Intersectoral Interactions: 
Agriculture, Rural Industry & State Industry

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ABSTRACT

We attempt to model important elements of intersectoral interaction in developing economies. In Asia, rural manufacturing industries are an important source of national income and employment growth. A growing rural industrial sector provides inputs and markets for agriculture, which in turn provides inputs and markets for rural industry. As the mutually supportive linkages between rural industry and agriculture develop, the size of both sectors increases. Under certain conditions rural industry could grow more rapidly than agriculture, resulting in the structural transformation of the rural sector. But the growth of rural industry may hurt the state-owned industrial sector if there is competition for resources and product markets. To protect state enterprises, some countries suppress the growth of the non-state rural industries. Unfortunately this can hurt an economy overall. We show how the growth rates of agriculture and rural industry may decline, and, surprisingly, how the growth of state industry might fall if rural industry is suppressed. This is especially so if agriculture supports state industry. By suppressing rural industry, agriculture is hurt. The decline in agriculture then hurts state industry, undermining the objective of protecting state industry. Depending on the magnitude of the relevant impacts, intervention to protect state industry may or may not be optimal, leaving governments with difficult policy decisions.

1 Introduction

We begin our modelling of intersectoral interactions in a developing country with a simple two-sector development process involving agriculture and rural industry. If both sectors support one another, then their growth rates will be higher than if they grow independently. But growth in non-state rural manufacturing industries can adversely affect state manufacturing industries. Competition between rural and state industry is the focus of Section 3, which develops a three-sector model. Given the prospect of state industry succumbing to competitive pressures from rural industry, one response is for governments to protect state industry by restricting rural industry.

But as Section 4 attempts to show, direct suppression of rural industry may indirectly harm state industry. This stems from the positive links between rural industry and agriculture: when rural industry declines, so does agriculture. Since agriculture helps to support state industry by providing inputs and establishing market linkages, state industry is potentially disadvantaged by the restrictions imposed on rural industry.

The severity of the choice facing governments becomes clear. They must either allow rural industry to prosper and compete with state industry, or, in suppressing rural industry to release resources for state industry, suffer the adverse economic consequences of indirectly harming agriculture (and possibly even state industry itself), while at the same time retarding the structural transformation of the rural economy.

2 Linkages between rural industry and agriculture

Rural industries are important in helping agriculture to grow. A mutualistic relationship develops as rural industry grows, generates more employment and raises off-farm incomes. Part of the rising incomes are spent on or remitted to the agricultural sector. Farm incomes rise and farmers are able to increase their expenditures on inputs, such as transport services and farm equipment, and on consumer goods, such as electrical appliances and housing, much of which is provided by rural industry. The increased availability of consumer goods provides incentives for farmers to produce more to buy consumer goods. A virtuous circle emerges in which rural industry and agriculture expand in tandem.

The beneficial intersectoral relations are readily modelled. Consider first a sector without intersectoral interactions. Denote the number of firms in this sector by \( X_1 \), where all firms in a sector are assumed to be identical. Let the sector grow at an intrinsic rate \( r_1 \), which reflects the rate at which the sector would grow without the inhibiting effects of resource competition from other sectors. This rate may depend on factors such as the macro- or microeconomic environment and is assumed to be constant.

As economic activity consumes available resources, a physical limit to the number of firms that may exist in this sector is approached. Refer to this physical limit as the carrying capacity, \( K_1 \) (defined as the number of firms that resources may support indefinitely). The carrying capacity has useful economic interpretations:
property rights and governance, hard or soft budget constraints, operational autonomy and intersectoral competitive pressures, for example, may influence the efficiency with which resources are used and determine the carrying capacity of a given sector.

