Testing Alternative Models of Volatility in Currency Futures Markets

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Abstract Numerous studies of spot market volatility exist, but few studies have concentrated on currency futures returns. The search for effective hedging strategies, the use of circuit breakers in currency markets, and a heightened interest in currency forecasting techniques highlight the importance of modelling volatility in currency futures markets. The underlying relationship between spot and futures contracts means that movements in one market can be captured by corresponding changes in the other. In this paper, the volatility in Australian dollar futures is examined using Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models. Models of the volatility in Australian dollar futures returns are based on two standard hypotheses of futures prices. It is shown that the volatility in futures returns is strongly affected by volatility in the underlying spot market and volatility in the foreign risk-free interest rate, but not by volatility in the domestic risk-free interest rate.

1. INTRODUCTION

Studies related to price volatility have recently been the focus of much research in stock markets and foreign exchange markets. Lately, interest in futures market volatility has become more apparent in light of the sharp volatility seen in many foreign exchange markets. The increased activity of multinational corporations, the development of global financial markets, and the prominent role of international fund managers have resulted in the need for managing exchange rate volatility through the use of currency derivatives. This need has introduced into the market a large number of derivative products that has witnessed large volumes of trading activity that have matched those of the underlying currencies.

In light of the importance of volatility in financial markets, a seminal contribution to the study of stock market volatility was Schwert (1989). He sought to establish which economic variables are highly correlated with volatility in returns, and found little evidence that volatility in economic fundamentals had a discernible influence on stock market returns. Another study by Lamoureux and Lastrapes (1990) assumed that volatility was influenced both by past forecast errors (GARCH) and by the volume of trading, where volume was interpreted as measuring the arrival of new information. Lamoureux and Lastrapes (1989) conjectured that, in general, GARCH effects in many other studies are really measuring the persistence in the arrival of new information.

Few studies have focused on modelling volatility in currency futures markets. Many studies of currency futures deal with issues related to market efficiency and the development of optimal dynamic hedge ratios (see Kroner and Sultan (1993)), but few studies investigate the volatility of futures returns. In this paper, models of volatility are formulated for Australian dollar futures contracts traded on the International Monetary Market of the Chicago Mercantile to gain a better understanding of the determinants of volatility in currency futures returns.

In Section 2, models of volatility for Australian dollar futures contracts traded on the International Monetary Market of the Chicago Mercantile Exchange are formulated. Section 3 describes the data. In Section 4, results of nonstationarity and cointegration tests are analysed. Empirical results are presented in Section 5. Some concluding remarks are given in Section 6.

2. MODELS OF VOLATILITY FOR CURRENCY FUTURES CONTRACTS

In Sequeira and McAleer (1999), models of Australian dollar futures contracts are based on the two main hypotheses for pricing currency futures contracts, namely the Cost-of-Carry and Risk Premium hypotheses. The Cost-of-Carry model is based on the theoretical formulation derived by Amin and Jarrow (1991) within the Heath et al. (1992) framework. Based on this framework, Sequeira and McAleer (1999) derive models for the Cost-of-Carry and Risk

Premium hypotheses. The Cost-of-Carry hypothesis can be represented as follows:

$$f_t = s_t + r_t^d - r_t^f + \theta_t \qquad (1)$$

where f_t is the logarithm of price of a one-period ahead futures contract at time t, s_t is the logarithm of spot price at time $t+1$, r_t^d is the domestic risk-free rate of interest, r_t^f is the foreign risk-free rate of interest, and θ_t is the nonstochastic adjustment term for the marking to market feature of futures contracts. Assuming that the marking to market term is stationary, equation (1) can be rewritten after eliminating the marking to market term. The reason for this deletion is that Park and Phillips (1989) show that a stationary variable can be omitted from a cointegrating regression without affecting the consistency of the coefficient estimates or the power of hypothesis testing procedures. Consequently, equation (1) can be rewritten in either of the following forms:

$$\Delta f_t = \Delta s_t + \Delta r_t^d + \Delta r_t^f - (f_{t-1} - s_{t-1}) + \ldots \\ \ldots + (r_{t-1}^f - r_{t-1}^d) \qquad (2)$$

$$\Delta f_t = \Delta s_t + \Delta r_t^d + \Delta r_t^f - \ldots \\ \ldots (f_{t-1} - s_{t-1} - r_{t-1}^f - r_{t-1}^d). \qquad (3)$$

Equation (2) represents Case 1, which describes the futures returns as being explained by spot returns, changes in the domestic and foreign risk-free rates of interest, and two error-correction terms representing the basis (that is, the difference between the spot and futures price) and the interest rate differential between the foreign and domestic risk-free rates of interest. Equation (3), on the other hand, represents Case 2, which is similar to equation (2), with the exception that the two error-correction terms are replaced by one error-correction term among all four variables that comprise the Cost-of-Carry model.

