A New Definition of Soil Hydraulic Similarity for Physically Based Modelling of Soil Water Transport on Watershed Scale

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Abstract Watersheds on a regional scale exhibit a considerable variability of soil hydraulic properties. Due to the lack of relevant observation records physically based hydrological modelling on this scale has to account for this variability. Despite comprehensive research efforts in this field yet no physically sound methodology is available that allows for an upscaling of soil hydraulic properties using data of various origin (e.g. lab measurements, soil survey data). In order to contribute to this situation we extend the current definition of soil hydraulic similarity. The concept consists of a physically based model together with a standardized experimental setup. The model describes the hydraulic variability of soils by the means of two independend, measurable parameters namely the porosity and the density of the soils and permits the use of commonly used soil models e.g. VAN GENUCHTEN/MUALEM. Within classes of soils of similar hydraulic behaviour the variability is described by scaling the two similarity parameters. Thus, our concept allows the introduction of "internal" soil hydraulic heterogeneity into physically based soil moisture transport models by the means of the two similarity parameters. The methodology requires measurements of soil hydraulic functions (water retention characteristics and conductivity) but it also uses other accessable data (soil maps, soil survey data).

1 Introduction

Physically based deterministic models that describe soil water dynamics on the basis of the RICHARDS equation require the soil hydraulic functions ($h(\theta)$; $K(\theta)$) as input parameters. The exact determination of these characteristics for a watershed is an impossible task. Therefore, modelling soil water transport on field or catchment scale is (due to the lack of a physically based upscaling procedures) based on homogenized or "effective" (e. g. SMITH and DIEKKRÜGER[1993]) parameters that are often derived from few small (100ccm) soil samples. In order to contribute to an improvement of this situation we developed an integrated strategy that consists out of three main parts. 1. a stochastic soil water transport model that allows to incorporate two independend stochastic parameters directly within the calculations. 2. We derive the soil hydraulic properties from a combination of physical and numerical experiments by a standardized experimental setup. To estimate "meaningfull" data with respect to the envisaged scale we use sample sizes up to several m^3. 3. Several applications of the concept of geometric similarity from MILLER & MILLER [1956] proofed that a single scaling factor describes the natural heterogeneity of soils unsufficiently (e.g. JURY and ROTH [1990]). For this reason we extended the concept of geometric similarity by a second independend and measurable scaling factor. The results are classes of soils with a similar hydraulic behaviour. The variability within one class is described by scaling the two similarity parameters.

2 The Soil Model

BUCKINGHAM [1907] and RICHARDS [1931] extended the law of DARCY for unsteady flow in unsaturated conditions. The application of the RICHARDS equation requires knowledge of the soil hydraulic properties of unsaturated soils namely the matrixpotential (h), the volumetric water contents (θ) and the hydraulic conductivity (K). The interdependencies of these properties are mainly controlled by the pore characteristics of the soil. Unfortunately the pores of a natural soil are irregularely shaped, tortuous and interconnected. Therefore, a physically exact description of all properties governing flow in such a pore system is impossible. Most authors e.g. BURDINE [1953]; BROOKS & COREY [1966], MUALEM[1976] represent the real soil pore system by bundles of cylindrical tubes with different diameters. This leads to the pore- volume distribution function $U(d)$ (see figure 1), with d = effective pore diameter.
$U(d)$ is derived from measured soil water retention data $h(d)$, $\theta(d)$

$$\theta = \theta_r + U(d) \tag{1}$$

$$h = \frac{2a^2}{d} \tag{2}$$

and yield the effective (water filled) pore diameters which are required for the calculation of the hydraulic conductivity ($K(\theta)$).

