Numerical Experiments on Double Diffusively Induced Gravity Currents

Hideki Nagashima and Jiro Yoshida:

Department of Ocean Sciences, Tokyo University of Fisheries,

4-5-7 Kohnan, Minato-ku, Tokyo 108-8477, Japan

ABSTRACT The behavior of double diffusively induced gravity current is investigated numerically. First, double diffusive lock exchange flow is produced with the initial density differences between each side of the barrier wall. After the lock-gate is withdrawn, the light hot salty water flows over the heavy cold fresh water with transporting salt as salt finger convection through the interface between the fluids. The interface is not horizontal and inclined up towards the heavier fluid with the triangle shape. This behavior is clearly different from the lock exchange flow of single component. In particular, the gravity current is induced even if the density difference is exactly equal to zero. These results are well compared with the laboratory experiments by Yoshida et al. [1987]. Second, we released cold/fresh water on to the ambient warm/salty water, by parameterizing the effective heat and salt transport in double diffusive convection by an experimentally determined flux law due to Linden [1974]. It is found that the behavior of the current is well described by the combination of three parameters. One is the turbulent Prandtl number ε ; when ε is large, the density current is suppressed. A second parameter is the turbulent Rayleigh number Ra ; when Ra is large, the induced convection becomes vigorous and the resulting density current becomes strong. The last parameter is the Turner number $R\rho$; when $R\rho$ is sufficiently small, the activity of double diffusive convection becomes large.

1. INTRODUCTION

The behavior of gravity current of a single property such as temperature or salinity alone has been intensively studied in the wide variety of environment, such as in the ocean, atmosphere, volcano and so on (for details, see Simpson [1983]). In the case of two properties having different diffusivities, however, double diffusive convection should affect the behavior of the gravity current.. Maxworthy [1983] used salt and sugar as density contribution components to the environment for convenience. He released salt (sugar) water on to the homogeneous sugar (salt) water, which corresponds to the oceanic situation that cold/fresh (warm/salty) water on to warm/salty (cold/fresh) water. He found that the velocity of the current is considerably reduced by double diffusive convection. Interesting phenomena observed in his experiment was that, when the density differences of two solutions were extremely low, a secondary current was generated near the bottom of tank by the descending plume from the original surface current. This secondary current was also observed when the double diffusive current was produced by the classical dam break method (Yoshida et al. [1987]).

In the present paper, we examine numerically two typical examples of double diffusive gravity currents. First, in section 2, numerical experiments of lock exchange flow are carried out and compared with the previous laboratory experiment by Yoshida et al. [1987].

We simulate a double diffusively induced current of the case when the density difference is exactly equal to zero.

Second, in section 3, we restricted our attention to the case when the cold/fresh water is released on to the warm/salty water as a typical example of behaviors of ice melting water and spreading of rain pools. The results of the numerical experiments are compared with those of Maxworthy's experiment.

2. DOUBLE DIFFUSIVE LOCK EXCHANGE FLOW

2.1. Governing Equations

The governing equations are as follows:

$$\frac{\partial \xi}{\partial t} + J(\phi, \xi) = g(-\alpha T_x + \beta S_x) + \nu \nabla^2 \xi \quad (1)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z} = K_T \nabla^2 T \quad (2)$$

$$\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x} + w\frac{\partial S}{\partial z} = K_S \nabla^2 S \quad (3)$$

$$\xi = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = \nabla^2 \phi . \quad (4)$$

Here, t is time, u and w velocities in the x, z directions, respectively, g is the gravitational acceleration and ϕ is stream function. T and S are temperature and salinity, respectively. ν, K_T and K_S are viscosity, and the diffusivities of T and S, respectively. J is the Jacobian, ξ is the vorticity, α is the expansion coefficient of heat and β is the contraction coefficient of salt.

(2) and (3) represent the conservation of heat and salt. We focus on the fingering convection and parameterize the vertical heat flux using Stern's formula, where heat flux is parameterized in terms of density flux ratio $\gamma = \frac{\alpha F_T}{\beta F_S} = \frac{\alpha K_{TV} \partial T / \partial z}{\beta K_{SV} \partial S / \partial z}$.

