FUTURES MARKETS

Several recently published empirical papers analyse aspects of non-ferrous metals spot and futures markets, with the majority focusing on the LME. The copper market is frequently the subject of empirical analysis, as are other non-ferrous metals markets, including those for aluminium, aluminium alloy, lead, nickel, tin and zinc. Properties of precious metals markets, namely gold, silver, platinum and palladium, are also investigated. Empirical research involving the copper spot and futures markets can be classified into four areas: market efficiency; the theory of storage and cost-of-carry model; price volatility and risk; and other aspects of metals markets.

The majority of non-ferrous metal futures market empirical research published over the last two decades relates to the efficient market hypothesis. Approaches to market efficiency in non-ferrous metals markets include models of the unbiased expectations hypothesis, investigation of the properties of forecast errors, tests of restrictions imposed on regression models by the efficient market hypothesis, modelling and/or detecting the existence of time varying risk premia, efficiency motivated tests for cointegration between price series, and the investigation of flow parities between metals prices using cointegration. Evidence on the efficiency of the LME copper futures market provided by cointegration models is mixed. Some analyses support the efficient market hypothesis, while others contradict these results by rejecting efficiency for metals futures markets.

Implications of the theory of storage, and related cost-of-carry model, for non-ferrous metals futures have been examined through modelling of the convenience yield, convenience yield and dispersion premium, analysis of inventory and excess demand effects on the futures basis, and cointegration modelling of the cost-of-carry relationship. Tests on non-ferrous metals including copper support the proposition in Fama and French (1988) that the marginal convenience yield on inventory declines at higher levels of inventory, but at a decreasing rate. Evidence to support the cost-of-carry model in explaining 3-month LME lead futures prices was presented in Heaney (1998).

Empirical studies of price volatility and risk in non-ferrous metals markets include modelling the volatility of spot and futures prices using a random walk model, or various GARCH processes such as GARCH, AGARCH, EGARCH, FIGARCH, and the GJR model, and the analysis of the risk to return relationship in futures markets using a CAPM approach.

Other aspects of copper markets analysed recently include the relationship between margin requirements and market participation, lead-lag relationships between copper futures markets, and manipulation of the copper futures market on the LME.

3 MODELS OF FUTURES PRICES

This paper aims to model the relationship between the futures price and the spot price variables. There are two popular theories for the pricing of futures contracts, from which to motivate a relationship between futures and spot prices, namely the risk premium hypothesis and the theory of storage.

The risk premium hypothesis presumes the risk and return relationship that is commonly proposed for other asset markets is applicable to futures markets, and states that, under market efficiency and rational expectations, the futures price is equal to the expected future spot price plus a risk premium. The risk premium hypothesis can be represented as:

$$(1) \qquad f_{t+k|t} = E_t(s_{t+k}) + \pi_{t+k|t}$$

where $f_{t+k|t}$ is the k-period log futures price at time t, conditional on information available at time t, $E_t(s_{t+k})$ is the expectation at time t of the log spot price at time t+k conditional on information available at time t, and $\pi_{t+k|t}$ is the expected risk premium at time t for a futures contract maturing at t+k, given information at time t. Setting k=1 and assuming expectations are rational, an empirical form of the risk premium model can be specified as:

$$(2) \qquad f_t = \alpha_0 + \alpha_1 s_{t+1} + \alpha_2 \pi_t + \varepsilon_t .$$

where f_t is the log futures price at t for a contract maturing in t+1, s_{t+1} is the log spot price in period t+1, and ε_t is a white noise error term. However, the expected risk premium, π_t, is frequently not a measurable or observable variable.

The cost-of-carry model (COC) uses a no-arbitrage argument by factoring in the carrying costs involved in holding an underlying asset until maturity. For commodity futures contracts, the underlying asset is the physical commodity. Carrying costs within models of commodity futures pricing include interest costs, a risk premium for holding stocks, and storage costs net of convenience yield. Convenience yield is the return due to holding inventory or stocks. This return accrues to an agent or firm because holding stocks of a commodity may reduce transactions costs involved with frequent deliveries of an input in a firm's production process, or may provide the flexibility to meet unexpected demand. The cost-of-carry argument justifies the futures price as being equal to the current spot price minus net carrying costs, and the model can be written as:

$$(3) \qquad f_t = s_t + r_t - c_t + \theta_t$$

where r_t is the risk-free interest rate, c_t refers to storage costs net of the convenience yield, and θ_t is the marking-to-market term.