Of interest in this paper is the modelling of the second moments of futures returns. Futures returns are obtained by taking the variance of equation (2) for Case 1 and the variance of equation (3) for Case 2, as follows:

Case 1

$$\text{var}(\Delta f_t) = \text{var}(\Delta s_t) + \text{var}(\Delta r_t^d) + \text{var}(\Delta r_t^f) \ldots \\ \ldots + \text{var}(f_{t-1} - s_{t-1}) + \text{var}(r_{t-1}^f - r_{t-1}^d) + \text{cov. terms} \qquad (4)$$

Case 2

$$\text{var}(\Delta f_t) = \text{var}(\Delta s_t) + \text{var}(\Delta r_t^d) + \text{var}(\Delta r_t^f) \ldots \\ + \text{var}(f_{t-1} - s_{t-1} - r_{t-1}^f - r_{t-1}^d) + \text{cov. terms.} \qquad (5)$$

The empirical formulations corresponding to equations (4) and (5) can be rewritten as equations (6) and (7), respectively:

$$\text{var}(\Delta f_t) = \alpha_0 + \alpha_1 \text{ var}(\Delta s_t) + \alpha_2 \text{ var}(\Delta r_t^d) + \ldots \\ + \alpha_3 \text{ var}(\Delta r_t^f) + \alpha_4 \text{ var}(f_{t-1} - s_{t-1}) + \ldots \\ \ldots + \alpha_5 \text{ var}(r_{t-1}^f - r_{t-1}^d) + \varepsilon_t \qquad (6)$$

$$\text{var}(\Delta f_t) = \beta_0 + \beta_1 \text{ var}(\Delta s_t) + \ldots \\ \ldots + \beta_2 \text{ var}(\Delta r_t^d) + + \beta_3 \text{ var}(\Delta r_t^f) + \ldots \qquad (7) \\ \ldots + \beta_4 \text{ var}(f_{t-1} - s_{t-1} - r_{t-1}^f - r_{t-1}^d) + \varepsilon_t .$$

Equations (6) and (7) are denoted as Cost-of-Carry Volatility models. In Case 1, the conditional variance of the futures returns is comprised of the conditional variances of the underlying spot currency, the domestic and foreign risk-free interest rates, the interest rate differential, and the basis. For Case 2, the conditional variance of futures returns are driven by the same conditional variances, with the exception that the conditional variances of the interest rate differential and basis are replaced by the conditional variance of a single error-correction term describing the long run relationship between futures and spot prices, and domestic and foreign risk-free interest rates.

In a similar manner, the Risk Premium hypothesis is based on the analysis in Fama and Farber (1979) and the theoretical model of Stulz (1981). This model is represented as:

$$f_t = s_t + \pi_t \qquad (8)$$

where π_t is the expected (logarithmic) risk premium at time $t+1$. Assuming that the risk premium is also stationary, and invoking the same result from Park and Phillips (1989), Sequeira and McAleer (1999) derive the Risk Premium model as:

$$\Delta f_t = \Delta s_t - (f_{t-1} - s_{t-1}). \qquad (9)$$

The second moment of the future returns according to the Risk Premium hypothesis is given by:

$$\mathrm{var}(\Delta f_t) = \mathrm{var}(\Delta s_t) + \mathrm{var}(f_{t-1} - s_{t-1})... \\ ... - 2\mathrm{cov}(\Delta s_t, (f_t - s_{t-1})) \tag{10}$$

To simplify the empirical formulation of the model, we assume that the covariance term is subsumed into the error term, so that the conditional variance of the futures returns based on the Risk Premium hypothesis is given by:

$$\mathrm{var}(\Delta f_t) = \delta_0 + \delta_1 \mathrm{var}(\Delta s_t)... \\ ... - \delta_4 \mathrm{var}(f_{t-1} - s_{t-1}) + \varepsilon_t \tag{11}$$

Equation (11) is denoted as the Risk Premium Volatility model. The model of futures returns in equation (11) is nested within the Cost-of-Carry volatility model in equation (6). In order for equation (6) to reduce to (11), it is necessary that the conditional variance of the interest rate differential, and the conditional variances of the changes in the domestic and foreign risk-free interest rates, be eliminated from equation (6).