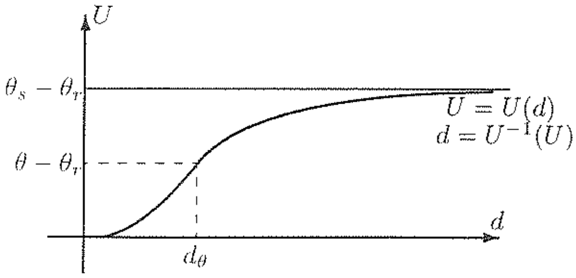

fig. 1

This leads to different conductivity models with respect to the different approaches to incorporate the tortuosity of the natural pore system.

– Cylindrical tubes:

$$K(\theta) = k \int_0^{\theta-\theta_r} d^2 \mathrm{d}U(d) \tag{3}$$

– BURDINE:

$$K(\theta) = k' \left(U\left(d_\theta\right)\right)^2 \int_0^{\theta-\theta_r} d^2 \mathrm{d}U(d) \tag{4}$$

– MUALEM:

$$K(\theta) = k'' \left(U\left(d_\theta\right)\right)^n \left(\int_0^{\theta-\theta_r} d \mathrm{d}U(d)\right)^2 \tag{5}$$

We likewise consider the the real pore system as bundles of cylindrical tubes with different diameters. However, in our approach $U(d)$ is not only used for the evaluation of pore diameters by fitting $U(d)$ to measured data. For providing the possibility of including more "hard" information in this kind of model studies our concept introduces at this stage two physical parameters which can be measured rather easily namely δ representing the dry bulk density of the soil and λ the porosity of the soil.

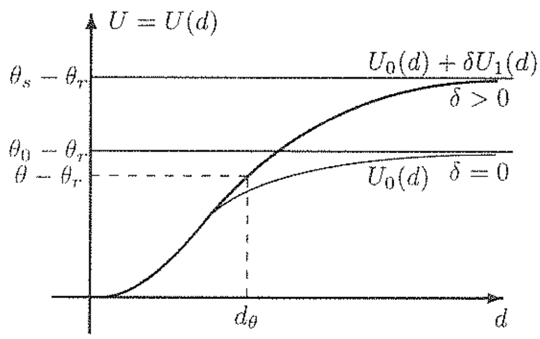

fig. 2

3 The extended concept of soil hydraulic similarity

The concept of (geometric) similarity of MILLER & MILLER[1956] employs a similarity parameter λ to the pore size distribution

$$U(d) = U_0\left(\frac{d}{\lambda}\right). \tag{6}$$

A consequence of equation (6)are the well known transformation properties

$$d = d_0 \cdot \lambda, \quad \theta = \theta_0, \quad h = h_0/\lambda, \quad K = K_0 \cdot \lambda^2. \tag{7}$$

Extending the idea of strict geometrical similarity (congruence) to a bivariate concept of "hydraulic similarity" we introduce a variable pore volume depending on the bulk density which is represented by a compression number $\delta \geq 0$. This leads to the new similarity concept (cf. fig. 2)

$$U(d) = U_0\left(\frac{d}{\lambda}\right) + \delta U_1\left(\frac{d}{\lambda}\right) \tag{8}$$

Equation (8) reflects the consideration that changes of the bulk density δ of a soil effect the macroporosity and not the microporosity of a soil.

Instead of (7) we get more complex transformation properties derived from (1–5) which can be resolved numerically.

The decomposition of the moisture content to the incompressible (θ_0) and the compressible part (θ_1) of the pore volume results from

$$\begin{aligned}
\theta &= \theta_0 + \delta\theta_1 \\
\theta_0 &= U_0\left(\frac{d}{\lambda}\right) + \theta_r \\
\theta_1 &= U_1\left(\frac{d}{\lambda}\right)
\end{aligned}$$

If δ is a small parameter an asymptotic expansion of transformation properties reads

$$\begin{aligned}
d &\approx \lambda\left[d_0 - \delta\frac{U_1(d_0)}{U_0'(d_0)}\right] && (9) \\
h &\approx \frac{h_0}{\lambda}\left[1 + \delta \cdot d_0\frac{U_1(d_0)}{U_0'(d_0)}\right] && (10) \\
K &= \lambda^2\left[K_0 + \delta K_1\right] && (11)
\end{aligned}$$

where for the capillarity model it yields

$$\begin{aligned}
K_0 &= k \cdot \int_0^{\theta_0-\theta_r} d^2 \mathrm{d}U_0\left(d\right) \\
K_1 &= k \cdot \int_0^{\theta_1} d^2 \mathrm{d}U_1\left(d\right)
\end{aligned}$$

