Diagnostic Tests of GARCH Against E-GARCH

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Abstract: This paper considers diagnostic tests of the GARCH and E-GARCH models against each other, based on a linear weighting of the competing conditional variances. The asymptotic distributions and power functions of the diagnostic tests are examined. Alternative weighting schemes are also analysed.

1. INTRODUCTION

Various volatilities, such as asset returns, stock returns and exchange rates, are believed to change over time. Modelling time-varying volatility has been one of the most important research topics in various economic and financial applications over the last fifteen years. The first development to capture such volatility was the autoregressive conditional heteroskedasticity (ARCH) model of Engle [1982]. Following Engle's seminal contribution, many different ARCH-type models have been proposed; see, for example, ARMA-ARCH [Weiss, 1984], GARCH [Bollerslev, 1986], CHARMA [Tsay, 1987], E-GARCH [Nelson, 1989], Threshold ARCH [Zakoian, 1994], and double threshold ARCH [Li and Li, 1996], among others (for a survey of recent theoretical results, see Li et al. [1999]). Without doubt, two of the more interesting and widely used models in the ARCH family are Bollerslev's GARCH and Nelson's E-GARCH.

The GARCH model has two quite attractive features. First, it can capture the persistence of volatility. A substantial body of empirical evidence has helped to explain various economic and financial phenomena (see, for example, Engle and Bollerslev [1986a, b], Bollerslev et al. [1992, 1994], and Bollerslev and Mikkelsen [1996]). Second, GARCH is mathematically and computationally straightforward, as compared with some other ARCH-type models. Many theoretical results, including the statistical properties of the model and the large sample properties of some estimation methods, are now available, and these provide a solid foundation for applications of the model. However, as argued by Nelson [1989], the GARCH model has several drawbacks, including an inability to capture asymmetric volatility and to impose nonnegativity restrictions.

In order to avoid these shortcomings, Nelson [1989] proposed the E-GARCH model. The GARCH and E-GARCH models are non-nested (or separate) and the volatilities modelled by these two models should be substantially different form each other. However, as the true feature of the volatility is not known in practice, it is also not known whether the true model is GARCH or E-GARCH when a series of economic or financial data are observed. This suggests the motivation for developing diagnostic tests of the GARCH and E-GARCH models.

A primary aim of this paper is to develop and examine the asymptotic properties of diagnostic tests of the non-nested GARCH and E-GARCH models. The non-nested testing methodology was developed almost four decades ago, and has been demonstrated to be a powerful tool for testing such models (see McAleer [1995] for a recent review). However, virtually all non-nested tests have been developed for the functional forms of the regression, or for the conditional means. This paper adapts the non-nested procedure for testing the conditional variances of different models, in particular, to develop non-nested diagnostic tests of the GARCH and E-GARCH models. The asymptotic distributions and power functions of the diagnostic tests are derived. Alternative weighting schemes are also examined.

The paper is organised as follows. Section 2 presents the GARCH and E-GARCH models, and the non-nested diagnostic testing procedures. Section 3 presents the non-nested diagnostic tests of the GARCH and E-GARCH models, and examines their asymptotic distributions and power functions. Section 4 discusses non-nested tests based on alternative weighting schemes. Some concluding remarks are given in Section 5. The proofs of all theorems can be found in Ling and McAleer [1996b].

2. NON-NESTED DIAGNOSTIC TESTING PROCEDURE

Suppose that $\{\varepsilon_t\}$ is the time series process of interest. One possible specification for ε_t is the GARCH(p,q) model, namely:

$$H_0\colon\ \varepsilon_t = z_{0t} h_t^{1/2} \qquad (2.1)$$
$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i}$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$ and $\beta_i \geq 0$; $\{z_{0t}\}$ is a series of independently and identically distributed (i.i.d.) random variables with mean zero and variance one; and $t = 1, \ldots, n$. It is assumed that $\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1$, which ensures that the GARCH model is strictly stationary and ergodic, and $E\varepsilon_t^2 < \infty$ (see Bollerslev [1986] and Ling and Li [1997]).