The logistic growth of the sector is given by:

\[ \dot{X}_i = (r_i - (r_i / K_i) X_i) X_i. \]

But sectors do not grow independently of others. In particular, consider the mutually supportive articulation between two sectors (i.e., agriculture and rural industry):

\[ \dot{X}_i = (r_i - a_{ij} X_i + a_{ji} X_j) X_i. \]

(1)

The notation has been simplified, with the intrasectoral competition coefficient denoted by \( a_{ii} = r_i / K_i \) and the \( a_{ij} \) terms representing the intersectoral interaction coefficients, such that \( a_{ij} > 0, \ i = 1, 2, \ i \neq j \). The interactions are beneficial, so that \( a_{ij} \) shows the positive effect of a production unit from sector \( j \) on a unit from sector \( i \). The equilibria are given by:

\[ X_i^* = \frac{K_i + a_{ij} K_j}{1 - a_{ij} a_{ji}}, \]

(2)

where \( a_{ij} = a_{ij} / a_{ii} \). It is easy to show that the \( X_i^* \) are strictly positive if \( r_i a_{ji} > r_j a_{ij} \). Assuming both sectors can persist in the absence of interaction, the phase diagram for the stable mutualistic system is presented in Figure 1.

**Figure 1 Phase diagram**

The equilibrium point is \( E = (X_1^*, X_2^*) \). The slope of the \( X_1 = 0 \) isocline is \( a_{12} / a_{21} \) and that of \( X_2 = 0 \) is \( a_{21} / a_{12} \). The larger the beneficial interaction and the weaker the intrasectoral competition, the larger is the equilibrium size of the benefiting sector. Note that \( X_1^* > r_i / a_{ii} = K_i \); i.e., both sectors are larger than they would be in the absence of mutually beneficial interactions. The stability of the equilibrium is discussed in Appendix 1.

**PROPOSITION 1** An increase in intrasectoral competition in agriculture, an increase in the beneficial effect of agriculture on rural industry, a fall in intrasectoral competition in rural industry, or a fall in the beneficial effect of rural industry on agriculture is sufficient for structural transformation on the basis of agricultural-rural industrial articulation.

**Proof.** Structural transformation implies that the ratio of agricultural to rural industrial output falls over time. Define \( \bar{X} \) as the ratio of the equilibrium values of agriculture to rural industry:

\[ \bar{X} = \frac{X_1^*}{X_2^*} = \frac{K_1 + a_{12} K_2}{K_2 + a_{21} K_1}. \]

Recalling that the intrasectoral and intersectoral coefficients are strictly positive, as are the carrying capacities, \( \partial \bar{X} / \partial a_{11} < 0, \partial \bar{X} / \partial a_{12} > 0, \partial \bar{X} / \partial a_{22} > 0 \) and \( \partial \bar{X} / \partial a_{21} < 0 \). A fall in \( \bar{X} \) over time is consistent with increases in \( a_{11} \) and \( a_{21} \), and decreases in \( a_{12} \) and \( a_{22} \) over time.

**Remark.** The intrasectoral competition coefficient in agriculture, \( a_{11} \), is likely to rise as fewer resources become available to agriculture with the resource flow to rural industry. As rural industry purchases fewer agricultural products or as it changes the composition of its output to supply fewer inputs to agriculture, the positive feedback coefficient from rural industry to agriculture, \( a_{12} \), is likely to fall. An increase in the carrying capacity of rural industry, \( K_2 \), decreases \( a_{22} \), the intrasectoral competition coefficient. This could result from the flow of resources from agriculture to industry with the easing of restrictions on rural enterprises. As agriculture expands in output value, the beneficial effect of agriculture on rural industry, \( a_{21} \), may rise as increasingly wealthier farmers purchase inputs and consumer goods from rural industry. (Note that an increase in \( a_{21} \) is consistent with a rising \( a_{11} \) since agricultural output value could increase in absolute terms even with a labour outflow if agricultural productivity rises enough.) The effect of these changes is to decrease the per unit growth rate of the agricultural sector and raise that of rural industry.

