Stochastic Volatility Structures and Intra-day Asset Price Dynamics

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Abstract The behaviour of financial asset price data when observed intra-day is quite different from these same processes observed from day to day and longer sampling intervals. Volatility estimates obtained from intra-day observed data are badly distorted if anomalies and intra-day trading patterns are not accounted for in the estimation process. In this paper I consider conditional volatility estimators as special cases of a general stochastic volatility structure. The theoretical asymptotic distribution of the measurement error process for these estimators is considered for particular features observed in intra-day financial asset price processes. Specifically, I consider the effects of (i) induced serial correlation in returns processes, (ii) excess kurtosis in the underlying unconditional distribution of returns, (iii) market anomalies such as market opening and closing effects and (iv) failure to account for intra-day trading patterns. These issues are considered with applications in option pricing/trading strategies and the constant/dynamic hedging frameworks in mind.

1. INTRODUCTION

One issue considered in Nelson (1990) is whether it is possible to formulate an ARCH data generation process that is similar to the true process. The distribution of the sample paths generated by the ARCH structure and the underlying diffusion process are assumed to be "close" for increasingly finer discretizations of the observation interval. Maximum likelihood estimates are difficult to obtain from stochastic differential equations of time-varying volatility common in the finance literature. If the results in Nelson hold for `real time' data when ARCH structures approximate a diffusion process then these ARCH structures may be usefully employed in option pricing equations. In this paper I consider the ARCH structure as a special case of a general stochastic volatility structure. One advantage of an ARCH structure over a general stochastic volatility structure lies in computational simplicity. In the ARCH structure it is not necessary for the underlying processes to be stationary or ergodic. The crucial assumption in an option pricing context is that these assumed processes approach a diffusion limit. These assumed diffusion limits have been derived for processes assumed to be observed from day to day records.

The specific concern in this paper is the effect on the asymptotic distribution of the measurement error process and on parameter estimates, obtained from the Generalised ARCH (GARCH(1,1)) equations for the conditional variance, as the observation interval approaches transactions records ($d \rightarrow 0$). Three issues are considered for cases where the diffusion limit may not be achieved at these observation intervals. The first issue is the effect of mis-specifying the dynamics of the first moment generating equation on resultant GARCH(1,1) parameter estimates. The second issue is the effect on measures of persistence obtained from the GARCH structure when increasing kurtosis is induced in the underlying unconditional distribution as $d \rightarrow 0$. This leads to a third issue which is concerned with evaluating effects of inclusion of weighting (mixing) variables on parameter estimates obtained from these GARCH(1,1) equations. If these mixing variables are important then standard GARCH equation estimates will be seriously distorted. These mixing variables may proxy the level of activity within particular markets or account for common volatility of assets trading in the same market.

2.STOCHASTIC VOLATILITY AND GARCH

Nelson and Foster (1994) derive and discuss properties for the ARCH process as the observation interval reduces to daily records ($h \rightarrow 0$) when the underlying process is driven by an assumed continuous diffusion process. Nelson and Foster (1991) generalised a Markov process with two state variables, ${}_hX_t$ and ${}_h\sigma_t^2$, only one of which ${}_hX_t$ is ever directly observable. The conditional variance ${}_h\sigma_t^2$ is defined conditional on the increments in ${}_hX_t$ per unit time and conditional on an information set ${}_h\varsigma_t$. Modifying the notation from h to d (to account for intra-day discretely observed data) and employing the notation d→0 to indicate reduction in the observation interval from above, when their assumptions 2, 3 and 1' hold, when d is small, ${}_dX_t, \varphi({}_d\sigma_t^2))$ is referred to as a near diffusion if for any T,

$$0 \le T \le \infty, ({}_dX_t, \varphi({}_d\sigma_t^2))_{0\le t\le T} \Rightarrow (X_t, \varphi(\sigma_t^2))_{0\le t\le T}.$$