A popular alternative specification to GARCH is the E-GARCH(r,s) model, which is defined by:

$$\mathrm{H}_1\colon\ \varepsilon_t = z_{1t} g_t^{1/2} \qquad (2.2)$$
$$\ln g_t = \omega + (1 - \sum_{i=1}^{r} \phi_i B^i)^{-1} (1 + \sum_{i=1}^{s} \psi_i B^i) u(\varepsilon_{t-1})$$

where

$$u(\varepsilon_t) = \theta \varepsilon_t / g_t^{1/2} + \gamma\left(|\varepsilon_t| / g_t^{1/2} - E(|\varepsilon_t| / g_t^{1/2})\right)$$
$$= \theta z_{1t} + \gamma\left(|z_{1t}| - E(|z_{1t}|)\right)$$

and $\{z_{1t}\}$ is a series of i.i.d. random variables with mean zero and variance one. It is assumed that γ and θ are not both equal to zero and $\sum_{k=0}^{\infty} \rho_k^2 < \infty$, where $(1 - \sum_{i=1}^{r} \phi_i B^i)^{-1} (1 + \sum_{i=1}^{s} B^s) = \sum_{k=0}^{\infty} \rho_k^2 B^k$. This is the condition for the strict stationarity, ergodicity, and covariance stationarity of $\ln g_t^2$ (see Nelson [1989]). From (2.1)-(2.2), it is clear that H_0 and H_1 are non-nested, in that neither can be obtained from the other by the imposition of suitable parametric restrictions.

In a similar spirit to that of Davidson and MacKinnon [1981], MacKinnon et al. [1983], and Bera and McAleer [1989], we can construct the auxiliary ARCH-type model given by the following linear weighting of the competing conditional variances:

$$H_L\colon\ \varepsilon_t = z_t f_t^{1/2} \qquad (2.3)$$
$$f_t = (1 - \delta) h_t + \delta g_t$$

where $\{z_t\}$ is a series of i.i.d. random variables with mean zero and variance one. If H_0 is true, then $\delta = 0$, and $\delta = 1$ if H_1 is true. Now let $\hat{\beta}_n$ be the maximum likelihood estimator (MLE) of β in (2.2). Denote $g_t(\hat{\beta}_n)$ by $\hat{g}_t$ where $\beta = (\omega, \phi_1, \ldots, \phi_r, \psi_1, \ldots, \psi_s, \theta, \gamma)'$, and let A' denote the transpose of the vector or matrix A. H_L can be approximated by the following model:

$$H_{0L}\colon\ \varepsilon_t = z_t f_{0t}^{1/2} \qquad (2.4)$$
$$f_{0t} = (1 - \delta) h_t + \delta \hat{g}_t\,.$$

It can be seen that $\hat{g}_t$ is a function of $\{\varepsilon_{t-1}, \ldots, \varepsilon_0, \varepsilon_{-1}, \ldots\}$, and hence is independent of z_t because the influence of any particular error term on the estimates tends to zero as the sample size approaches infinity (this argument is similar to that in Davidson and MacKinnon [1981]). In practice, it is usually assumed that the pre-sample values, i.e. ε_t for $t \leq 0$, are zero. This assumption does not affect the asymptotic properties of the estimators or tests (see Bollerslev [1986] and Weiss [1986]). Under H_0, ε_t is strictly stationary and ergodic, and has a finite unconditional variance.

Using maximum likelihood estimation, we can obtain the joint estimators, $\hat{\delta}_n$ and $\hat{\alpha}_n$, of δ and α, where $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_q)'$. Under H_0, we can derive the asymptotic distribution of $\hat{\delta}_n$ in order to test H_0.

