Testing for Random Effects in Panel Data

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Abstract: This paper has three objectives. First, it shows that the variable addition tests proposed by Baltagi (1995b, 1996a, b, 1997) to test for random effects in panel data models use variables that are correlated with the testing equation's error even under the null hypothesis of no random effects making his suggested procedure inappropriate. Second, it develops some simple modifications of Baltagi's testing procedure that are asymptotically valid. Third, some Monte Carlo evidence is provided on the small sample properties of some of these tests and existing tests proposed in the literature. The small sample properties of the original and modified Baltagi tests are found to be relatively poor. In contrast, the standard Breusch-Pagan (1980) two- sided and Honda (1985) one-sided tests for random effects have good small sample properties, as does a test for first order serial correlation.

1. INTRODUCTION

The purpose of this paper is threefold. First, this paper shows that the added variables that Baltagi (1995b, 1996a, b, 1997) proposed for testing for random effects in panel data models are correlated with the testing equation's error even under the null hypothesis making the procedure inappropriate. Second, it develops some modifications of Baltagi's testing procedure to make the procedure valid asymptotically. Third, it provides some limited Monte Carlo evidence on the small sample properties of some of these tests and existing tests in the literature.

The plan of the paper is as follows. In section 2, the individual random effects model and its standard estimator are defined. Following a brief survey of existing tests for random individual effects, Baltagi's (1995b, 1996a, b, 1997) tests, their problems and some suggested modifications are discussed in section 3. Some limited Monte Carlo evidence on the small sample properties of some of the tests is provided in section 4.

2. MODEL AND NOTATION

Consider the following linear regression model for panel data:

$$y_{it} = a + x_{it}' b + u_{it} + v_i, t = 1,.., T; i = 1,.., N, \quad (1)$$

where x_{it} is presumed to be a kx1 vector of non-stochastic explanatory variables, b is a kx1 vector of unknown parameters, a is a scalar, u_{it} is assumed to be iid(0, σ_u^2), and the properties of v_i will be discussed shortly. For simplicity, the panel is assumed to be balanced (see Baltagi (1995a, chapter 9) on estimation of unbalanced panels).

There are three standard models that arise from (1): (a) the pooled model where there is no difference in the regression parameters across agents ($v_i = 0, \forall i$); (b) the one-way fixed effect model which allows the constant in the regression model to differ across agents (that is, the v_i are treated as fixed constants); and (c) the one-way random effect model which treats the v_i 's as iid(0, σ_v^2) variates.

In the one-way random effects model, the equation's true error $\eta_{it} = u_{it} + v_i$ has a particular form of correlation, namely, $E(\eta_{it}\eta_{is}) = \sigma_v^2 \neq 0, t \neq s$, while $E(\eta_{it}\eta_{js}) = 0, i \neq j$ (see Baltagi (1995a)). In this case, the standard method of estimation is to apply the Fuller and Battese (1973, 1974) transformation to this model to convert the error to one which satisfies the assumptions of the standard linear regression model. This transformation is as follows:

$$y_{it} - c\,y_{i.} = a(1-c) + (x'_{it} - c\,x'_{i.})b + \eta_{it} - c\,\eta_{i.}, \quad (2)$$

where $c = 1 - \sigma_u / \sigma_1$, $\sigma_1^2 = T\sigma_v^2 + \sigma_u^2$, and $y_{i.}$, $x'_{i.}$ and $\eta_{i.}$ refer to the group means of each variable for each i, that is, $y_{i.} = (1/T)\sum_{t=1}^{T} y_{it}$. As is obvious from the definition of c, c satisfies the restriction $0 \le c \le 1$, and when c=0 there are no random effects (σ_v^2=0).

3. TESTS FOR RANDOM EFFECTS

Breusch and Pagan (1980) suggested a test for testing for random individual effects against the null of the pooled model based on the Lagrange Multiplier (LM) principle. If $\hat{u}_{it}$ are the residuals obtained from estimating (1) by ordinary least squares (OLS) and $\hat{u}_{i.} = (1/T)\sum_{t=1}^{T} \hat{u}_{it}$, then the Breusch-Pagan (1980) LM test for random individual effects ($H_0 : \sigma_v^2 = 0$) can be written as

$$LM = b^2 s^2, \quad (3)$$

where $b^2 = NT/\{2(T-1)\}$ and

$$s = [\Sigma_i \{\hat{u}_{i.} T\}^2 / \Sigma_i \Sigma_t \hat{u}_{it}^2] - 1.$$

Under the null hypothesis of no random individual effects, this LM test is asymptotically distributed as a $\chi^2_{(1)}$ variate.