If we assume these data generating processes are near diffusions then the general discrete time stochastic volatility structure, defined in Nelson and Foster (1991), may be described using the following modified notation:

$$\begin{bmatrix} {}_dX_{(k+1)d} \\ \varphi({}_d\sigma^2{}_{(k+1)d}) \end{bmatrix} = \begin{bmatrix} {}_dX_{kd} \\ \varphi({}_d\sigma_{kd}^2) \end{bmatrix} + d\begin{bmatrix} \mu({}_dX_{kd}, \sigma_{kd}) \\ \lambda({}_dX_{kd}, {}_d\sigma_{kd}) \end{bmatrix} +$$

$$d^{1/2}\begin{bmatrix} {}_d\sigma_{kd}^2 & \Lambda_{\varphi,x}({}_dX_{kd}, {}_d\sigma_{kd}) \\ \Lambda_{\varphi,x}({}_dX_{kd}, {}_d\sigma_{kd}) & \Lambda^2({}_dX_{kd}, {}_d\sigma_{kd}) \end{bmatrix}^{1/2} \begin{bmatrix} {}_dZ_{1,kd} \\ {}_dZ_{2,kd} \end{bmatrix} \quad (1)$$

$({}_dZ_{1,kd}, {}_dZ_{2,kd})_{k=0,\infty}$ is i.i.d. mean zero and identity covariance matrix. In equation (1) d is the size of the observation interval, X may describe the asset price return and σ^2 the volatility of the process. It is not necessary to assume the data generating processes are stationary or egodic but the crucial assumption is that the data generating processes are near diffusions.

In the ARCH specification ${}_dZ_{2,kd}$ is a function of ${}_dZ_{1,kd}$ so that ${}_d\sigma_{kd}^2({}_dh_{kd})$ can be inferred from past values of the one observable process ${}_dX_{kd}$. This is not true for a general stochastic volatility structure where there are two driving noise terms.

For the first order Markov ARCH structure a strictly increasing function of estimates ${}_d\hat{\sigma}_t^2({}_d\hat{h}_t)$ of the conditional variance process ${}_d\sigma_t^2({}_dh_t)$ is defined as $\phi(\sigma^2)$ and estimates of the conditional mean per unit of time of the increments in X and $\phi(\sigma^2)$ are defined as $\hat{\mu}(x,\hat{\sigma})$ and $\hat{\kappa}(x,\hat{\sigma})$. Estimates of ${}_d\sigma_{kd}^2$ are updated by the recursion

$$\phi({}_d\hat{\sigma}^2_{(k+1)d}) = \phi({}_d\hat{\sigma}^2_{kd}) + d\,\hat{\kappa}({}_dX_{kd}, {}_d\hat{\sigma}_{kd}) + d^{1/2}a({}_dX_{kd}, {}_d\hat{\sigma}_{kd})g({}_d\hat{Z}_{1,kd}, {}_dX_{kd}, {}_d\hat{\sigma}^2_{kd}) \quad (2)$$

$\hat{\kappa}(.), a(.), \hat{\mu}(.)$ and $g(.)$ are continuous on bounded $(\varphi(\sigma^2), x)$ sets and $g(z_1, x, \sigma^2)$ assumed continuous everywhere with the first three derivatives of g with respect to z_1 well defined and bounded. The function $g({}_dZ_{1,kd}, .)$ is normalised to have mean zero and unit conditional variance. Non-zero drifts in $\phi({}_d\sigma_{kd}^2)$ are allowed for in the $\hat{\kappa}(.)$ term and non-unit conditional variances accounted for in the a(.) term. The second term on the right measures the change in $\phi({}_d\sigma_{kd}^2)$ forecast by the ARCH structure while the last term measures the surprise change.

The GARCH(1,1) structure is obtained by setting

$$\phi(\sigma^2) = \sigma^2) = \sigma^2, \hat{\kappa}(x,\sigma) = (\omega - \theta\sigma^2),$$
$$g(z_1) = (z_1^2 - 1)/SD(z_1^2),$$
and $a(x,\sigma) = \alpha.\sigma^2.SD(z_1^2)$.

The parameters ω, θ and α index a family of GARCH(1,1) structures. In a similar manner the EGARCH volatility structure can be defined. That is by selecting $\phi(.), \hat{\kappa}(.), g(.)$ and $a(.)$ various 'filters' can be defined within this framework.