This formula is used in (2). Thus, the equation (2) is reduced to:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z} = K_T \nabla^2 T + \frac{\beta}{\alpha}\gamma K_{SV}\frac{\partial^2 S}{\partial z^2} \qquad (5)$$

The equation sets (1) and (3) to (5) are solved numerically by using the finite difference method.

2.2. Model

. The dimension of the model tank is 0.5m long and 0.06m deep. These are the same dimension of the previous laboratory experiments (Yoshida et al [1987]). Lighter hot-salty water is filled in the left half of the tank, and heavy cold fresh water in the right half. The rigid rid surface is assumed, and the slippery boundary condition is used on the every wall. The value of the parameters used here are $K_{SV} = 10^{-5} m^2/s$, $\gamma = 0.56$ and $K_T = 1.38 \times 10^{-7} m^2/s$, $K_S = 1.38 \times 10^{-9} m^2/s$, which are the molecular values of horizontal diffusivities. $\nu = 10^{-5} m^2/s$. We calculate five cases of the initial density difference ($4.0, 2.0, 1.0, 0.5, 0.02k\ g/m^3$) for different three density anomalies due to salt ($\beta \Delta S = 0.0$, 0.0435, 0.1). Here, $K_{SV} = 10^{-5} m^2/s$ is taken to be large to express the enhanced vertical transport of salt associated with salt finger convection. Then, ν is taken to be larger in order of magnitude than the usual molecular values to set the turbulent Prandtl number to be *O(1)*.

2.3. Results and Discussion

A typical example of numerical calculations is shown in Figure 1. This is the case of $\beta \Delta S = 0.1$ and. $\Delta\rho = 1.0 kg/m^3$, and shows density distributions at every four seconds after the withdrawal of the barrier. Figure 2 shows the pattern of stream function and the distributions of temperature, salinity and density fields at 16 seconds after the withdrawal of the barrier. From these figures, we can see that the hot/salty (cold/fresh) water flows over (under) the cold/fresh (warm/salty) water with making the gravity head and the behavior of the current shows a triangle shape which is also found in the laboratory experiment (Yoshida et al. [1987]). These features are clearly different from the lock exchange flow of single component case. The current speed is

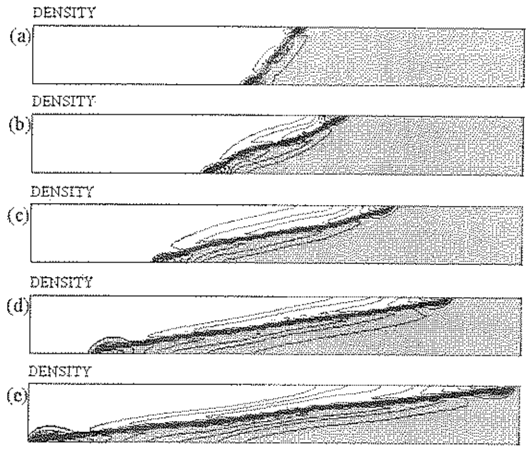

Figure 1. A typical flow pattern of the lock exchange flow, where $\beta \Delta S = 0.1$ and. Figures are density fields at every four seconds, (a) 4sec (b)8sec, (c)12sec, (d)16sec and (e)20sec, after withdrawal of the barrier .

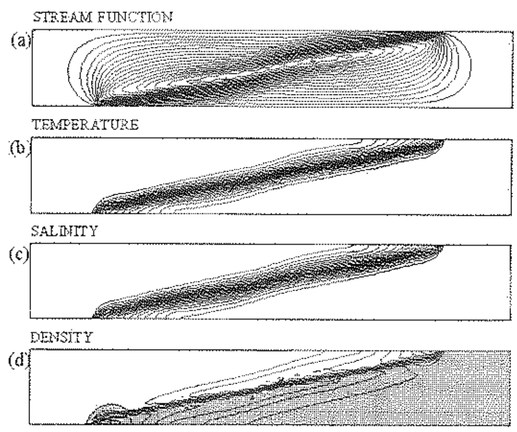

Figure 2. (a) stream function, (b)temperature, (c)salinity and (d)density fields at 16 seconds after the withdrawal of the barrier. Parameters are same to those in Figure1.

