Are the Electricity Price and Demand Cointegrated?

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Abstract: This study tries to find an answer to the question: "Are the electricity price and demand cointegrated?" There are few studies in the literature that tried to find if there exists a relationship between these two variables. It is well known that the volatility of the prices and demand is very high. Finding a relationship between the price and demand is a key issue for the electricity operators. Few questions have to be answered: if the price and demand are stationary, which is the best way to predict them; if the series are non-stationary, is there any cointegration relation between them.

1 INTRODUCTION

In the last years, the concept of cointegration has attained considerable popularity among econometricians. The aim of this paper is to test using the theory of cointegration the short and long-run relationship between energy demand and the price.

Many economic time series appear to be non-stationary and to drift over the time. The non-stationary series are transformed in stationary series by differencing or detrending.

Two series are said to be cointegrated if each of them taken individually is non-stationary with a unit root, and a stationary linear combination of these series exists. Therefore the individual components are tied together in some long-run equilibrium.

2 METHODOLOGY

2.1 Some useful terminology

A time series is said to be **stationary** if the mean, variance and covariance remain constant over time.

A time series is said to be **integrated of order d**, written as **I(d)**, if, after being differenced **d times**, it becomes a stationary series.

Two different kinds of trends can appear in a series: stochastic or deterministic. A stochastic trend can be removed by first-differencing, and therefore the initial time series, Y_t, is referred as a **difference stationary process**.

A deterministic trend cannot be removed by first-differencing, and Y_t is referred to as a **trend stationary process**. After the trend is removed from the series, a stationary series is obtained.

If the series is trend stationary it also means that it is not an integrated process. If the series are not stationary, then it is possible that a linear combination of integrated variables that is stationary; such variables are said to be cointegrated.

Many series entail such cointegrating relationship. Cointegration refers to a linear combination of non-stationary variables.

Engle and Granger (1987) introduced the concept of cointegration and proposed a methodology to test this.

First a verification of a "Unit Root Process" was made using Dickey-Fuller tests. The hypothesis which are tested are:

(1)
$$H_0 : a_1 = 1 (y_t \quad unit \quad root \quad process)$$

(2)
$$H_1 : a_1 < 1 (y_t \quad not \quad a \quad unit \quad root)$$

for the model:

(3) $$Y_t = a_1 Y_{t-1} + \varepsilon_t$$

or:

(4) $$H_0 : \gamma = 1 (y_t \quad unit \quad root \quad process)$$

(5)
$H_1 : \gamma < 1 (y_t \quad not \quad a \quad unit \quad root)$

for the model:

(6) $$\Delta y_t = \gamma y_{t-1} + \varepsilon_t$$

The null hypothesis of a unit root is tested using the Augmented Dickey-Fuller test. For different number of lags, the following equations are estimated and the null hypothesis of a unit root is tested.

Dickey and Fuller (1979) suggested three different regressions that can be estimated using OLS to test the presence of a unit root:

Regression 1: (no intercept, no trend):

(7) $$\Delta y_t = \rho y_{t-1} + \sum_{i=1}^{m} \gamma_i \Delta y_{t-i} + \varepsilon_t$$

Regression 2: (only intercept, no trend):

(8) $$\Delta y_t = \alpha + \rho y_{t-1} + \sum_{i=1}^{m} \gamma_i \Delta y_{t-i} + \varepsilon_t$$

Regression 3: (both intercept and trend):

(9) $$\Delta y_t = \alpha + \rho y_{t-1} + \beta t + \sum_{i=1}^{m} \gamma_i \Delta y_{t-i} + \varepsilon_t$$

After the series are tested using the Augmented Dickey-Fuller tests, and the stationarity or non-stationarity of the series determined, than the cointegration relation has to be tested and the error correction models constructed.

There is a simple relationship between vector autoregression (VAR) and cointegration. In the case of two variables, the characteristic roots of the matrix of coefficients in the VAR model have to be calculated.