Other conditions in Nelson and Foster (1991) define the rate at which the normalised measurement error process ${}_dQ_t$ mean reverts relative to ${}_dX_t, {}_d\sigma_t^2)$. It becomes white noise as d→0 on a standard time scale but operates on a faster time scale, mean-reverting more rapidly. Asymptotically optimal choice of a (.), and $\phi(.)$ given g(.) can be considered with respect to minimising the asymptotic variance of the measurement error. This is considered on a faster time scale $(T+\tau)$ than T. The asymptotically optimal choice of g (.) depends upon the assumed relationship between Z_1 and Z_2. In the ARCH structure Z_2 is a function of Z_1 so that the level driving ${}_d\sigma_t^2$ can be recovered from shocks driving ${}_dX_t^2$. In the absence of further structure in the equation specified for σ_t^2 we are unable to recover information about changes in σ_t^2. Their discussion is strictly in terms of constructing a sequence of optimal ARCH filters to minimise the asymptotic variance of the asymptotic distribution of the measurement errors. This approach is not the same as choosing an ARCH structure that minimises the measurement error variance for each d.

The asymptotic distribution of the measurement error process, for large τ and small d,

$$\left[{}_d\hat{\sigma}^2_{T+\tau d^{1/2}} - {}_d\sigma^2_{T+\tau d^{1/2}} \,\middle|\, ({}_dQ_T, {}_d\sigma^2_T, {}_dX_T)) \right] = (q, \sigma^2, x)$$

derivatives evaluated as $\phi'({}_d\sigma^2_T)$ and $\varphi'({}_d\sigma^2_T)$ etc and the notation simplified as ϕ' and φ' is approximately normal with mean

$$d^{1/2}\frac{(2\sigma^2\phi')[\hat{\kappa}_d/\phi' - a^2\phi''/2(\phi')^3 - \lambda_d/\varphi' + \Lambda^2\phi''/2(\phi')^3]}{\{a.E[Z_1.g_z]\}} +$$

$$+$$

$$\frac{2a\sigma.[\mu_d - \hat{\mu}_d].E[g_z]}{\{a.E[Z_1.g_z]\}}$$

and variance

$$d^{1/2}\frac{(2\sigma^2\phi').(a^2/\phi'] + \lambda^2/[\varphi']^2 - 2a\Lambda.Cov(Z_2,g)/[\sigma'][\varphi']}{a.E[Z_1.g_z]}$$

General results in Nelson and Foster (1991 and 1994) for the GARCH(1,1) structure are that GARCH(1,1) can be more accurately measured firstly the less variable and the smaller is ${}_d\sigma^2_t$, second the thinner the tails of Z_1 and third the more the true data generating mechanism resembles an ARCH structure as opposed to a stochastic volatility structure. If the true data generating process is GARCH(1,1) then $\text{Corr}(Z_1^2, Z_2) = 1$.

As d→0 the first result will generally hold and the second can be checked from the unconditional distribution of the returns process. The latter result is the most difficult to evaluate. Now reconsider some assumptions necessary to obtain these results and reasons these assumptions may not hold when d→0.

(i) Mis-specification of the difference between the estimated and true drift in mean, $[{}_d\hat{\mu}_t - {}_d\mu_t]$, is assumed fixed as d→0 so that effects of mis-specifying this drift has an effect that vanishes at rate $d^{1/2}$ and is negligible asymptotically. These terms do not appear in the expression for the variance of the asymptotic distribution of the measurement error. As d→0, the effect of bid/ask bounce and order splitting in futures price processes and non-trading induced effects on market indices becomes more severe. Mis-specification of the drift in the mean is not constant. Whether this effect transfers to estimates of conditional variances is an empirical issue.

(ii) The conditional variance of the increments in ${}_d\sigma^2_t$ involves the fourth moment of ${}_dZ_{1,kd}$ so that the influence of this fourth moment remains as the diffusion limit is approached. Excess kurtosis is a feature of intra-day financial price changes.

(iii) Values of $\hat{\kappa}_d$ and λ_d are considered fixed as d→0 so that effects of mis-specifying the drift in σ^2 has an effect that also vanishes at rate $d^{1/2}$. As well, although these drift terms enter the expression for the asymptotic bias of the measurement error these also do not appear in the expression for the asymptotic variance. The term g_z represents part of the 'surprise' change in the recursion defined in equation (2) and is directly linked to departures from normality observed in point (ii). These departures from normality can be generated by extremes in Z_1 induced by large jumps in the underlying distribution. In this case first and second derivatives of ϕ may be discontinuous throughout the sample space as well. Then the expression for the bias in this asymptotic distribution of the measurement errors may be explosive.