### 3 Competition with state industry

State industry is now explicitly added to the sectoral analysis in a three-sector model. Assume that rural industry and agriculture continue to exhibit positive feedback, as do agriculture and state industry. This section focuses on the interactions between rural and state industry, where it is assumed that the two industrial sectors compete with one another in net terms (i.e., negative interactions). The concept of intersectoral competition is now widened to include competition in output markets as well as for resources.
The interrelationships between the three sectors can be modelled in terms of the familiar linear growth equations:

\[
\begin{align*}
\dot{X}_1 &= (r_1 - a_{11}X_1 + a_{12}X_2 + a_{13}X_3)X_1 \\
\dot{X}_2 &= (r_2 - a_{21}X_1 - a_{22}X_2 - a_{23}X_3)X_2 \\
\dot{X}_3 &= (r_3 + a_{31}X_1 - a_{32}X_2 - a_{33}X_3)X_3.
\end{align*}
\] (4)

The subscript 1 denotes agriculture, 2 denotes rural industry, and 3 represents state industry. As reflected in the signs of the coefficients, \(a_{ij} (i,j=1,2,3)\), the system (4) models beneficial interactions between agriculture and rural industry and between agriculture and state industry, while rural and state industry compete with one another.

The system (4) may now be used to derive the following proposition:

**PROPOSITION 2** Given the system (4), a necessary condition for state industry to decline is that:

\[
\frac{a_{11}}{a_{12}} > 0 \quad \text{and} \quad \frac{a_{31}}{a_{32}} > 0.
\]

**Proof.** See Appendix 2.

**Remark.** The first condition in Proposition 2 is more likely to be satisfied the higher the intrinsic growth rate of rural industry \((r_2)\) relative to that of state industry \((r_3)\), the lower the intrasectoral competition in rural industry \((a_{22})\) and the higher the intersectoral competition from rural industry on state industry \((a_{12})\). The coefficient \(a_{32}\) is likely to be high given the intense rivalry between the two sectors, while \(a_{22}\) is probably relatively low due to the large flow of resources to rural industry, which increases rural industry’s carrying capacity.

**Remark.** The second condition is more likely to be satisfied the higher the beneficial impact of agriculture on rural industry \((a_{12})\) relative to agriculture’s impact on state industry \((a_{31})\). Coefficient \(a_{12}\) is likely to be relatively high as farmers purchase the producer and consumer goods offered by rural enterprises.

Proposition 2 presents governments with a dilemma. A growing and prospering rural industrial sector puts competitive pressure on state industry, which faces the prospect of a decline. The other side of the dilemma is that rural industry can be a very dynamic and important sector in a developing country’s modernisation drive in terms of output, employment and tax revenues. Either the objective of protecting an inefficient state sector must be downgraded or rural industry must face the potential for further suppression by the state.

### 4 Suppression of rural industry

In seeking to avoid widespread unemployment and urban unrest attending a decline in state industry, the Chinese government, for example, has demonstrated sufficient political resolve to retrench rural industry; in fact this was one of the goals of China’s 1989 austerity program (Puttermann 1992:480), although state industry also suffered as a result. Two million rural enterprises were closed or taken over by other firms (Zweig 1992:422).

This section seeks to show that such a strategy has the potential to harm state industry: given the positive feedback between rural industry and agriculture, the restrictions placed on rural enterprises hurt agriculture, which in turn indirectly hurts state industry, given the mutualism between state industry and agriculture.

In the following model, the size of the state industrial sector is fixed and the focus is solely on the mutualistic relationship between agriculture and rural industry. The model of agricultural and rural industrial growth is given in the generalised form:

\[
\begin{align*}
\dot{X}_1 &= h_1(X_1, X_2)X_1 \\
\dot{X}_2 &= h_2(X_1, X_2, \beta)X_2.
\end{align*}
\] (5)

\(\beta\) is a measure of government suppression of rural industry, with higher values of \(\beta\) reflecting increasing suppression.