(10) $$y_t = \alpha y_{t-1} + \beta x_{t-1} + \eta_t$$

(11) $$x_t = \delta y_{t-1} + \xi x_{t-1} + \varepsilon_t$$

The matrix of coefficients is:

(12) $$A = \begin{bmatrix} \alpha \beta \\ \delta \xi \end{bmatrix}$$

with the polynomial equation:

(13) $$(\alpha - \lambda)(\xi - \lambda) - \delta\beta = 0$$

or the equivalent:

(14) $$\lambda^2 - (\alpha + \xi)\lambda + (\alpha\xi - \delta\beta) = 0$$

which has the solutions (characteristic roots) λ_1 and λ_2. There are three possible cases for the characteristics roots to be in, ie:

1. both roots equal to unity, the two series are than both I(1), but not cointegrated.
2. if neither of the roots is unity, the series are stationary.
3. otherwise, the series are cointegrated.

The "**direct regression**" is given by the following equation:

(15) $$y_t = \alpha + \beta x_t + u_t$$

The "**reverse regression**" is given by the next equation:

(16) $$x_t = \pi + \omega y_t + v_t$$

The residuals from the "direct regression" and "reverse regression" are denoted by u_t and v_t respectively. In order to determine if the two series are cointegrated, a Dickey-Fuller test on the residuals u_t and v_t is performed. If $\{Y_t\}$ and $\{X_t\}$ are cointegrated than the residuals has to be stationary.

After these tests are conducted, than the residuals from the direct regression can be used to estimate the Error Correction Model (ECM) as follows:

(17) $$\Delta y_t = a_0 + a_1 u_{t-1} + \sum_{i=1}^{n} \alpha_{11}(i) \Delta y_{t-i} + \sum_{i=1}^{n} \alpha_{12}(i) \Delta x_{t-i} + \varepsilon_t$$

$$\Delta x_t = b_0 + b_1 u_{t-1} + \sum_{i=1}^{n} \alpha_{21}(i)\Delta y_{t-i} + \sum_{i=1}^{n} \alpha_{22}(i)\Delta x_{t-i} + \zeta_t \quad (18)$$

3 EMPIRICAL RESULTS

3.1 Data

The data consists in daily observations of the energy consumption (Y_t) and prices (X_t) during the period 4 May 1997 and 12 December 1998 for the Victorian Market.

Data was downloaded from Internet from VPX site. Graphs of the daily prices and energy consumption are shown in **Figure 1** and **Figure 2**.

As it can be seen in the graphs price seems to be stationary, and demand is a seasonal variable.

Figure 1

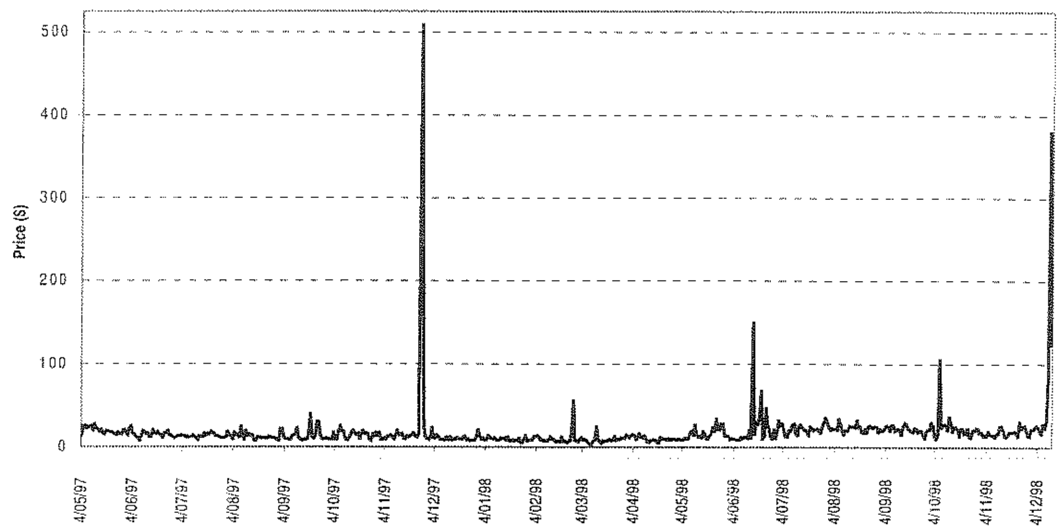
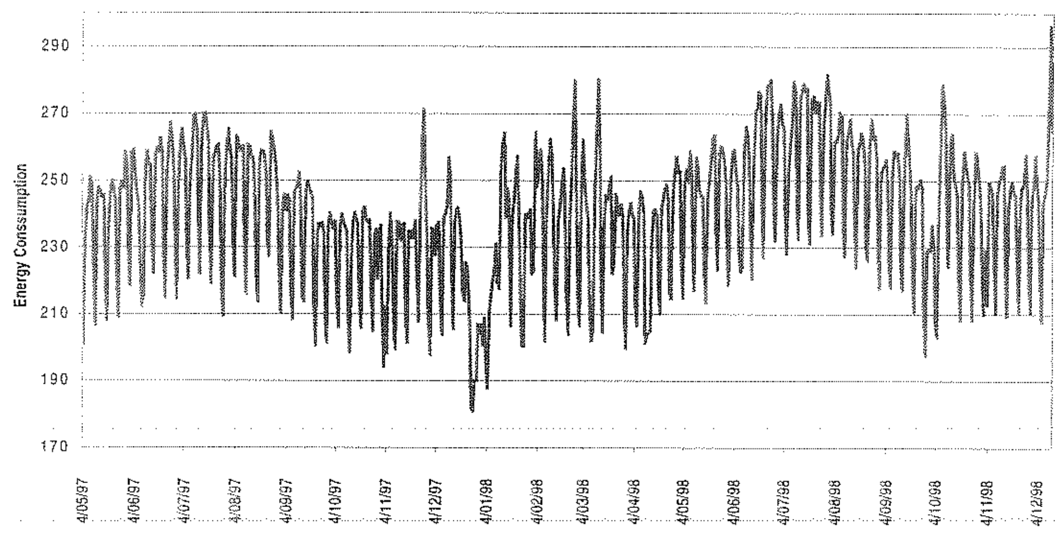