(iv) The ARCH specification of the drift in mean and variance only enters the $0_p(d^{1/2})$ terms of the measurement error. Asymptotically, the differences in the conditional variance specifications are more important, appearing in the $0_p(d^{1/4})$ terms. If the conditional variance specification is not correct then the measurement error variance is affected. This is because matching the ARCH and true variance of the variance cannot proceed.

3. SERIAL CORRELATION IN RETURNS

The simplest GARCH structure derived from equation (1) for the conditional variance is the GARCH(1,1):

$$h_t = \omega + \alpha_1\varepsilon^2_{t-1} + \beta_1 h_{t-1} \qquad (3)$$

$h_t(\sigma^2_t)$ the conditional variance at time t, ε^2_{t-1} are squared unconditional shocks generated from any assumed first moment equation and $0 \le \alpha_1, \beta_1 \le 1$ and $\alpha_1 + \beta_1 \le 1$. This parameterisation is a parsimonious representation of an ARCH(p') process where a geometrically declining weighting pattern on lags of ε^2 is imposed. This is easily seen by successive substitution for $h_{t-j}(j=1,\dots,J)$ as $J\to\infty$,

Consider an AR(1) equation in the log of the levels, with the autoregressive parameter, ϕ_1 fixed at 1, as representing one mis-specified spot asset price process. An alternative specification in the presence of non-synchronous induced serial correlation may be an AR(1) equation for the logarithmic return, with the autoregressive parameter bounded $-1<\rho_1<1$. A further alternative in the presence of bid/ask bounce induced effects may be a MA(1) augmented return equation, with the MA parameter bounded. $-1<\theta_1<0$.

Define s_t as the return on the asset. Taking expected values then the unconditional variance when an AR(1) levels equation is the mis-specified representation and ϕ_1 set equal to one is

$$E(s_t^2)=E(a_t^2). \tag{4}$$

When the correct equation is the moving average MA(1) return representation the unconditional variance relative to shocks generated via equation (4) is,

$$E(s_t^2)_{MA}=E[(1+\theta^2)a_t^2] \tag{5}$$

When the correct equation is the autoregressive AR(1) return representation the unconditional variance relative to shocks generated via equation (4) is,

$$E(s_t^2)_{AR}=E\left[\left(\frac{1-\rho_1}{1+\rho_1}\right)(a_t^2)\right]. \tag{6}$$

The conditional variance from a GARCH(1,1) structure for the AR(1) levels equation can be rewritten as,

$$h_t(s)=\omega(1+\beta_1+..+\beta_1^J)+\alpha_1(a_{t-1}^2+\beta_1 a_{t-2}^2+. .+\beta_1^J a_{t-J-1}^2)+\text{Remainder} \tag{7}$$

If the MA(1) return equation is the correct representation then, relative to the conditional variance equation from equation (4),

$$h_t(s)_{MA}=\omega(1+\beta_1+..+\beta_1^J)+\alpha_1(1+\theta^2)a_{t-1}^2+\beta(1+\theta^2)a_{t-2}^2+.. \beta_1^J(1+\theta^2)a_{t-J-1}^2)+\text{Remainder} \tag{8}$$

If the AR(1) return equation is the correct representation then, relative to the conditional variance equation from equation (4),

$$h_t(s)_{AR}=\omega(1+\beta_1+..+\beta_1^J)+\alpha_1(\left(\frac{1-\rho_1}{1+\rho_1}\right)a_{t-1}^2+\beta_1\left(\frac{1-\rho_1}{1+\rho_1}\right)a_{t-2}^2+.. \beta_1^J\left(\frac{1-\rho_1}{1+\rho_1}\right)a_{t-J-1}^2)+\text{Remainder} \tag{9}$$

If ω,α_1 and β_1 were equivalent in equations (7), (8) and (9), then when the conditional variance is driven by the MA(1) return equation, with θ_1 negative, $h_t(s)_{MA}>h_t(s)$, and when the conditional variance is driven by am AR(1) return equation, with ρ_1 negative, $h_t(s)_{AR}>h_t(s)$, and with ρ_1 positive, $h_t(s)_{AR}<h_t(s)$. However, given the scaling factor in equation (8) relative to equation (9) the potential for distortions to GARCH parameter estimates is greater when the underlying process is driven by an AR(1) return equation relative to a MA(1) return equation.