Unlike the Cost-of-Carry volatility model in Case 1, the Cost-of-Carry volatility model in Case 2 does not nest the Risk Premium volatility model. Appropriate non-nested tests are required to determine the volatility model that best describes Australian dollar futures returns.

3. DATA

The futures contracts used in this study are Australian dollar futures traded on the International Monetary Market (IMM) of the Chicago Mercantile Exchange. Data on futures and spot prices of the Australian dollar are in natural logarithms, while data on the US 90-day Treasury spot and Australian 90-day bank accepted bill rates are in levels. The foreign risk-free interest rate is represented by 90-day bank accepted bill rates, with the US Treasury bill rate used as the domestic risk-free interest rate.

Due to the nature of futures contracts, price data obtained on futures contracts reflect a "stale price effect" when a single futures contract is analyzed. This effect is the occurrence of a dramatic fall in the open interest and trading activity as the maturity date of the particular contract is reached. Prices of futures contracts in the last days prior to maturity are said to be "stale". To overcome this effect, the analysis of futures prices is typically performed using several contracts over a longer time span. This approach will, however, result in overlapping contracts since, on any trading day, several contracts with different maturities may be traded simultaneously.

The issue as to the handling of overlapping contracts in futures data, coupled with the stale price effect, remains unresolved. Different approaches have been proposed and used in various studies. Hakkio (1981) applied the certainty equivalence theory of the term structure of interest rates and the hypothesis of interest rate parity to obtain a simple expression relating the six-month forward premium to the expected future one-month forward premium. His proposed theory, however, imposes non-linear cross-equation restrictions on the parameters of the model. Clark (1973), on the other hand, constructed a continuous time series of prices and volumes, and defined a contract that matured a fixed distance into the future. This fixed distance was taken to be the average time to maturity of all futures in the market. To define an average future, Clark constructed a weight function $W(\tau)$, where τ is the time distance between the current period and the maturity of existing contracts.

In this paper, the futures price data cover the period between 13 November 1989 and 17 July 1996. A continuous time series of futures prices is obtained by rolling over the current futures contract two weeks before maturity. Contracts are linked by excluding the last two weeks prior to delivery of the current contract, using volume data as a guide. Nearby contracts tend to dominate other contracts traded on the same day in terms of the higher trading volumes observed. This is the result of investors with long-term commitments rolling-over their hedge positions into the nearby contract just before the maturity date to avoid often erratic price movements. Following this procedure, a total of 1621 observations on the Australian dollar futures series are obtained. An examination of these time series movements reveals structural breaks in the futures price series at observations 190 and 937 in the sample period. Consequently, the empirical analysis is based on an analysis of the following three sample sets:

Set 1: full sample with two structural breaks;
Set 2: full sample without structural breaks;
Set 3: three sub-samples.

4. NON-STATIONARITY AND COINTEGRATION TESTS

4.1 Non-stationarity Tests

Four sets of variables, namely Australian dollar futures prices, Australian dollar spot prices, and domestic and foreign risk-free interest rates, are tested for unit roots using the augmented Dickey-Fuller (ADF) test. Results of the ADF test for all four variables and their first differences for the different sample sets are presented in Tables 1 and 2.

Two different types of unit root tests are applied to the three sample sets. Perron's (1989) unit root testing procedure is employed for sample set 1 to determine the order of integration of all four variables in the presence of two structural breaks. Augmented Dickey-Fuller (ADF) tests are used for sample sets 2 and 3, when the sample involves either the full sample without the sample breaks, as in set 2, or when the analysis is conducted as three separate sub-samples, as in set 3.

A Newey-West covariance matrix is used to calculate several of the unit root tests when Lagrange Multiplier diagnostics for the presence of serial correlation and heteroskedasticity are found to be significant. The results suggest the presence of a unit root in all four variables in all sample sets. Tests applied to the first differences of the variables strongly reject the null hypothesis of a unit root, which implies that the variables are integrated of order one, I(1).