The marking-to-market term represents the process by which, at the end of each trading day, the daily gain or loss from holding a futures contract is transferred between traders. LME contracts are not marked-to-market, so that the marking-to-market term is zero. Daily profits and losses from holding a contract on the LME accumulate until the contract maturity date. With the marking-to-market term, θ_t, set to zero, an empirical specification of the model for LME futures contracts can be represented as:

$$(4) \qquad f_t = \beta_0 + \beta_1 s_t + \beta_2 r_t + \beta_3 c_t + \phi_t$$

where ϕ_t is a white noise error term. However, the storage cost net of convenience yield is not an observable variable. An alternative specification of the cost-of-carry model is:

$$(5) \qquad f_t = s_t + r_t + w_t + l_t$$

where w_t represents storage costs over the period t to t+1, and l_t refers to stock level effects which include convenience yield and a premium for the risk due to holding stocks.

Stock level effects, l_t, have been modelled by Heaney (1998) for the LME lead market, and the same specification is used in equation (6):

$$(6) \qquad l_t = \delta i_t - \gamma$$

where i_t is the log of the inventory or stock level, and γ is a constant parameter of the model. The restriction on the parameter of the stock level, $\delta > 0$, ensures the model is consistent with the behaviour of the convenience yield and risk premium effect in Working (1949). Storage costs in equation (5), w_t, are assumed to be constant, as is consistent with the recent literature. Thus, for empirical modelling, the Cost-of-Carry model can be specified as:

$$(7) \qquad f_t = \eta_0 + \eta_1 s_t + \eta_2 r_t + \eta_3 i_t + v_t$$

where v_t is a zero mean stationary error term.

Chow et al. (1999) note that most of the theoretical futures pricing literature does not take into account common time series properties of financial data, particularly the existence of stochastic trends, or unit roots in the price levels. In addition, cointegration provides a linear framework in which the cost-of-carry and risk premium relationships may be directly tested when the interest rate, stock and price levels contain stochastic trends. A stationary variable can be omitted from a cointegrating regression without affecting the consistency of the coefficient estimates or the power of the statistical procedures for hypothesis testing (Park and Phillips, 1989). Storage cost, convenience yield and risk premium variables have been traditionally considered as covariance stationary in the recent literature (Chow et al., 1999), although there have been some arguments advanced for a non-stationary convenience yield. Under the above assumptions, the models in equations (2) and (4) can be considered nested within the model in equation (7). The empirical analysis will consider these three models. In each model, the spot price effect on the futures price is expected to be positive and close to one. The theory of storage implies that the effect of the interest rate in equations (4) and (7), as a cost of storage, should also be positive.

4 DATA

Daily data for the LME spot price and 3-month contract settlement price covering the period 3 January 1989 to 30 September 1998 are obtained from the LME. Prices quoted by the LME prior to July 1993 are denominated in British Pounds. These spot and 3-month futures prices are converted from British Pounds to US Dollars using the spot and 3-month US Dollar to British Pound exchange rates, respectively. After July 1993, prices are quoted in US Dollars. Plots of the LME copper spot and futures prices are given in Figures 1 and 2.

The LME holds significant stocks of copper in official LME warehouses in Europe, the United States, Japan and Singapore. Data on official stock levels are also obtained from the LME. Stock levels are recorded on a weekly basis from 6 January 1989 to 26 April 1990, a twice-weekly basis from 30 April 1990 to 30 March 1997, and daily for the remainder of the sample, namely 1 April 1997 to 30 September 1998. A daily series of stock levels is constructed by assuming daily observations are identical to the weekly stock level quote for the relevant week. Where stock level quotes are twice-weekly, the Tuesday quotation is assumed to apply to Monday and Tuesday, while the Friday observation applies to Wednesday, Thursday and Friday. The stock level is plotted in Figure 3.

An appropriate proxy for the risk-free interest rate must be determined. Contracts on the LME are denominated in US Dollars, so that a US Dollar interest rate is required. The US 3-month Treasury bill secondary market rate is used. A secondary market rate is an appropriate proxy because of its similarity to the "notional" risk free rate faced by market participants in the LME copper market. Rates in the secondary market are available to participants in the LME metals market. A sample of daily observations from 3 January 1989 to 30 September 1998, to correspond with the LME data, are obtained from the Federal Reserve Bank of St. Louis. Figure 4 plots the US 3-month Treasury bill secondary market rate.

LME spot and futures prices are expressed in natural logarithms, as is the stock level variable. The risk-free interest rate proxy is expressed in levels.