Since this test does not take account of the one-sided nature of the alternative $\sigma_v^2 > 0$, Honda (1985) suggested using bs to test the null hypothesis $\sigma_v^2 = 0$. By showing that asymptotically bs ~ N(0, 1) under the null hypothesis, Honda (1985) proposed a one-sided testing procedure for random individual effects using this statistic with the upper tail as the rejection region.

In a series of papers, Baltagi (1995b, 1996a, b, 1997) has suggested a regression approach based on a Gauss-Newton expansion of the Fuller-Battese type estimator to derive tests for random individual effects; random time effects; random individual and time effects; and random individual effects and serial correlation. The regressions to be estimated in each case are all of the form:

$$y_{it} = a + x'_{it} b + z'_{it} c + u_{it}, \quad (4)$$

where z'_{it} is the variable(s) to be added. If only one variable is to be added, its significance is tested using a t-test, while if two variables are added their significance is tested using an 'F-type' test. For random individual effects, the suggested added variable is $\hat{u}_{i.}$ (Baltagi (1995b, 1996a)). For random time effects, the suggested added variable is $\hat{u}_{.t} = (1/N)\sum_{i=1}^{N} \hat{u}_{it}$ (Baltagi (1996b)). For random individual and time effects, the suggested added variables are $\hat{u}_{i.}$ and $\hat{u}_{.t}$ (Baltagi (1996b)). For serial correlation and random individual effects, the suggested added variables are $\hat{u}_{i.}$ and $\hat{u}_{it-1}$ (Baltagi (1997)).

Each of the tests involves adding one or more variables to the null model (1) (with $v_i = 0, \forall i$) and testing their significance. Although Baltagi develops these tests assuming that the regression component of the model is nonlinear in the parameters, this confuses the simple essence of the test and its problems.

Since the derivation of each of these tests is essentially the same, the random individual effects case is explained. Equation (2) offers a hint for one possible way to test whether there are any random effects, that is, by testing c= 0. Since $\eta_{i.} = y_{i.} - a - x'_{i.} b$, (2) can be rearranged as

$$y_{it} = a + x_{it}' b + c\,\eta_{i.} + (\eta_{it} - c\,\eta_{i.}), \qquad (5)$$

Under the null hypothesis that c=0, $\eta_{it} = u_{it}$, and $\eta_{i.} = u_{i.}$, so that (5) can be rewritten as

$$y_{it} = a + x_{it}' b + c\,u_{i.} + u_{it}. \qquad (6)$$

If $u_{i.}$ was observed, OLS could be applied to (6), and the significance of $u_{i.}$ tested.

Since $u_{i.}$ is not observed, an obvious alternative is to replace $u_{i.}$ by the residuals obtained by applying OLS to (1), $\hat{u}_{it}$, which gives

$$y_{it} = a + x_{it}' b + c\,\hat{u}_{i.} + u_{it} + c(u_{i.} - \hat{u}_{i.}). \qquad (7)$$

If a regression model contains a generated regressor, then, in general, this will cause a loss of efficiency and problems with inference (see Pagan (1984b, 1986) and McKenzie and McAleer (1997)). However, since (7) is used to test c=0, the final term in (7) that causes the generated regressor problem disappears under the null hypothesis. It is obvious that (7) is the basis for Baltagi's (1995b, 1996a) test for random individual effects.

For the testing procedure based on OLS applied to (7) to be valid requires that all the regressors in (7) be uncorrelated with the error term under the null hypothesis (see Pagan and Hall (1983), Pagan (1984a) and Godfrey (1988)). The correlation between the regressor $u_{i.}$ and u_{it} in (6) is $E(u_{i.}u_{it}) = \sigma_u^2 / T \neq 0$ and $\sum_t \sum_i E(u_{i.}u_{it}) = N\sigma_u^2$, so that OLS applied to (6) will be inconsistent even under H_0 because the variable being added, $u_{i.}$, is correlated with the error term. The use of an estimate $\hat{u}_{i.}$ to replace $u_{i.}$ will not avoid this problem. By similar reasoning, adding $\hat{u}_{.t}$ to (1) will cause the same sort of problem.