4.2 Cointegration tests

Cointegration relationships among the four variables for the three sample sets are examined using Johansen's (1991) test and the results are presented in Table 3. For sample set 1, two cointegrating vectors were obtained, representing the long-run relationship between the futures and spot prices (i.e. the basis), and the long-run relationship between the domestic and foreign risk-free rates of interest (i.e. the interest rate differential). In sample sets 2, 3B and 3C, one cointegrating vector was obtained, suggesting an error-correction term among the futures and spot prices, and foreign and domestic risk-free interest rates. No cointegrating relationship was obtained for sample set 3A.

5. EMPIRICAL RESULTS

From the results of the cointegration tests, estimates of the error-correction terms in each sample set are obtained. GARCH (1,1) processes are estimated for each of the error-correction terms. Conditional variances for the error-correction terms in the Risk Premium Volatility models are denoted as CAECM, CBECM, CDECM and CEECM, for sample sets 1, 2, 3B, and 3C, respectively. For Cost-of-Carry Volatility models, the conditional variances are denoted CAECM4 and CAECM5 for set 1, CBECM1 for set 2, DECM1 for set 3B, and EECM1 for set 3C.

Similarly, conditional variances of the futures and spot returns, and domestic and foreign risk-free interest rates are estimated from their respective mean equations using a standard GARCH(1,1) formulation. These are denoted CVARF, CVARS, CVARRD, CVARRF for the conditional variances of the futures and spot returns, and the domestic and foreign risk-free interest rates, respectively.

Tables 3 presents the estimates of the Risk Premium Volatility models while table 4 presents the estimates for Cost-of-Carry Volatility models for Australian dollar futures returns. Newey-West adjusted covariance matrices are used when LM tests for serial correlation and heteroskedasticity are found to be significant.

In all sample sets, the conditional variances of the spot returns and of the rate of change of the foreign risk-free interest rate are significant in explaining the conditional variance of futures returns. For set 3B, the conditional variance of the error-correction term was found to be significant in explaining the conditional variance of the futures returns. Much of the conditional volatility in Australian dollar futures returns is, therefore, explained by the conditional volatility in the Australian dollar spot returns and in the 90-day Bank Accepted Bill rate movements. Changes in the US risk free interest rates are not significant, and do not appear to affect the conditional volatility in Australian dollar futures returns.

Another notable result is the apparent lack of significance in the conditional variance of the interest rate differential between foreign and domestic risk-free rates, and the conditional variance of the basis. Thus, it can be seen that interest rate parity is maintained in Australian dollar markets due to the insignificance of the conditional variance of the interest rate differential and the basis. No significant changes emerge from the two long-run relationships for the interest rate differential and the basis. The conditional volatility in futures returns appears to be driven mainly by conditional volatility in the spot market and the associated foreign risk-free interest rate.

These results are important in terms of their contribution to an understanding of the behaviour of currency futures markets. Whether currency futures are transacted for speculative or hedging purposes, the underlying factors that drive the futures returns, and invariably the speculative trader's decision, are the spot market and foreign risk-free interest rate volatilities. For Australian dollar futures contracts, the returns are determined by the conditional volatilities in the underlying Australian dollar spot rate and the Australian 90 day bank accepted bill rate market. It was noted that volatility in the US market does not appear to have any significant effect on the volatility in Australian dollar futures returns.

6. CONCLUSION

Volatility in futures markets is an important consideration for investors who actively operate in currency futures markets, either for speculative or hedging purposes. Although extensive research has been devoted to understanding volatility in stock markets, there are comparatively few studies in futures markets. The growth potential of futures contracts and derivatives, in general, is expected to generate interest in developing models of volatility for these derivatives. Such models will be useful in assisting investors to evaluate the risks involved in investing in these financial instruments.

In line with these developments, this paper has developed models of volatility based on two standard hypotheses of futures prices. From these models, the important factors that explain volatility in currency futures markets were examined. The empirical results obtained suggest that volatility in futures markets is due essentially to volatilities in the underlying spot market and in the foreign risk-free interest rate. These results also indicate that volatility in the domestic risk-free interest rate does not appear to affect the volatility of futures returns. The findings of this paper provide some evidence that the volatility in futures returns are driven by volatilities in the underlying spot returns and in the foreign risk-free interest rate.