5 NON-STATIONARITY AND UNIT ROOTS

Structural breaks are evident in plots of the four variables over the full sample (Figures 1 to 4). It has been established that the presence of structural breaks affects tests of non-stationarity. Augmented Dickey-Fuller and Phillips-Perron tests are generally biased toward the non-rejection of a unit root (see Perron, 1989). Examination of the data reveals structural breaks at observations numbered 1185 and 1484, which refer to 27 October 1993 and 20 January 1995, respectively. The following stationarity testing and cointegration analysis is based on four sample sets:

Full sample: observations 1 to 2394, without explicitly modelling structural breaks;
Sub-sample A: observations 1 to 1185;
Sub-sample B: observations 1186 to 1484;
Sub-sample C: observations 1485 to 2394.

In the full sample, there are assumed to be no structural breaks, and testing of the single full sample is conducted accordingly. Testing of sub-samples A, B and C explicitly accommodates the exogenously specified structural breaks.

The augmented Dickey-Fuller (ADF) test is used to test for the presence of unit roots in each of the four variables in the full sample and sub-sample sets A, B and C. The ADF(p) statistic for a unit root in x_t is given by the t-ratio of the ordinary least squares estimate of β in the auxiliary regression:

$$(8) \qquad \Delta x_t = \alpha + \gamma t + \beta x_{t-1} + \sum_{i=1}^{p} \delta_i \Delta x_{t-i} + v_t$$

where Δx_t is the first difference of x_t, t is a deterministic trend term, and v_t is a stationary error term. The distributional properties of the error term are non-standard. Simulated critical values provided by MacKinnon (1991) are used to determine the significance of the ADF test statistics.

Plots of the data show the possibility of a deterministic trend in all four series (see Figures 1 to 4). Where a trend is present in the data, the test statistics and critical values for the ADF test are substantially different when the auxiliary regression is estimated with and without the trend term. Both the ADF tests with and without trend are considered for determining the order of integration the logarithms of each data series. Where inclusion of the trend term makes a substantial difference to the test statistic, the ADF with trend is used. Plots of the first differences for each variable (Figures 5 to 8) show that there are no deterministic trends in the first differences of the data. Inclusion of the trend term in the ADF regression makes little difference to the test statistic in every case, so that the ADF test without trend is used for the first differences of each series.

As the data are daily, unit root testing is conducted with lag lengths of 0 (DF test) to 6 (ADF(6) test). The Akaike Information Criterion, Schwarz Bayesian Criterion and the Hannan-Quinn Criterion are used to select the optimal lag length. The results of the unit root testing procedure for each sample are presented for the logarithms of each series in Table 1 and for logarithmic first differences in Table 2. The unit root tests suggest that each series is integrated of order 1, or I(1), within the full sample, and for the three sub-samples.

6 COINTEGRATION TESTS AND ESTIMATION RESULTS

Tests for the number of cointegrating relationships between the four variables in equation (7), the futures price, spot price, stock level and interest rate, were conducted for all four samples using the Johansen maximum likelihood procedure with an unrestricted intercept and an unrestricted trend term. VAR lag lengths from 1 to 6 were used. For the majority of cointegration tests, the choice of VAR lag length had no discernible effect on the number of cointegrating vectors using the trace and maximal eigenvalue statistics. The parameter estimates of the cointegrating vectors were also stable over the choice of VAR lag length. As daily data are used, a VAR lag length of 5 is chosen to ensure the time series properties of the data are reflected in the modelling procedure.

For the full sample and sub-samples A and C, both the trace and maximal eigenvalue statistics suggest the presence of one cointegrating vector for the four variables. Both statistics imply no cointegrating relationships exist between the four variables within sub-sample B (see Table 3), so that there is no long run relationship among the four variables within sub-sample B.

Cointegration tests indicate the existence of one long run relationship among the four variables for each of the full sample, and sub-samples A and C. The cointegrating vector for each sample, normalised on the futures price, is given in Table 4. In each long run relationship, the coefficient of the spot price is positive and close to one. The stock coefficient is positive in each case, as is required for equation (6). The interest rate coefficient is negative for the full sample and sub-sample A, but is positive for sub-sample C. The sign of the interest rate coefficient for the full sample is not consistent with the theory of storage. However, the cost-of-carry model of equation (4) may alternatively be viewed as a special case of the risk premium hypothesis, in which the interest rate is the risk premium (see Chow et al., 1999). Under this interpretation, the interest rate would have a negative effect. Over the full sample, the magnitude is small, but the effect of the interest rate is larger for the sub-samples, particularly for sub-sample C. A joint test of zero coefficients on all the endogenous variables in the model is conducted for each sample period. The LR statistic is significant in each case, rejecting the null hypothesis.