also larger than that of a single component case. Another interesting feature of double diffusive gravity current is found in the case when $\Delta\rho$ is exactly equal to zero. Yoshida ET al.[1987] concluded in their laboratory experiments that the gravity current became stronger with increasing $\beta \Delta S$ when $\Delta \rho$ was very small. Furthermore, they suggested that the double diffusive effect produced inhomogeneity of density field and induced the gravity current even if initial density difference ($\Delta \rho$) was exactly equal to zero. Since, it is impossible to make $\Delta \rho = 0$ exactly in the laboratory experiment, such an argument has not become clear. The numerical result for this case shows that the double diffusive effect induces the density anomaly and produces the gravity current as shown in Figure 3.

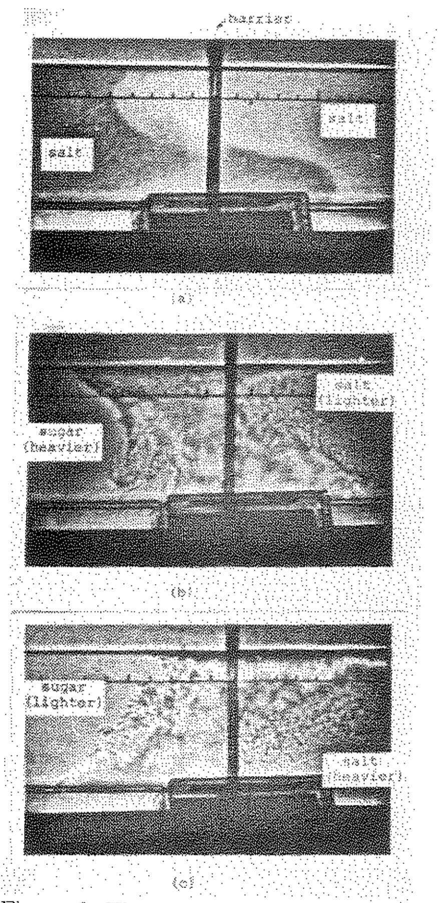

Figure 3. The results of laboratory experiment of a double diffusive lock exchange flow for the case of very small density difference. The flow fields after the withdrawal of the barrier are shown for (a) one-component case (b)diffusive case and (c) finger case.
(After Yoshida teal. [1987])

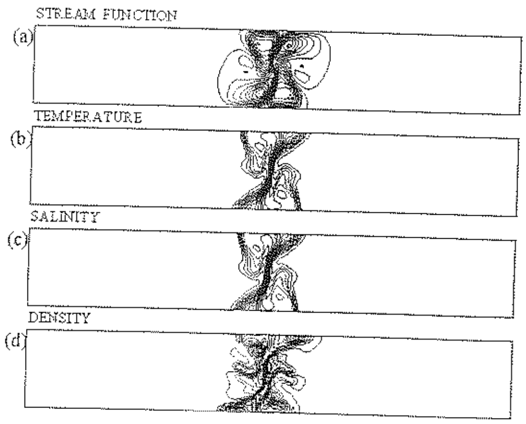

Figure 4. Stream function, temperature, salinity and density fields at 20 seconds after withdrawal of the barrier in the case of $\beta \Delta S = 0.1$ and the initial density difference is exactly equal to zero ($\Delta \rho = 0$). It is clearly seen in the figure that density anomaly is induced and density current is generated.

3. COLD/FRESH WATER RELEASE ON THE SURFACE OF DENSE HOT/SALTY WATER

3.1. Formulation

In this case, as the so-called diffusive type convection is expected, the vertical salt flux is paremetarized as follows:

$$\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x} + w\frac{\partial S}{\partial z}$$

$$= K_{SH}\frac{\partial^2 S}{\partial x^2} + \frac{\partial}{\partial z}\left(\frac{\alpha}{\beta}\gamma K_{TV}\frac{\partial T}{\partial z}\right). \qquad (6)$$

In this conservation equation, the horizontal and vertical diffusivities are described separately. This generalized description of diffusion was introduced by considering the possibility of inhomogeneity in turbulent diffusion. However, in the present calculation, for simplicity, the ratio of vertical to horizontal diffusion is taken to be unity.