Figure 2



3.2 Results

The three regression lines, given by the equation (7)-(9), were applied to both price and demand. The t-ratio for ρ is computed and compared with the values calculated by Dickey and available in the next tabular form.

It can be seen from the tables that:

- For price: the null hypothesis of a unit root was rejected at all conventional significance level for all three equations.
- For demand: the null hypothesis of a unit root was rejected at all conventional significance level for equations (8) and (9) and cannot be rejected for equation (7).

Empirical cumulative distribution for t for a sample size of 500				
		Significance Level		
		99%	95.00%	90%
Model	Model 1	-2.58	-1.95	-1.62
	Model 2	-3.44	-2.87	-2.57
	Model 3	-3.98	-3.42	-3.13

Table 1

- For deseasonalised demand: the null hypothesis of a unit root was rejected at all conventional significance level for equations (8) and (9) and cannot be rejected for equation (7).

The results are presented in **Figure 3** for Price, **Figure 4** for Demand and **Figure 5** for the deseasonalised demand. .

Figure 3

Coefficients and t-ratio						
	Equations					
X_t	(7)		(8)		(9)	
	coefficients	t	coefficients	t	coefficients	t
α			11.197		7.312	
γ	-0.435	-12.55	-0.61192	-15.84	0.013741	-16
β					-0.62129	

Figure 4

Coefficients and t-ratio						
	Equations					
Y_t	(7)		(8)		(9)	
	coefficients	t	coefficients	t	coefficients	t
α			91.9		91.7	
γ	-0.00233	-0.76	-0.382	-11.78	-0.388	-11.9
β					0.0061	

Figure 5

Coefficients and t-ratio						
	Equations					
Y_t	(7)		(8)		(9)	
	coefficients	t	coefficients	t	coefficients	t
α			37.6		38.4	
γ	-0.00017	-0.12	-0.156	-6.87	-0.163	-7.06
β					0.00313	

Therefore it can be concluded that price and demand are stationary variables. The deseasonalised demand is also a stationary series.

Price and demand are I(0) processes and therefore cointegration cannot be studied further. A conclusion regarding the long-run relationship cannot be drawn.

4 CONCLUSION

In this study the theory of cointegration was tested for the energy consumption and price in the Victorian Electricity Market.

Unexpected, both series were find to be stationary and therefore an error correction model could not be applied.

If the price and demand are stationary than which is the best way to find a relationship between them in order to be able to forecast the future values.

5 BIBLIOGRAPHY

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