Likelihood ratio tests are conducted in the presence of restrictions on the general model (see Table 5). Restrictions according to the model of equation (2) delete LME stocks and interest rates from the model, while those from equation (4) delete only LME stocks from the model. Finally, the general model with interest rates excluded is also considered.

For the full sample, the LR tests imply that either the stock variable or interest rate variable can be excluded from the model, but not both. The model in equation (4) is not rejected, providing support for this specification of the cost-of-carry model. Similarly, the no-interest-rate model is also not rejected, but the LR test rejects the validity of restrictions implied by the risk premium hypothesis, as specified by equation (2). For sub-sample A, the LR test supports the exclusion of both stocks and interest rates from the model, providing support for the risk premium model. The tests for sub-sample C suggest all four variables must be included for a long run model to exist, thereby supporting the cost-of-carry specification in equation (7).

Likelihood ratio tests of the equality of the stock and interest rate parameters indicated rejection of the null for the full sample and sub-sample A, but not sub-sample C. The validity of restricting stocks and interest rates to have an equal and opposite effect was rejected for sub-sample C, but not for the full sample or sub-sample A.

7 CONCLUSION

Based on the risk premium and cost-of-carry models, where the futures price, spot price, interest rate, and stock level variables all contain stochastic trends, a framework for estimating a long run pricing model for copper futures prices was specified. After testing for non-stationarity, assuming no structural break and explicitly accommodating two exogenously specified structural breaks, all four variables were found to be integrated of order 1. One long run model was found to exist in the full sample, and in sub-samples A and C. Tests provided support for the cost-of-carry model in the full sample, the risk premium hypothesis in sub-sample A, and the cost-of-carry hypothesis in sub-sample C.

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Figure 1. LME Copper Spot Price