The density flux ratio γ is usually a function of the stability ratio of the double diffusive system in terms of the Turner number $R\rho$. From the laboratory experiment of Linden [1974], γ is given as

$$\gamma = \frac{(R\rho - 1)^{3/2} + 0.5R\rho}{10(R\rho - 1)^{3/2} + 0.5} \quad where, R\rho = \frac{\beta \Delta S}{\alpha \Delta T}.$$

(7)

Here, ΔS and ΔT are the vertical salinity and temperature difference.

3.2. Non-Dimensional Form

We make the non-dimensional form of above equations by introducing the following scales of length scale H (Depth of the calculating domain), temperature scale ΔT (initial temperature difference), salinity scale ΔS (initial salinity difference) and reduced gravity $g^*(=g\Delta\rho/\rho = -g\alpha\Delta T(1-R\rho))$.

Based on these scales, the dimensional variables are transformed by the following relations

$$x = Hx',\ z = Hz',\ u=\sqrt{g^* H}\cdot u',\ w=\sqrt{g^* H}\cdot w',\ \phi=\sqrt{g^* H}\cdot H\phi'$$

$$\xi = \sqrt{g^* H}/H\cdot \xi',\ T=\Delta T\cdot T',\ S=\Delta S\cdot S',\ t=H/\sqrt{g^* H}\cdot t'$$

Where, (') represents non-dimensional variables. After these procedures, we have the following non-dimensional equations :

$$\frac{\partial \xi}{\partial t}+u\frac{\partial \xi}{\partial x}+w\frac{\partial \xi}{\partial z} = -\frac{1}{1-R\rho}\left(-\frac{\partial T}{\partial x}+R\rho\frac{\partial S}{\partial x}\right)+\varepsilon Ra^{-\frac{1}{2}}\nabla^2\xi \quad (8)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z} = Ra^{-\frac{1}{2}}\left(\frac{\partial^2 T}{\partial x^2}+\delta\frac{\partial^2 T}{\partial z^2}\right) \quad (9)$$

$$\frac{\partial S}{\partial t}+u\frac{\partial S}{\partial x}+w\frac{\partial S}{\partial z} = Ra^{-\frac{1}{2}}\left(\tau^{-1}\frac{\partial^2 S}{\partial x^2}+\frac{\delta}{R\rho}\frac{\partial}{\partial z}\left(\gamma\frac{\partial T}{\partial z}\right)\right) \quad (10)$$

$$\xi = \frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}=\nabla^2\phi. \quad (11)$$

In these non-dimensional equations, we have the following five dominant parameters which govern the motion of the current: $R\rho=\frac{\beta\Delta S}{\alpha\Delta T}$ (Turner Number), $Ra=\frac{g^* H^3}{K_{TH}^2}$, (Turbulent Rayleigh Number), $\varepsilon=\frac{\nu}{K_{TH}}$ (Turbulent Prandtl Number),

$$\tau=\frac{K_{SH}}{K_{TH}} \text{ and } \delta=\frac{K_{TV}}{K_{TH}}.$$

In the present numerical experiment, the last two parameters are fixed to be unity. Ra is taken to be 10^4, 10^6, 10^7; ε is 1, 10 and 100 ; $R\rho$ is 1.01 and 2. With these parameters changed in various combinations, equations (8) to (11) are solved numerically by using the finite difference method. The model domain is shown in Fig.ure5, where the horizontal grid number is 201 and the vertical one is 25. Cold fresh water, which occupies half of the depth and one-tenth of the horizontal length, are set on the hot salty water. All fluid is initially at rest. All boundary conditions are rigid lid and no-slip.

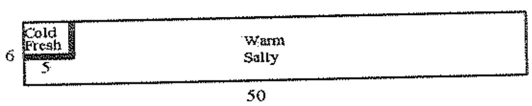

Figure 5. Schematic view of the model for cold/fresh water release on hot/salty water. Numerals in the figure represent the non-dimensional length of the domain.

3.3 Results and discussion

A typical example of numerical calculation is shown in Fig.6 and Fig.7 where the values of τ,δ, $R\rho$ and Ra are 1.0,1.0,2.0 and 10^7, respectively.