There are two relatively obvious ways around this problem. One is to slightly modify the variable being added to remove the portion that is causing the problem, u_{it}. Define $u_{it}^* = u_{i.} - u_{it}/T$, and consider

$$y_{it} = a + x_{it}' b + c\,u_{it}^* + u_{it} + c(u_{i.} - u_{it}^*). \qquad (8)$$

It is easy to show if the null hypothesis of no random effects is correct, then $E(u_{it}^* u_{it}) = 0$. Thus, (8) can then be estimated by ordinary least squares, and will give consistent estimates and correct inference under H_0. The other alternative is to estimate (6) using an instrumental variable (IV) technique with u_{it}^* used as the instrument for $u_{i.}$. Since the equation's error in each case is u_{it} under H_0, the standard errors from either the first OLS regression or the second IV regression will be appropriate.

This argument assumes that $u_{i.}$ and u_{it}^* can be observed. Suppose that u_{it}^* is replaced by $\hat{u}_{it}^* = \hat{u}_{i.} - (1/T)\hat{u}_{it}$. Equation (8) can then be rewritten under H_0 as:

$$y_{it} = a + x_{it}' b + c\,\hat{u}_{it}^* + u_{it}. \qquad (9)$$

A test of c=0 can then be implemented by testing the significance of $\hat{u}_{it}^*$ in (9) using a t-test. A test based on the sort of modification is called an adjusted Baltagi test. By similar reasoning, the obvious replacement for $\hat{u}_{.t}$ suggested by Baltagi (1996b) to test for random time effects is $\hat{u}_{it}^+ = \hat{u}_{.t} - (1/N)\hat{u}_{it}$.

Writing the model in the form of (7) or (9) also allows us to make use of the information that $c \geq 0$ by performing a one-sided test rather than a two-sided test.

A totally equivalent testing equation to (9) is

$$\hat{u}_{it} = a^* + x_{it}' b^* + c \hat{u}_{it}^* + u_{it} , \qquad (10)$$

where $a^* = a - \hat{a}$, $b^* = b - \hat{b}$, and $\hat{a}$ and $\hat{b}$ are the OLS estimators of a and b in (1). Since $\hat{u}_{it}^*$ is asymptotically uncorrelated with the other regressors in (10), the equation can be rewritten as

$$\hat{u}_{it} = c \hat{u}_{it}^* + w_{it} , \qquad (11)$$

where $w_{it} = a^* + x_{it}' b^* + u_{it}$. A t-test of c=0 in (11) would also be an asymptotically valid test. A test based on this type of equation is called a modified Baltagi test.

As a result, instead of the added variables suggested by Baltagi the following variables are proposed. For random individual effects, the suggested added variable is $\hat{u}_{it}^*$. For random time effects, the suggested added variable is $\hat{u}_{it}^+$. For random individual and time effects, the suggested added variables are $\hat{u}_{it}^*$ and $\hat{u}_{it}^+$. For serial correlation and random individual effects, the suggested added variables are $\hat{u}_{it}^*$ and $\hat{u}_{it-1}$.

4. MONTE CARLO SIMULATION

The design of the Monte Carlo simulation follows what has become the standard design suggested by Nerlove (1971). That is,

$$y_{it} = a + x_{it} b + u_{it} + v_i , t = 1,.., T; i = 1,.., N , \qquad (12)$$

$$x_{it} = 0.1t + 0.5 x_{it-1} + \omega_{it} , \qquad (13)$$

where ω_{it} is generated as a uniformly distributed random variable on the interval [-0.5, 0.5], $x_{i0} = 5 + 10 \omega_{i0}$, $u_{it} \sim N(0, (1-\rho)\sigma^2)$ and $v_i \sim N(0, \rho \sigma^2)$. Throughout the experiments, a=5, b=0.5, and σ^2=20. However, ρ is varied over the set (0.0, 0.01, 0.1, 0.2) so random individual effects are generated. The number of individuals (N) was varied over the set (25, 50, 75, 100), and the number of time periods (T) was varied over the set (5, 10, 20). Table 1 reports the proportion of rejections based on 5000 replications for a selected subset of these cases. The nominal size for all the tests is set at 5%. The maximum standard errors of the estimates of Type I errors and powers are, respectively, $\sqrt{0.05(1-0.05)/5000} \approx 0.003$ and $\sqrt{0.5(1-0.5)/5000} \approx 0.007$.