Figure 2. LME Copper 3-Month Futures Price

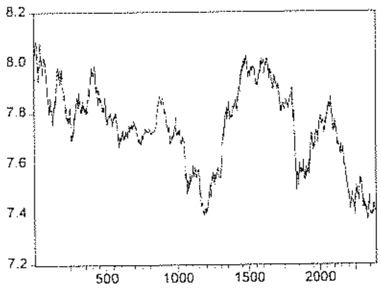

Figure 3. LME Official Copper Stocks

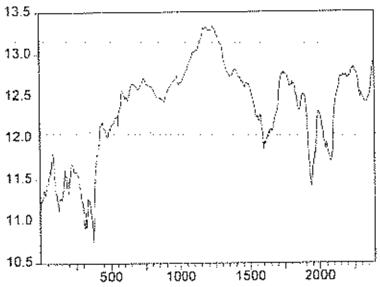

Figure 4. US 3-Month Treasury Bill Rate

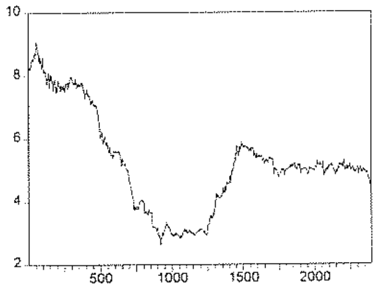

Figure 5. Logarithmic First Difference of LME Copper Spot Price

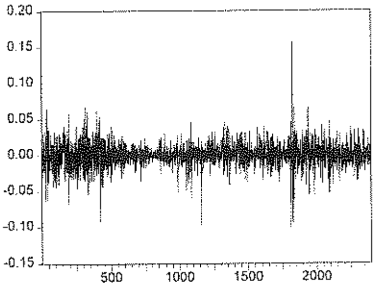

Figure 6. Logarithmic First Difference of LME 3-Month Copper Futures Price

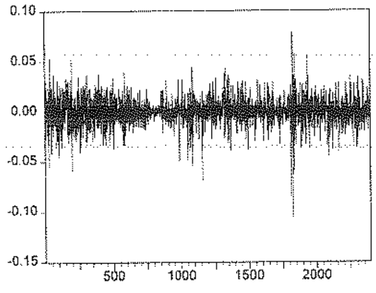

Figure 7. Logarithmic First Difference of LME Official Copper Stocks

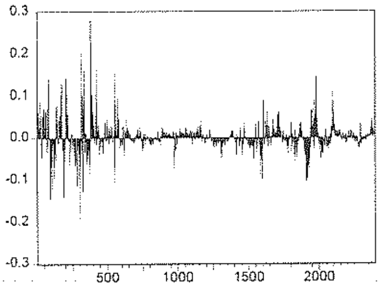

Figure 8. First Difference of US 3-Month Treasury Bill Rate

Table 1. Unit Root Tests of Logarithms of Variables

Sample Set	Non-Stationarity Test	Copper Spot Price	Copper Futures Price	LME Stocks	Interest Rate
Full Sample Observations 1-2394	Deterministic Trend ?	No	No	No	Yes
	ADF Lag Length	1	1	5	2
	ADF Statistic	-2.006	-1.664	-2.305	-1.303
	Critical Value	-2.863	-2.863	-2.863	-3.414
Sub-sample A Observations 1-1185	Deterministic Trend ?	Yes	Yes	Yes	Yes
	ADF Lag Length	1	1	5	1
	ADF Statistic	-2.761	-2.005	-2.661	-1.731
	Critical Value	-3.416	-3.416	-3.416	-3.416
Sub-sample B Observations 1186-1484	Deterministic Trend ?	Yes	Yes	Yes	Yes
	ADF Lag Length	0	0	5	2
	ADF Statistic	-3.156	-2.945	-1.421	-3.114
	Critical Value	-3.426	-3.426	-3.426	-3.426
Sub-sample C Observations 1485-2394	Deterministic Trend ?	Yes	Yes	Yes	Yes
	ADF Lag Length	2	1	5	6
	ADF Statistic	-2.148	-2.048	-2.907	-2.133
	Critical Value	-3.417	-3.417	-3.417	-3.417

Table 2. Unit Root Tests of Logarithmic First Differences of Variables

Sample Set	Non-Stationarity Test	Copper Spot Price	Copper Futures Price	LME Stocks	Interest Rate
Full Sample Observations 1-2394	Deterministic Trend ?	No	No	No	No
	ADF Lag Length	0	0	4	1
	ADF Statistic	-55.622	-55.899	-12.220	-35.066
	Critical Value	-2.863	-2.863	-2.863	-2.863
Sub-sample A Observations 1-1185	Deterministic Trend ?	No	No	No	No
	ADF Lag Length	0	0	4	0
	ADF Statistic	-40.116	-41.147	-10.072	-30.499
	Critical Value	-3.416	-3.416	-3.416	-3.416
Sub-sample B Observations 1186-1484	Deterministic Trend ?	No	No	No	No
	ADF Lag Length	1	1	4	1
	ADF Statistic	-14.712	-14.422	-4.857	-13.106
	Critical Value	-2.871	-2.871	-2.871	-2.871
Sub-sample C Observations 1485-2394	Deterministic Trend ?	No	No	No	No
	ADF Lag Length	1	0	4	5
	ADF Statistic	-24.756	-33.830	-5.503	-14.188
	Critical Value	-3.417	-3.417	-3.417	-3.417

Table 3. Cointegration Tests for the General Model

Test Statistic	Full Sample Observations 1-2394	Sub-sample A Observations 1-1185	Sub-sample B Observations 1186-1484	Sub-sample C Observations 1485-2394
Trace	1	1	0	1
Eigenvalue	1	1	0	1

Table 4. Cointegrating Vectors for the General Model

Variable	Full Sample Observations 1-2394	Sub-sample A Observations 1-1185	Sub-sample C Observations 1485-2394
Copper Spot Price	0.980	1.043	1.070
LME Stocks	0.019	0.011	0.053
Interest Rate	-0.005	-0.018	0.059
LR	56.754 (3)	30.384 (3)	31.244 (3)
Prob Value	0.000	0.000	0.000

Notes: The endogenous variable is the copper futures price. The LR statistic is the joint test of zero coefficients on all the variables in the model. The degrees of freedom of the tests are given in parentheses.

Table 5. Tests of Restrictions on the General Model

Variable	Full Sample Observations 1-2394			Sub-Sample A Observations 1-1185			Sub-Sample C Observations 1485-2394		
	Model (2)	Model (4)	No Interest Rate	Model (2)	Model (4)	No Interest Rate	Model (2)	Model (4)	No Interest Rate
Spot Price	0.941	0.974	0.977	1.155	1.028	1.161	1.106	1.065	1.011
LME Stocks	0.000	0.000	0.029	0.000	0.000	0.026	0.000	0.000	0.055
Interest Rate	0.000	-0.010	0.000	0.000	-0.020	0.000	0.000	0.052	0.000
LR	19.112 (2)	3.802 (1)	1.970 (1)	5.645 (2)	0.677 (1)	3.583 (1)	16.210 (2)	14.146 (1)	7.089 (1)
Prob Value	0.000	0.051	0.160	0.059	0.411	0.058	0.000	0.000	0.008

Notes: The endogenous variable is the copper futures price. The LR statistic tests the validity of the zero restriction(s) imposed on the model. The degrees of freedom of the tests are given in parentheses.