Under H_1, the estimator of δ in (2.4) will converge to 1. However, as the t-statistic for testing $\delta = 0$ in (2.4) is conditional on the truth of H_0, it is valid only for testing H_0. In order to derive a test for H_1, consider the following auxiliary ARCH-type model:

$$H_{1L}:\ \varepsilon_t = z_t f_{1t}^{1/2} \qquad (2.5)$$
$$f_{1t} = (1-\delta)g_t + \delta\hat{h}_t$$

where $\hat{h}_t$ denote $h_t(\hat{\alpha}_n)$ and $\hat{\alpha}_n$ is the MLE of α. The MLE of δ in (2.5) can be used to test H_1 as the null, namely $\delta = 0$.

Unlike the typical linear and nonlinear regression models considered in the literature, the estimation of (2.4) or (2.5) is more complicated. Consider quasi-maximum likelihood estimation of (2.4). Under the regularity conditions given in Bollerslev and Wooldridge [1992, Theorem 2.1] or White [1994, Theorem 6.2], there exist a series of consistent MLEs which are asymptotically normal or satisfy condition (3.2) of the following section. Unfortunately, the verification of these regularity conditions can be difficult, even for the GARCH and E-GARCH models. A weaker regularity condition is available only for the GARCH(1,1) model (see Lee and Hansen [1994] and Lumsdaine [1996]). For the general ARCH(p) model, Ling and McAleer [1999a] showed asymptotic normality under the second moment condition, while the finite fourth-moment condition is required for the general GARCH(p,q) model (see Ling and Li [1997]). The latter is a strong condition and may not always be satisfied in practical applications. For the E-GARCH model, no regularity condition has yet been established. In what follows, it is assumed that all of the appropriate regularity conditions are satisfied.

3. ASYMPTOTIC PROPERTIES OF THE NON-NESTED TESTS

In this section, we present the asymptotic distribution of the non-nested test under the null, H_0: GARCH, and the corresponding power function under the alternative, H_1: E-GARCH. The problem is symmetric, and the case of testing the E-GARCH null against the GARCH alternative is also examined.

Consider the (conditional) quasi-log-likelihood function of model (2.4), i.e. H_{0L}:

$$L(\delta,\alpha) = -\frac{1}{2n}\sum_{t=1}^{n}\log f_{0t} - \frac{1}{2n}\sum_{t=1}^{n}\frac{\varepsilon_t^2}{f_{0t}}. \qquad (3.1)$$

Suppose that $(\hat{\delta}_n, \hat{\alpha}_n')$ are a series of MLEs of (δ,α'), such that:

$$\sqrt{n}\begin{pmatrix}\hat{\delta}_n - \delta \\ \hat{\alpha}_n - \alpha\end{pmatrix} = -\sqrt{n}B^{-1}\begin{pmatrix}\dfrac{\partial L(\delta,\alpha)}{\partial\delta} \\ \dfrac{\partial L(\delta,\alpha)}{\partial\alpha}\end{pmatrix} + o_p(1) \qquad (3.2)$$

where $\partial L(\delta,\alpha)/\partial\delta$ and $\partial L(\delta,\alpha)/\partial\alpha$ are the first-order derivatives of $L(\delta,\alpha)$ with respect to δ and α, respectively, and $B = \partial L^2(\delta,\alpha)/\partial(\delta,\alpha)\partial(\delta,\alpha)'$ is the corresponding second-order derivative of $L(\delta,\alpha)$.

Denote $F = (a_1,\ldots,a_n)'$, with $a_t = (\partial h_t/\partial\alpha)/\sqrt{2}h_t$; $e = (e_1,\ldots,e_n)'$, with $e_t = (\varepsilon_t^2/h_t - 1)/\sqrt{2}$; and $R = (r_1,\ldots,r_n)'$, with $r_t = (\hat{g}_t - h_t)/\sqrt{2}h_t$. It follows that:

$$\sqrt{n}\hat{\delta}_n = \sqrt{n}R'M_0 e/\|M_0 R\|^2 + o_p(1) \qquad (3.3)$$

where
$M_0 = I - F(F'F)^{-1}F'$.