The effect of a change in \(\beta\) on the equilibrium values of \(x_1\) and \(x_2\) is given in the following proposition:

**PROPOSITION 3** Let the model of agriculture and rural industry be given by (5), in which the derivatives satisfy:

\[
\begin{align*}
\frac{\partial h_1}{\partial X_1} < 0 & \quad \frac{\partial h_2}{\partial X_2} < 0 \quad \frac{\partial h_1}{\partial X_1} > 0 \quad \frac{\partial h_2}{\partial X_2} > 0 \quad \frac{\partial h_1}{\partial \beta} < 0 \quad \frac{\partial h_2}{\partial \beta} < 0
\end{align*}
\]

for all \(X_1, X_2, \beta\). For \(\beta \geq 0\), let \((X_{1^*}, X_{2^*})\) represent the intersection of the curves:

\[
h_1(X_1, X_2) = 0, \quad h_2(X_1, X_2, \beta) = 0.
\]

If:

\[
\frac{\partial h_1}{\partial X_1} \frac{\partial h_2}{\partial X_2} - \frac{\partial h_1}{\partial X_2} \frac{\partial h_2}{\partial X_1} > 0
\]

for all \(\beta\), then \((X_{1^*}, X_{2^*})\) is a stable equilibrium of the dynamic system (5). Further, both \(X_{1^*}\) and \(X_{2^*}\) are decreasing functions of \(\beta\).

**Proof.** See Appendix 3.

Thus, assuming a mutualistic relationship between agriculture and rural industry, and intrasectoral competition in both, it follows that suppression of rural...
industry by the government is damaging. The effect of the suppression is to lower both rural industrial and agricultural output.

**Remark.** All other things being equal, the decline in agriculture in turn reduces the size of the state industrial sector, due to the positive linkage effects between the two sectors. On the other hand, the suppression of rural industry releases resources to state industry, increases state industry's carrying capacity and, all other things being equal, raises the growth rate of state industry. Thus, whether the policy of protecting state industry by suppressing rural enterprises harms state industry overall depends on the relative sizes of the two opposing effects.

A corollary of Proposition 3 is given in the following proposition, such that decreases in rural industrial and agricultural output may inhibit the structural transformation of the rural economy.

**PROPOSITION 4** Consider the system (5) and the assumptions concerning the signs of the derivatives. Let \( X_1^*, X_2^* \) represent the stable equilibrium when \( \beta = 0 \). Assuming that \( X_1^* > X_2^* \) and that \( \dot{h}_1 \mid \dot{h}_2 < 0, \dot{h}_2 \mid \dot{X}_2 < 0 \), then for \( \beta > 0 \):

\[
\frac{X_1}{X_2} > \frac{X_1^*}{X_2^*}.
\]

That is, government suppression of rural industry leads to a rise in the ratio of agriculture to rural industry.

**Proof.** See Appendix 4.

Note the assumptions needed in this proof. The proof assumes that the initial size of agriculture is higher than that of rural industry. This is the case for China, as it is for many other developing economies, especially if we consider the total number of workers in each sector. The assumption that:

\[
\frac{\dot{h}_1}{\dot{h}_2} < 1
\]

demands that the positive impact of rural industry on agriculture be lower than the effects of intrasectoral competition in agriculture.

Proposition 4 has important implications for the growth of the rural sector as a whole. Assume that structural transformation arising from an initial intersectoral disequilibrium increases economic growth. The increase results from the reallocation of inputs from less productive to more productive sectors of the economy (see Putterman 1992:467 for a related view). That is, given an initial intersectoral disequilibrium where the marginal product of a resource is lower in one sector than another, the reallocation of resources to the intersectoral equilibrium maximises aggregate output.

If Proposition 4 and the assumption that structural transformation contributes to aggregate economic growth hold, it follows that the suppression of rural industry must decrease the growth rate of the rural sector overall. (In our three-sector model, we define the rural sector to be the sum of the rural industrial and agricultural sectors.) The direct impact is that rural industrial output falls and drags down agricultural output, given the complementary linkages between the two sectors. This effect is reinforced by the reversal of the path of structural transformation, as the number of agricultural to rural industrial firms rises.

**5 Policy implications**

The suppression of rural industry to protect state industry has two adverse consequences: state industry faces the possibility of being harmed indirectly through the rural industry-agriculture-state industry linkages, and the structural transformation of the rural economy is impeded. The harm done to rural industry may be reduced by the selective targeting of individual rural enterprises to be discriminated against, as opposed to a general sector-wide retrenchment. For example, rural enterprises with weak or non-existent linkages with agriculture could be targeted for close-down.