4. PERSISTENCE, CO-PERSISTENCE AND NON-NORMALITY

Now define ε_t as shocks from any of the assumed first moment equations from section 3.

In the univariate GARCH(1,1) structure h_t converges and is strictly stationary if $E[\ln(\beta_1+\alpha_1 z_{t-i}^2)]<0$. Then $\Sigma_{i=1,kd}\ln(\beta_1+\alpha_1 z_{t-i}^2)$ is a random walk with negative drift diverging to $-\infty$ as the observation interval reduces.

Defining the true processes for the differences in the natural logarithm of the spot index price, i_t, and the natural logarithm of the futures price, f_t, as:

$$\begin{aligned} i_t &= \gamma_1\xi_t+\eta_{it} \\ f_t &= \gamma_2\xi_t+\eta_{ft} \end{aligned} \tag{10}$$

the common 'news' factor ξ_t is IGARCH, in the co-persistence structure, while the idiosynchratic parts are assumed jointly independent and independent of ξ_t and not IGARCH. The individual processes have infinite unconditional variance. If a linear combination is not IGARCH then the unconditional variance of the linear combination is finite and a constant hedge ratio (defined below) leads to substantial reduction in portfolio risk.

A non optimal restricted linear combination is the basis change defined as the difference between the change in the log of the index futures price and change in the log of the spot index level. This implied portfolio is short 1 unit of the spot for every unit long in the futures.

If there are 'news factors' $\xi_{ft} \neq \xi_{it}$ then the constant hedge ratio may not exist. Define these processes as,

$$i_t = \gamma_1 \xi_{it} + \eta_{it}$$
$$f_t = \gamma_2 \xi_{ft} + \eta_{ft} \qquad (11)$$

The estimated constant hedge ratio is short γ units of the spot for every 1 unit long in the futures is,

$$\hat{\gamma} = \left[\frac{c\hat{o}v(f,i)}{v\hat{a}r(i)} \right]\Bigg|_t = \left[\hat{\gamma}_2 \left[\frac{\hat{\rho}\sqrt{v\hat{a}r[\xi_f]\, v\hat{a}r[\xi_i]}}{v\hat{a}r[\xi_i]} \right] + v\hat{a}r(\eta_i) \right]\Bigg|_t \qquad (12)$$

$\hat{\rho}$ is the correlation between ξ_{ft} and ξ_{it}.

When both ξ_f and ξ_i follow IGARCH processes and no common factor structure exists then estimates of the constant hedge diverge. Ghose and Kroner (1994) investigate this case.

When ξ_f follows an IGARCH process but ξ_i is weak GARCH then the estimated constant hedge ratio cannot be evaluated. There are two problems:

(a) the estimated sample variance of ξ_f in equation (12) is infinite as $T \to \infty$ and

(b) $\hat{\rho} = \left[\left[c\hat{o}v[\xi_f \xi_i]\right] / \sqrt{v\hat{a}r[\xi_f]}\sqrt{v\hat{a}r[\xi_i]}\right]\Big|_t$ so that

there is no linear combination of ξ_f and ξ_i which can provide a stationary unconditional variance.

When observing spot index and futures prices over successively finer intervals the co-persistence structure may not hold for at least two further reasons. This argument relates directly to the horizon t+kd for the hedging strategy. This argument also relates directly to distortions possibly induced onto a dynamic hedging strategy, as d→0.

Perverse behaviour can be observed in spot index level changes as over sampling becomes severe. The smoothing effect due to a large proportion of the portfolio entering the non and thin trading group generates a smoothly evolving process with short lived shocks generated by irregular news effects.

General results assume that the z_t's are drawn from a continuous distribution. When sampling futures price data at high frequency then the discrete nature of the price recording mechanism guarantees that there are dis-continuities in return generating processes. The distribution of the z_t's can become extremely peaked due to multiple small price changes and can have very long thin tails due to abrupt shifts in the distribution. As $d \to 0, -\infty < E[\ln(\beta_1 + \alpha_1 z_{t-i}^2)] < +\infty$ for a large range of values for $0 \le \alpha_1, \beta_1 \le 1$ and $\alpha_1 + \beta_1 < 1$ and this depends on the distribution induced by over sampling and resultant reduction in $(\alpha_1 + \beta_1)$. In the limit, $E[(z_t)^4]/[E(z_t^2)]^2 \to \infty$. Then even numbered higher moments of z_t are unbounded as d→0. This over sampling can lead to two extreme perverse effects generated by bid/ask bounce or zero price changes. The effect depends upon liquidity in the respective markets.