When ρ=0, the rejection proportions indicate estimated probabilities of type I errors. The only tests with sizes being acceptable (within 2 standard deviations of the nominal size for a majority of the combinations of N and T in Table 1) are the Breusch-Pagan (1980) test for random individual effects (PI), Honda's (1985) one-sided variation of the PI test (HI), and an LM test for first-order serial correlation (SC). The severe impact of the correlation between the added variable and the testing equation's error for Baltagi's tests is easily observed for the one-sided versions of Baltagi's original tests (BI1, BT1, BJ and BIS). For three of these tests (BI1, BJ and BIS), the rejection proportion is unity regardless of the value of ρ! While the estimated sizes of the modified and adjusted Baltagi tests are significantly better than those for the original Baltagi tests, the suggested modifications of Baltagi's tests for random individual effects (MI1 and AI1), for joint random individual and time effects (MJ and AJ) , and for random individual effects and serial correlation (MIS and AIS) still have unacceptable large sizes. In contrast, it is interesting that the size of all the tests for random time effects (apart from the original Baltagi test) and the Breusch-Pagan joint test for random individual and time effects are on the conservative side.

When $\rho \neq 0$, the rejection proportions indicate size-unadjusted powers. The power performance of the Breusch Pagan and Honda tests for random individual effects (PI and HI) are quite similar. As might be expected, the LM test for random individual and time effects (BJ) does not perform quite as well as the tests designed for the particular alternative being examined (PI and HI). The tests for random time effects designed for a quite different type of random effects model do not perform at all well when the true alternative is the random individual effects model. In fact, power tends to be smaller than size!

One interesting result from the Monte Carlo study is that the test for first-order serial correlation has some power against the alternative of random individual effects. This is not surprising since this serial correlation test tests whether $E(u_{it} u_{it-1})$ in (6) is zero, while under the alternative of random individual effects $E(u_{it} u_{it-1}) = \sigma_v^2 \neq 0$.

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Table 1: Monte Carlo Simulation – Proportion of Rejections (Nominal Size 5%)

N	25	50	75	100	25	50	50	100	100
T	5	5	5	5	10	5	5	5	5
ρ	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.1	0.2
SC	0.4720*	0.0504*	0.0490*	0.0460*	0.0474*	0.2694	0.7260	0.4982	0.9544
PI	0.0448*	0.0458*	0.0436	0.0480*	0.0398	0.5236	0.9412	0.8060	0.9988
HI	0.0450*	0.0420	0.0462*	0.0466*	0.0448*	0.6160	0.9636	0.8720	0.9996
BI1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
AI1	0.0844	0.0920	0.0962	0.1004	0.0792	0.7370	0.9822	0.9304	0.9998
MI1	0.0848	0.0914	0.0962	0.1000	0.0794	0.7374	0.9822	0.9306	0.9998
PT	0.0168	0.0188	0.0170	0.0202	0.0222	0.0102	0.0058	0.0096	0.0048
HT	0.0250	0.0294	0.0250	0.0296	0.0312	0.0152	0.0100	0.0164	0.0090
BT1	0.6026	0.6074	0.6016	0.6100	0.9754	0.5568	0.5330	0.5642	0.5296
AT1	0.0286	0.0306	0.0264	0.0322	0.0426	0.0176	0.0104	0.0168	0.0108
MT1	0.0272	0.0300	0.0248	0.0292	0.0414	0.0152	0.0100	0.0162	0.0088
PJ	0.0306	0.0316	0.0316	0.0354	0.0342	0.4260	0.9076	0.7290	0.9974
BJ	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
AJ	0.4084	0.3952	0.3940	0.3999	0.3180	0.7302	0.9710	0.9090	0.9998
MJ	0.3954	0.3836	0.3810	0.3856	0.3126	0.7208	0.9696	0.9074	0.9998
BIS	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
AIS	0.1324	0.1272	0.1214	0.1244	0.1480	0.5786	0.9536	0.8504	0.9994
MIS	0.1330	0.1272	0.1222	0.1244	0.1484	0.5800	0.9536	0.8506	0.9994

Notes: (1) SC refers to an LM test for first-order serial correlation; for the remaining entries the coding is as follows: the first letters P, H, B, A and M refer to the Breusch-Pagan LM test, Honda one-sided LM test, Baltagi test, adjusted Baltagi test, and modified Baltagi test (based on equation (11)), respectively; the second letters I,T and J refer to a test for random individual effects, a test for random time effects, and a joint test for random individual and time effects, respectively; and the third letter or numerals 1 and S refer to a one-sided test, and a test combined with a test for serial correlation, respectively.

(2) A '*' indicates the value is within 2 standard deviations of the nominal size of 0.05 when the standard deviation is computed as $\sqrt{0.05(1-0.05)/5000} \approx 0.003$. The maximum standard error of the power estimates is $\sqrt{0.5(1-0.5)/5000} \approx 0.007$.