Despite such policies, other aspects of the intersectoral competition problem remain difficult to resolve. There are resources which are used by almost all rural enterprises, such as energy and transport energy and transport. Shifts in product line are not likely to have a large impact on total use of such resources by a sector. The shifting of product lines entails producer and consumer welfare costs, as rural producers move against comparative advantage and market demand. Most importantly, the suppressing of rural industry diverts attention from the area most in need of overhaul - the inefficient state sector. A long term solution to the problem involves state sector industrial reform and greater privatisation.

The impetus for these changes comes from competition with rural enterprises. Singh and Jefferson suggest that the growth of the non-state sector (i.e., town and village enterprises) has led state industry to increase its productivity: "For every 10 percent increase in the non-state sector's share of industrial output, productivity in state industry - depending on the initial level of productivity - has risen by an average of 2.5 to 4.0 percent" (Singh and Jefferson 1994:7). These intersectoral competitive pressures raise the carrying capacity of state industry and therefore its growth rate. Removing the obligation of state enterprises to provide social services for its workers, greater input and output market flexibility, and the imposition of financial responsibility and accountability, would contribute significantly to easing the current problems of state industry.

To the extent that intersectoral competition is encountered in output markets, this competition might be reduced by redirecting sales to external markets. In
this regard, the three-sector model of (4) implicitly incorporates the international trade sector as a vent for reduced intersectoral competition, since the competition coefficients (for outputs, at least) tend to decline with the introduction of new, overseas outlets for the outputs of state and rural industries. Rural industries, for example, have played an active role in this area in becoming an important vehicle for China's export-led growth.

In the longer term, an increase in export markets for rural industrial output can contribute indirectly to the growth of state industry. Rural industry provides foreign exchange, with which more resources and technology may be imported to aid state industry. The short-run "zero-sum" scenario of rural-state industrial competition may partially give way to positive impacts provided by rural industry. As Rozelle (1994:385) suggests, policies that slow down rural industry impair the growth of the Chinese economy as a whole.

The growth of rural industry may even facilitate the transition to a privatised urban industrial sector. State industrial sector reform becomes more tenable politically when redundant state workers are able to find alternative employment. Given that the required educational levels of the workforce in rural enterprises are substantially above those in agriculture and are only slightly below the average levels in the state industrial sector, an expansion of rural industry may be a source of labour absorption as state industry is reformed and urban workers are displaced. Competitive pressure from state industry obliges rural enterprises to increase capital accumulation and technology, so that rural enterprises are likely to gain from the urban-rural migration of technically-trained urban workers.

Appendix 1 (Stability analysis)

The equilibrium \( E \) is stable if \( a_{11}a_{22} > a_{12}a_{21} \). To determine the stability of \( E \), let \( X_1 = X_1^* + x_1 \) and \( X_2 = X_2^* + x_2 \), where \( x_1 \) and \( x_2 \) are small. Linearise the system (1) in the neighbourhood of the fixed point, taking the first two terms of a Taylor series:

\[ \begin{align*}
    x_1 & = a_{11}x_1 + bx_2 \\
    x_2 & = cx_1 + dx_2.
\end{align*} \]

The coefficients are the partial derivatives evaluated at the fixed point. For example:

\[ \frac{\partial X_1}{\partial X_1} = -2a_{11}X_1 + a_{12}X_2. \]

When evaluated at the fixed point, and recalling that \( \eta = a_{11}X_1 - a_{12}X_2 \) in equilibrium:

\[ \begin{pmatrix} \frac{\partial X_1}{\partial X_1} \\ \frac{\partial X_2}{\partial X_2} \end{pmatrix} = -a_{11}X_1^*. \]

Thus:

\[ \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} = \begin{pmatrix} -a_{11}X_1^* & a_{12}X_1^* \\ a_{21}X_2^* & -a_{22}X_2^* \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \]