CASE 1:

$\alpha_1 \to 1, \alpha_1 + \beta_1 > 1$ and $E[\ln(\beta_1 + \alpha_1 z_{t-i}^2)] > 0$.

Over sampling approaches analysis of transactions. At this level bid/ask bounce and order splitting require an appropriate model

CASE 2:

Over sampling can produce many zero price changes in thin markets. In this latter case as d→0 then $\alpha_1 + \beta_1 \to 0$.

Conditional heteroskedasticity disappears as over sampling becomes severe. The effect on the basis change when there is a relatively illiquid futures market and in the presence of over sampling will be badly distorting.

5. WEIGHTED GARCH

Recall from equation (1) that $\hat{\kappa}$ is a function defining the estimated drift in $\{\phi({}_d\sigma_t^2)\}$ so that λ is a function defining true drift in $\{\varphi({}_d\sigma_t^2)\}$. In the ARCH structure the drift in $h_t(\sigma_t^2)$ in the diffusion limit is represented by $\hat{\kappa}/\phi' - a^2\phi''/2(\phi')^3$ whereas for the stochastic differential equation defined from assumptions 2,3 and 1' in Nelson and Foster (1991 & 1994) this diffusion limit is $\lambda/\varphi' + \Lambda^2\varphi''/2(\varphi')^3$. The effect on the expression for the bias in the asymptotic distribution of the measurement error process can be explosive if derivatives in the terms $a^2\phi''/2(\phi')^3 - \Lambda^2\varphi''/2(\varphi')^3$ cannot be evaluated because of discontinuities in the process. This can happen when important intra-day effects are neglected in the conditional variance equation specification. As well, the bias can diverge as d→0 if the ${}_dZ_{1,kd}$ terms are badly distorted.

Occasional large jumps in the underlying distribution contribute large $O_p(1)$ movements while the near diffusion components contribute small $O_p(d^{1/2})$ increments. When sampling intra-day financial data there are often many small price changes and these tend to be dominated by occasional large shifts in the underlying distribution.

Failure to account for intra-day effects (large shocks to the underlying distribution) can lead to a mixture of $O_p(1)$ and $O_p(d^{1/2})$ effects in the process. One approach is to specify this mixed process as a jump diffusion. An alternative is to account for these effects by incorporating activity measures in the specification of conditional variance equations.

It follows that failure to account for these $O_p(1)$ effects can lead to an explosive measurement error $[{}_d\hat{h}_t - {}_d h_t]$. However, in empirical applications this measurement error is unobservable since ${}_d h_t$ is unobservable. Failure to account for these jumps in the underlying distribution imply that the unweighted GARCH(1,1) structure cannot satisfy the necessary assumptions required to approximate a diffusion limit.

6. SUMMARY

Given that market anomalies such as market opening and market closing effects exist any volatility structure based on observations sampled on a daily basis will provide different volatility estimates.

Gannon (1996) demonstrated that sampling the process too finely does result in induced positive or negative serial correlation in return processes. However, the dominant factor identified in Gannon and Weatherill (1995), distorting unweighted GARCH estimates, was induced excess kurtosis in unconditional distributions of returns. Many small price changes are dominated by occasional large price changes. This effect leads to large jumps in the underlying distribution causing continuity assumptions necessary in deriving diffusion limit results for the unweighted GARCH structure to break down. Conditional volatility estimates may approach the (IGARCH) boundary and become explosive or conditional heteroskedasticity may disappear.

The relative importance of mis-specification of the second moment equation dynamics over mis-specification of first moment equation dynamics is the most important issue. If intra-day trading effects are important this has implications for smoothness and continuity assumptions necessary in deriving diffusion limit results for the unweighted GARCH structure. If these effects are severe then an implied lower sampling boundary needs to be imposed in order to obtain sensible results. Distortions to parameter estimates are most obvious when conditional second moment equations are mis-specified by failure to adequately account for observed intra-day trading patterns. These distortions can be observed across a wide class of financial asset and markets.

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