In Fig.6, sequential density distributions are shown. It is clearly seen that the light cold/fresh water flows over the hot/salty water with making gravity head. Note that time is in non-dimensional form, thus the real time is given by multiplying the dimension scale $H/\sqrt{g^* H}$, where $g^*=g\Delta\rho/\rho$. For example, the real time t^* is about *150* seconds when $\Delta\rho/\rho = 2\times10^{-3}$, $H=0.06m$ and $t=87.84$. The density field is not so different from that of single component case except that a wavy interface exists. However, temperature and salinity fields show clearly the occurrence of double diffusive convection as shown in Figure 7. Moreover, the stream function at the bottom of the figure shows alignment of convection cells below the surface. These cells are generated by the non-uniform density field associated with the difference of buoyancy flux through the diffusive interface and surroundings. The horizontal scale of cell is probably due to the length scale occupied initially by the cold/fresh water.

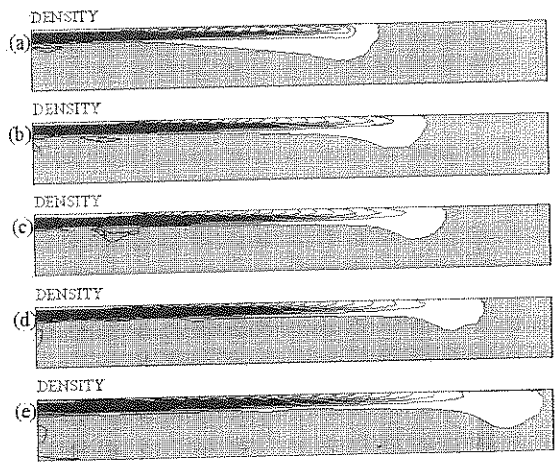

Figure 6. A typical flow pattern of gravity current when cold/fresh water is released on the hot/salty water, where $\tau=1$, $\delta=1$, $R\rho=2.0$, $Ra=10^7$ and $\varepsilon=10$. Density fields are shown for non-dimensional time t=27.45, 43.92, 54.90, 87.48 and 164.7. It is clearly seen that the density current is developed..

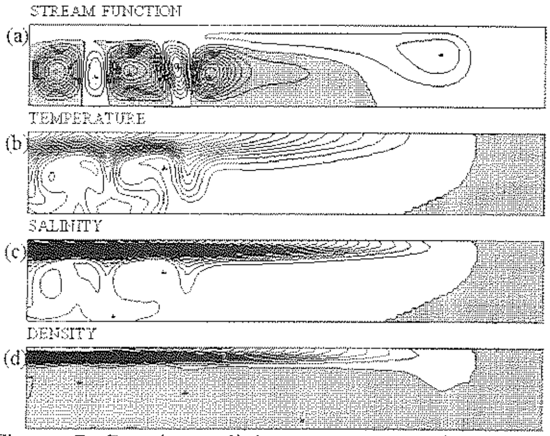

Figure 7. Density, salinity, temperature fields and stream function for $\tau = 1$, $\delta = 1$, $R\rho = 2.0$, $Ra = 10^7$ and $\varepsilon = 10$ at $t = 87.84$. Hatched area in density, salinity and temperature field shows the regions of the initial warm/salty water, and that in stream function shows the positive value region which corresponds to anti-clockwise circulation region.

In Figure 8, we show the case when $Ra = 10^6$ and other parameters are the same as in Figure 7. Since Ra is small, that is, the buoyancy anomaly is small or the effect of diffusion is large, the density current becomes weaker.

Next, in order to investigate the effect of viscosity, we take ε to be *100* and the other parameters are same as in Figure 7. The effect of viscosity suppresses clearly the density current as shown in Figure 9. The temperature and salinity fields, however, resemble those in Figure 8. This could be explained by examining Equations. (8) to (11), where ε only appears in the second term in the right hand side of (8) as a product with $Ra^{-1/2}$. This means that vorticity is diffused more when Ra becomes small and ε becomes large.