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Table 1: Unit root tests of levels of variables

Set	Test	Spot	Futures	US rates	Aust rates
1 (1-936)	Deterministic trend?	No	No	No	No
	Lag length	3	7	1	1
	Covariance Formula	OLS	OLS	OLS	NW
	ADF statistic	-2.63	-2.48	-1.39	-2.07
	Critical value ($\lambda = 0.2$)	-3.80	-3.80	-3.80	-3.80
1 (191-1621)	Deterministic trend?	No	No	No	No
	Lag length	3	3	9	1
	Covariance formula	OLS	OLS	OLS	NW
	ADF statistic	-3.59	-3.71	-1.22	-2.34
	Critical value ($\lambda = 0.6$)	-3.95	-3.95	-3.95	-3.95
2 (1-1621)	Deterministic trend?	No	No	Yes	Yes
	Lag length	3	3	0	4
	ADF statistic	-1.76	-1.78	-0.93	-2.26
	Critical value	-2.86	-2.86	-3.42	-3.42
3A (1-190)	Deterministic trend?	Yes	Yes	Yes	No
	Lag length	0	0	0	1
	ADF statistic	-1.42	-2.02	-2.64	-1.12
	Critical value	-3.44	-3.44	-3.44	-2.88
3B (191-937)	Deterministic trend?	Yes	Yes	Yes	Yes
	Lag length	3	0	1	0
	ADF statistic	-2.31	-2.63	-1.08	-1.19
	Critical value	-3.42	-3.42	-3.42	-3.42
3C (938-1621)	Deterministic trend?	No	Yes	Yes	Yes
	Lag length	0	5	6	5
	ADF statistic	-2.53	-2.73	-0.67	-0.49
	Critical value	-2.87	-3.42	-3.42	-3.42

Note: NW denotes the Newey-West covariance matrix formula. Unless otherwise specified, the OLS covariance formula is used in the calculation of the test statistics.

Table 2: Unit root tests of first difference

Set	Test	Spot	Futures	US rates	Aust rates
2	Lag length	2	2	4	2
	ADF statistic	24.92	-24.89	17.74	21.43
	Critical value	-2.86	-2.86	-2.86	-2.86
3A	Lag length	0	0	0	0
	ADF statistic	12.63	-13.26	14.04	11.09
	Critical value	-2.88	-2.88	-2.88	-2.88
3B	Lag length	2	2	0	0
	ADF statistic	17.28	-17.31	29.09	25.55
	Critical value	-2.87	-2.87	-2.87	-2.87
3C	Lag length	4	2	5	8
	ADF statistic	13.53	-16.71	11.76	-7.68
	Critical value	-2.87	-2.87	-2.87	-2.87

Note: The ADF tests are conducted without a time trend since the t-statistics with and without aTrend are not substantially different.

Table 3: Cointegration Tests for Risk Premium model

Test Statistic	Number of cointegrating vectors				
	Two structural breaks	No structural breaks	Sample set 3A	3B	3C
Maximal Eigenvalue	1	1	0	1	1
Trace	1	1	0	1	1

Table 4: Estimation results for Cost-of-Carry volatility models

Variables	Models 1	2	3B	3C
Constant	0.000 (0.596)	0.000 (0.536)	-0.0005* (0.011)	0.000* (0.000)
var(Δs_t)	0.7446* (0.000)	0.744* (0.000)	0.707* (0.000)	0.790* (0.000)
var(Δr_t^d)	0.000 (0.786)	0.000 (0.800)	0.0001 (0.0015)	-0.0005 (0.355)
var(Δr_t^f)	0.000* (0.009)	0.000* (0.008)	0.0005* (0.0015)	0.000* (0.020)
CAECM4	0.000 (0.556)	-	-	-
CAECM5	0.000 (0.863)	-	-	-
CBECM1	-	0.0001 (0.496)	-	-
CDECM1	-	-	0.0005* (0.0097)	-
CEECM1	-	-	-	0.326 (0.333)

Notes to Tables 4 and 5: Figures in parentheses are p-values; * denotes significance at the 5% level. Standard errors use Newey-West adjustments to the covariance matrix

Table 5: Estimation results for the Risk Premium volatility models

Variables	Sample sets 1	2	3B	3C
Constant	0.0000 (0.582)	0.0000 (0.578)	-0.0003 (0.057)	0.0000 (0.610)
var(Δs_t)	0.7753* (0.000)	0.7753* (0.000)	0.7437* (0.000)	0.7779* (0.000)
CAECM	0.0000 (0.541)	-	-	-
CBECM	-	0.0000 (0.537)	-	-
CDECM	-	-	0.0004 (0.051)	-
CEECM	-	-	-	-0.0000 (0.644)