It is necessary to find eigenvalues \( \lambda \) satisfying:

\[ \begin{vmatrix} -a_{11}X_1^* - \lambda & a_{12}X_1^* \\ a_{21}X_2^* & -a_{22}X_2^* - \lambda \end{vmatrix} = 0. \]

\[ 2\lambda = -(a_{11}X_1^* + a_{22}X_2^*) + \sqrt{(-a_{11}X_1^* - a_{12}X_1^*)^2 - 4(a_{11}a_{22}X_1^* X_2^* - a_{12}a_{21}X_1^* X_2^*)}. \]

The stability requirement hence becomes:

\[ a_{11}a_{22} > a_{12}a_{21}. \]

Remark. The positive intersectoral interactions, \( a_{12} \) and \( a_{21} \), are not stabilising. Stability derives from the self-regulatory effects, \( a_{11} \) and \( a_{22} \).

Appendix 2

The proof follows Hallam (1980). The proof by contradiction involves setting \( a_{22}a_{31} - a_{21}a_{32} = 0 \) and \( a_{21}r_3 - r_2a_{32} = 0 \). Consider the limiting \( (X_1, X_2, X_3) \) system, where \( x_3 \) is sufficiently close to zero. The \( x_2 \) and \( x_3 \) isopanes are derived by setting the respective per unit growth rates equal to zero, and their slopes are \( a_{22}/a_{21} \) and \( a_{32}/a_{31} \). Since the \( x_3 \) isopane is at least as steep as the \( X_3 \) isopane, then in \( R^3 \) above the \( x_3 \) isopane and close to the \( x_1x_2 \)-plane, \( x_3 \) must increase to keep the per unit growth rate of \( x_3 \) equal to zero. This contradicts the assumed decline in \( X_3 \).

To show that \( r_3a_{23} - a_{32}r_2 > 0 \), use the persistence function:

\[ V(t) = V(X_2(t), X_3(t)) = [X_2(t)]^{a_{23}} [X_3(t)]^{a_{22}}. \]

Along paths of (4),

\[ V = [(a_{22}r_3 - r_2a_{23}) + X_1(a_{22}a_{31} - a_{32}a_{21}) + X_3(a_{23}a_{32} - a_{22}a_{33})]. \]

Recall that it is assumed that \( X_3 \) is close to zero, \( a_{22}a_{31} - a_{21}a_{32} \geq 0 \) and \( a_{32}a_{23} - a_{23}a_{32} \geq 0 \). For sufficiently small \( X_3 \), it is true that:

\[ X_3(a_{23}a_{32} - a_{22}a_{33}) \geq \frac{(a_{22}r_3 - r_2a_{23})}{2}. \]
Therefore:
\[ V \geq \frac{(a_{22}b - a_{12}a_{22})V}{2} + X_1(a_{31}a_{22} - a_{21}a_{32})V \geq \frac{(a_{22}b - a_{12}a_{22})V}{2}. \]

Since \( V \) is weakly positive, this contradicts \( \lim_{t \to \infty} V(t) = 0 \).

Appendix 3

As noted in Appendix F, the intersection point of the curves \( h_0 = 0 \), \( h_0 = 0 \) is a stable equilibrium point of the dynamic system if:

\[ \frac{\partial h_1}{\partial X_1} \frac{\partial h_1}{\partial X_2} - \frac{\partial h_1}{\partial X_1} \frac{\partial h_2}{\partial X_2} > 0. \]

As \( \beta \) varies, these stable equilibrium points \( (X_1^\beta, X_2^\beta) \) vary and are functions of \( \beta \). Differentiate the system \( h_0 = 0 \), \( h_0 = 0 \) with respect to \( \beta \):

\[
\frac{\partial^2 h_1}{\partial X_1 \partial \beta} \frac{\partial h_1}{\partial X_2} + \frac{\partial^2 h_1}{\partial X_1 \partial \beta} \frac{\partial h_2}{\partial X_2} = 0
\]

\[
\frac{\partial^2 h_1}{\partial X_1 \partial \beta} \frac{\partial h_1}{\partial X_2} + \frac{\partial h_1}{\partial X_1} \frac{\partial^2 h_1}{\partial X_2 \partial \beta} + \frac{\partial h_2}{\partial X_2} \frac{\partial^2 h_1}{\partial X_2 \partial \beta} = 0
\]