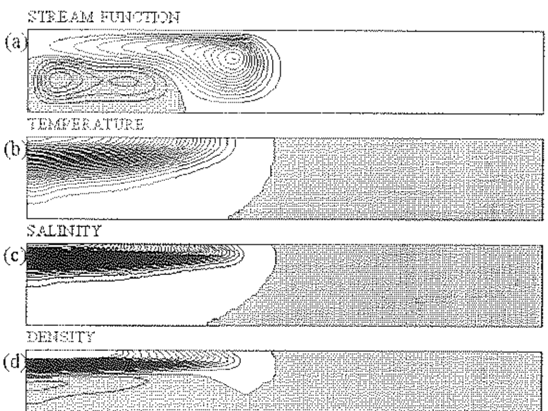

Figure 8. Same in Figure 7, but $Ra = 10^6$ and t=27.28

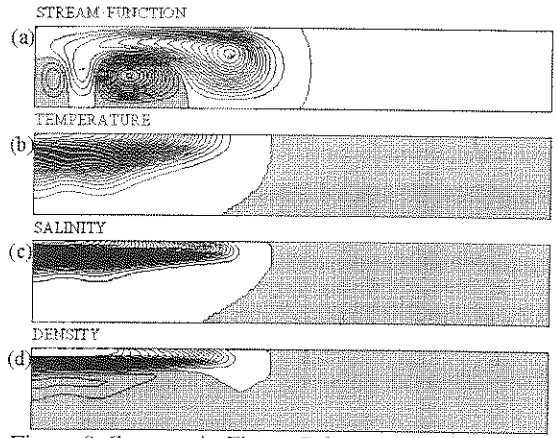

Figure 9. Same as in Figure 7, but $\varepsilon = 100$

4. CONCLUSIONS

Two kinds of numerical calculation on double diffusively induced gravity currents are carried out.

First, a lock exchange flow of salt finger type is carried out. The result shows the particular current behavior of triangle form of the interface between two fluids. The interface is not horizontal and inclines up towards the heavier fluid. When the density anomaly due to salt ($\beta \Delta S$) is small, the current behaves as a single component lock exchange flow. But when $\beta \Delta S$ is large, the above mentioned characteristics becomes conspicuous. Furthermore, the gravity current is induced even if the initial density anomaly is exactly equal to zero. These results are well compared with those of laboratory experiments by Yoshida et al.[1987].

Second, cold/fresh water is released on to the ambient warm/salty water. It is found that the phenomena are governed by the combination of three parameters. One is the turbulent Prandtl number (ε); when ε is large, the density current is suppressed. A second parameter is the turbulent Rayleigh number (Ra) defined by $Ra = g^* H^3 / K_{TH}^2$; when Ra is large, the induced convection becomes vigorous and the resulting density current becomes strong. The last parameter is the Turner number ($R\rho$) defined by $R\rho = \beta\Delta S/\alpha\Delta T$; when $R\rho$ is sufficiently small, the activity of double diffusive convection becomes large and enhances the gravity current.

5.REFERENCES

Linden, P.F., A note on the transport across a diffusive interface, *Deep Sea-Res.*, 21,283-287,1974.

Maxworthy, T., The dynamics of double diffusive

gravity currents, *J. Fluid Mech.,* 128, 259-282, 1983.
McDougall, T.J., Oceanic intrusions: Some limitations of the Ruddick and Turner(1979) mechanism, *Deep Sea Res.,* 33, 1653-1664, 1986.
Nagashima, H., J. Yoshida and M. Nagasaka, The behavior of double diffusive gravity current: Laboratory and numerical experiments. In *Double Diffusion in Oceanography: Proc. of a Meeting September 26-29, 1989*(1990), compiled by R. W. Schmitt, pp. 193-200, Woods Hole Oceanogr. Inst. Tech. Rept, WHOI-91-20, 1990
Simpson, J.E., *Gravity Currents: In the Environmental and the Laboratory,* , John Wiley and Sons, 244 pp. 1987
Thangam, T., and C.F. Chen, Salt-finger convection in the surface discharge of heated saline jets, *Geophys. Astrophys. Fluid Dyn.,* 18, 111-146, 1981.
Turner, J.S., *Buoyancy Effects in Fluids.* Cambridge Univ. Press., 367pp, 1973
Yoshida, J., H. Nagashima and Wen-Ju Ma, A double diffusive lock-exchange flow with small density difference, *Fluid Dynamics Res.,* 2, 205-215, 1987.