Let β^* be the limit of $\hat{\beta}_n$ in probability under H_0 as n goes to infinity, and $\bar{\beta}$ is an intermediate point between $\hat{\beta}_n$ and β^*. It is straightforward to show that $\sqrt{n}\hat{\delta}_n$ is asymptotically normal with mean zero and an asymptotic variance which may be estimated by:

$$nc/\|\hat{M}_0\hat{R}\|^2$$

where $c = (Ez_t^4 - 1)/2$, and $\hat{M}_0$ and $\hat{R}$ are, respectively, M_0 and R with all the parameters replaced by their corresponding estimates. Thus, we have the following theorem.

Theorem 3.1.
The t-statistic for $\hat{\delta}_n$ generated by (2.4) is asymptotically distributed as $N(0,1)$ if H_0 is true.

Denoting α^* as the limit of $\hat{\alpha}_n$ in probability under H_1 as n goes to infinity, we have the theorem.

Theorem 3.2.

The t-statistic for $\hat{\delta}_n$ generated by (2.4) is asymptotically distributed as $N\left(\left\|M_{10}R_1/\sqrt{c}\right\|, 1\right)$ under H_1, where M_{10} and R_1 are defined as M_0 and R with β^* and α replaced by β and α^*, respectively. The power function is asymptotically given by:

$$\Phi\left(\varsigma_\mu + \left\|M_{10}R_1/\sqrt{c}\right\|\right) + o(1)$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standardised normal and ζ_μ is the 100μ percentile of the standardised normal distribution.

For purposes of testing E-GARCH (that is, $\delta = 0$) in (2.5) as the null, the estimation procedure is similar to the above and hence is omitted. In this case, the asymptotic distribution of the t-statistic of $\hat{\delta}_n$ is the same as that given for testing GARCH (that is, $\delta = 0$) in (2.4).

Remark. From Theorem 3.2, the t-statistic from (2.4) rejects H_0 against H_1 with probability 1 when H_1 is true. A similar conclusion holds for the t-statistic from (2.5). In practice, as it is possible that both H_0 and H_1 are rejected using the two non-nested diagnostic tests from (2.4) and (2.5), the appropriate inference would be to reconsider both models. Thus, the non-nested tests from (2.4) and (2.5) are intended as simple and useful diagnostic tools. The results from both tests should provide some guidance for further empirical analysis.

4 ALTERNATIVE WEIGHTING SCHEMES

It is clear that auxiliary ARCH-type models can be constructed using different weighting functions. Consider the following two alternative forms:

$$H'_{0L}: \varepsilon_t = z_t f_{0t}^{1/2} \qquad (4.1)$$
$$f_{0t} = h_t^{1-\delta}\hat{g}_t^{\delta}$$

and

$$H''_{0L}: \varepsilon_t = z_t f_{0t}^{1/2} \qquad (4.2)$$
$$f_{0t} = (1-\delta)h_t^{1/2} + \delta\hat{g}_t^{1/2}.$$

In (4.1), the auxiliary ARCH-type model is given as a linear combination of the logarithms of the competing conditional variances, i.e. $\ln f_{0t} = (1-\delta)\ln h_t + \delta \ln \hat{g}_t$, whereas in (4.2), it is given as a linear combination of the competing conditional standard deviations. If H_0 is true, then $\delta = 0$ in each case, and if H_1 is true, then correspondingly $\delta = 1$. The asymptotic distribution of the MLE of δ can be obtained in each case for testing H_0.

First, consider the non-nested diagnostic test of H_0: GARCH (that is, $\delta = 0$) based on (4.1), which leads to the following theorems.

Theorem 4.1.