This is a linear system which may be solved for \( \frac{\partial X_1}{\partial \beta} \), \( \frac{\partial X_2}{\partial \beta} \):

\[
\frac{\partial X_1}{\partial \beta} = \frac{\frac{\partial h_1}{\partial X_1} \frac{\partial h_2}{\partial X_2} - \frac{\partial h_1}{\partial X_1} \frac{\partial h_1}{\partial X_2} \frac{\partial h_2}{\partial X_2}}{\frac{\partial h_1}{\partial X_1} \frac{\partial h_2}{\partial X_2} - \frac{\partial h_1}{\partial X_1} \frac{\partial h_1}{\partial X_2} \frac{\partial h_2}{\partial X_2}}
\]

\[
\frac{\partial X_2}{\partial \beta} = \frac{\frac{\partial h_1}{\partial X_1} \frac{\partial h_2}{\partial X_2} - \frac{\partial h_1}{\partial X_1} \frac{\partial h_1}{\partial X_2} \frac{\partial h_2}{\partial X_2}}{\frac{\partial h_1}{\partial X_1} \frac{\partial h_2}{\partial X_2} - \frac{\partial h_1}{\partial X_1} \frac{\partial h_1}{\partial X_2} \frac{\partial h_2}{\partial X_2}}
\]

Given the assumptions on the signs of the derivatives, including that:

\[
\frac{\partial h_1}{\partial X_1} \frac{\partial h_2}{\partial X_2} - \frac{\partial h_1}{\partial X_1} \frac{\partial h_1}{\partial X_2} \frac{\partial h_2}{\partial X_2} > 0.
\]

it follows that \( \frac{\partial X_1}{\partial \beta} < 0 \), \( \frac{\partial X_2}{\partial \beta} < 0 \).

Appendix 4

All equilibria \( (X_1^\beta, X_2^\beta) \) lie on the curve \( h_0(X_1, X_2) = 0 \). This equation implicitly defines \( X_1 \) as a function of \( X_2 \). The derivative \( \frac{\partial X_1}{\partial \beta} \) on this curve may be computed as follows. Differentiating with respect to \( X_2 \) and rearranging:

\[ \frac{\partial X_1}{\partial \beta} = -\frac{\partial^2 h_1}{\partial X_2 \partial \beta} \frac{\partial X_2}{\partial X_1}. \]

Let \( (X_1^\beta, X_2^\beta) \) be the equilibrium corresponding to some \( \beta > 0 \). Then, applying the mean-value theorem:

\[
\frac{X_1^\beta}{X_2^\beta} = \frac{X_1^\beta + (X_2^\beta - X_1^\beta) \frac{\partial X_1}{\partial X_2}}{X_2^\beta} \frac{\partial X_1}{\partial X_2} \frac{\partial X_2}{\partial X_1}, \quad \frac{X_1^\beta}{X_2^\beta} \leq c \leq X_2^\beta
\]

\[
\frac{X_1^\beta}{X_2^\beta} = \frac{X_1^\beta - (X_1^\beta - X_2^\beta) \frac{\partial X_1}{\partial X_2}}{X_2^\beta} \frac{\partial X_1}{\partial X_2} \frac{\partial X_2}{\partial X_1}.
\]

By Proposition 3, \( X_1^\beta - X_2^\beta \geq 0 \). By assumption:

\[
\frac{\partial X_1}{\partial X_2} = -\frac{\partial h_1}{\partial X_1} \frac{\partial h_2}{\partial X_2} > 1
\]

and thus \( (X_1^\beta - X_2^\beta) \frac{\partial X_1}{\partial X_2} \frac{\partial X_2}{\partial X_1} \leq X_1^\beta - X_2^\beta \). That is, a larger value is being subtracted from the denominator than from the numerator. With the assumption that \( X_1^\beta > X_2^\beta \), this is sufficient to give \( \frac{X_1^\beta}{X_2^\beta} \geq \frac{X_1^\beta}{X_2^\beta} \).

REFERENCES