The t-statistic for $\hat{\delta}_n$ generated by (4.1) is asymptotically distributed as $N(0,1)$ if H_0 is true.

Theorem 4.2.

The t-statistic for $\hat{\delta}_n$ generated by (4.1) is asymptotically distributed as $N\left(\tilde{R}_1' M_{10}R_1/(\sqrt{c}\|M_{10}R_1\|), 1\right)$ under H_1. The power function is asymptotically given by:

$$\Phi\left(\varsigma_\mu + \left|\tilde{R}_1' M_{10}R_1\right| / (\sqrt{c}\left\|M_{10}\tilde{R}_1\right\|)\right) + o(1)$$

where $\Phi(\cdot)$ and ζ_μ are defined as in Theorem 3.2, $\tilde{R} = (\tilde{r}_1, \ldots, \tilde{r}_n)'$, $\tilde{r}_t = \ln(\hat{g}_t/h_t)/\sqrt{2}$, and $\tilde{R}_1$ is $\tilde{R}$ with α and β^* replaced by α^* and β, respectively.

Remark. From Theorem 4.2, the t-statistic based on (4.1) still has asymptotic power of unity under H_1 if $\left|\tilde{R}_1' M_{10}R_1\right| \neq 0$. However, $\left|\tilde{R}_1' M_{10}R_1\right|$ may be zero, and in this case, the power function of the t-statistic based on (4.1) will be asymptotically $N(0,1)$. It is expected that the t-statistic from (4.1) will not be robust. Since M_{10} is an orthogonal projection matrix, $\left|\tilde{R}_1' M_{10}R_1\right| \leq \left\|\tilde{R}_1' M_{10}\right\| \cdot \left\|M_{10}R_1\right\|$. Thus, from Theorems 3.2 and 4.2, it follows that the t-statistic from (2.4) is more powerful than that from (4.1).

Now consider the alternative weighting scheme given as H''_{0L} in (4.2), which yields the following theorems.

Theorem 4.3.

The t-statistic for $\hat{\delta}_n$ generated by (4.2) is asymptotically distributed as $N(0,1)$ if H_0 is true.

Theorem 4.4.

The t-statistic for $\hat{\delta}_n$ generated by (4.2) is asymptotically distributed as $N\left(\tilde{\tilde{R}}_1' M_{10} R_1 / (\sqrt{c}\|M_{10}R_1\|), 1\right)$ under H_1. The power function is asymptotically given by:

$$\Phi\left(\varsigma_\mu + \left|\tilde{\tilde{R}}_1' M_{10} R_1\right| \Big/ \left(\sqrt{c}\left\|M_{10}\tilde{\tilde{R}}_1\right\|\right)\right) + o(1)$$

where $\Phi(\cdot)$ and ζ_μ are defined as in Theorem 3.2, and $\tilde{\tilde{R}}_1$ is $\tilde{\tilde{R}}$ with α and β^* replaced by α^* and β, respectively.

Remark. In a similar manner to the weighting scheme in (4.1), the power function of the t-statistic from (4.2) is asymptotically $N(0,1)$ if $\tilde{\tilde{R}}_1' M_{10} R_1 = 0$, and 1 in probability if $\tilde{\tilde{R}}_1' M_{10} R_1 \neq 0$. Thus, it is also not robust as compared with the linear weighting scheme given in (2.4). Similarly, since $\tilde{R}_1' M_{10} R_1 \leq \left\|\tilde{R}_1' M_{10}\right\| \cdot \left\|M_{10} R_1\right\|$, the t-statistic from (2.4) is more powerful than that from (4.2).

Consider the following more general weighting scheme:

$$H_{0L}^{\lambda} : \varepsilon_t = z_t f_{0t}^{1/2}$$
$$f_{0t}^{1/\lambda} = (1-\delta) h_t^{1/\lambda} + \delta g_t^{1/\lambda} \tag{4.3}$$

where $\lambda \neq 0$. It is clear that, when $\lambda = 1$, (4.3) reduces to (2.4); as $\lambda \to \infty$, (4.3) reduces to (4.1); and when $\lambda = 2$, (4.3) reduces to (4.2). Thus (4.3) will be referred to as the power family. If H_0 is true, then $\delta = 0$ in the power family, and if H_1 is true, then correspondingly $\delta = 1$.

Defining M_0 and c as in Section 3, and $R_\lambda = (r_{\lambda 1}, \ldots, r_{\lambda n})'$, with $r_{\lambda t} = \left(\lambda(h_t^{1/\lambda} - g_t^{1/\lambda}) / (\sqrt{2} h_t^{1/\lambda})\right)$, the asymptotic variance can be estimated by

$$nc \Big/ \left\|\hat{M}_0 \hat{R}_\lambda\right\|^2$$

where $\hat{M}_0$ and $\hat{R}_\lambda$ are, respectively, M_0 and R_λ with all the parameters replaced by their corresponding estimates.

As $\left|\tilde{R}_{\lambda 1}' M_{10} R_1\right| \Big/ \left(\sqrt{c}\left\|M_{10} R_{\lambda 1}\right\|\right) \leq \left\|M_{10} R_1 / \sqrt{c}\right\|$, where $R_{\lambda 1}$ is R_λ with α and β^* replaced by α^* and β, respectively, this determines the non-nested test with maximum power in finite samples when $\lambda = 1$. The above results are given in the following theorem.

Theorem 4.5.

(a) The t-statistic for $\hat{\delta}_n$ generated by (4.3) is asymptotically distributed as $N(0,1)$ if H_0 is true.

(b) The t-statistic for $\hat{\delta}_n$ generated by (4.3) is asymptotically distributed as $N(R_{\lambda 1}' M_{10} R_1 / (\sqrt{c}\|M_{10} R_1\|), 1)$ under H_1. The power function is asymptotically given by:

$$\Phi\left(\varsigma_\mu + \left|R_{\lambda 1}' M_{10} R_1\right| \Big/ \left(\sqrt{c}\left\|M_{10} R_{\lambda 1}\right\|\right)\right) + o(1)$$

where $\Phi(\cdot)$ and ζ_μ are defined as in Theorem 3.2.

(c) The test from (2.4) (that is, (4.3) with $\lambda = 1$) is the optimal non-nested test of H_0: $\delta = 0$ in the power family with respect to maximum power under H_1 in finite samples.

Remark. Note that $\left|R_{\lambda 1}' M_{10} R_1\right|$ may be zero unless $\lambda = 1$. In a similar manner to that given in the Remark for Theorem 4.4, the t-statistic for $\hat{\delta}_n$ from (4.3) may not be robust unless $\lambda = 1$. Moreover, it is clear that the t-statistic for $\hat{\delta}_n$ from (4.3) has asymptotic power of unity for all $\lambda \neq 0$ if $\left|R_{\lambda 1}' M_{10} R_1\right| \neq 0$. It should be noted that the optimal property of the non-nested diagnostic test in the power family given in Theorem 4.5 relates to differences in finite samples. The optimal property of the non-nested diagnostic test of H_1 from (2.5) can be obtained from a similarly defined power family.

5. CONCLUDING REMARKS

This paper has developed non-nested diagnostic tests of the GARCH and E-GARCH models against each other. It was shown that the t-statistic based on a linear weighting of the competing conditional variances is asymptotically normal and has asymptotic power of unity. The corresponding power function was also derived. In addition, the non-nested tests based on the weighting schemes in a power family were evaluated. The asymptotic distributions and power functions of the corresponding t-statistics were presented. It was demonstrated that the t-statistic based on a linear weighting of the competing conditional variances is robust and yields the maximum power in the power family in finite samples. Thus, the t-statistic based on the linear weighting scheme is recommended for practical purposes.

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