Using the different conductivity models leads to:

$$\begin{aligned}
K_0 &= k'(U_0(d))^2 \cdot \int_0^{\theta_0-\theta_r} d^2 \mathrm{d}U_0\left(d\right) \\
K_1 &= k'(U_0(d))^2 \cdot \int_0^{\theta_1} d^2 \mathrm{d}U_1\left(d\right)
\end{aligned}$$

or

$$\begin{aligned} K_0 &= k''(U_0(d))^n \cdot \int_0^{\theta_0-\theta_r} d^2 \mathrm{d}U_0\,(d) \\ K_1 &= k''(U_0(d))^n \cdot \int_0^{\theta_1} d^2 \mathrm{d}U_1\,(d) \end{aligned}$$

respectively. With respect to formulae (1–5) our hydraulic similarity concept requires the determination of the separate pore distribution functions $U_0(d)$ and $U_1(d)$ i.e. the form parameters describing the shape of these functional relationships. Using a parametrisation of the, e.g., VAN GENUCHTEN type leads to 6 parameters

$$U_0(d) = p_1 \left(1 + (p_2 d)^{-p_3}\right)^{1/p_3 - 1} \quad (12)$$

$$U_1(d) = p_4 \left(1 + (p_5 d)^{-p_6}\right)^{1/p_6 - 1} \quad (13)$$

From the experimental data the eight parameters θ_r, k_0 and $p_1, \ldots, p_6$ are evaluated for the reference soil representating the whole class of hydraulically similar soils. For each individual soil within this class only the two similarity parameters λ_j and δ_j have to be determined in order to account for the natural variability.

4 First statistical analysis

A first statistical analysis using data from the Las Cruces trench site experiment in New Mexico (WIERENGA et al.,[1989]) and soil hydraulic data from WENDROTH [1996] from several german experimental sites was performed in order to check the suitability of a principle feature of the proposed hydraulic similarity concept. This refers to answer the question if the heterogenity of the datasets can principally be described by two stochastically independend parameters. For this purpose we conducted a principal component analysis (PCA). The PCA investigates the distribution of the measured soil hydraulic properties with respect to the the variation of a similarity parameter. We employed the concept of MILLER & MILLER (geometric similarity) together with the VAN GENUCHTEN soil model. We than classified the different soil horizons from the New-Mexico data into classes of hydraulically similar soils. The data from WENDROTH were also classified into different types of soil. Each soil type represents one class of hydraulically similar soils. The results of the PCA conducted with these data sets are shown in table 1 & 2.

	1. PC	2. PC	Σ
Gley			
$K(h)$	100	0	100
$\theta(h)$	99.5	0.4	99.9
$\theta\|K$	100	0	100
Pelosol			
$K(h)$	99.1	0.9	100
$\theta(h)$	92.2	6.5	98.7
$\theta\|K$	99.1	0.9	100
Pseudo-gley			
$K(h)$	100	0	100
$\theta(h)$	89.1	9.4	98.5
$\theta\|K$	100	0	100
Braun-erde			
$K(h)$	99.6	0.4	100
$\theta(h)$	94.5	4.5	99.0
$\theta\|K$	99.6	0.4	100
Para-braun-erde			
$K(h)$	97.2	2.4	99.6
$\theta(h)$	65.7	29.4	95.1
$\theta\|K$	97.2	2.4	99.6

table 1 (Wendroth)

horizon	1. PC	2. PC	Σ	1. PC	2. PC	Σ
1						
$\theta(h)$	70.7	18.3	89.0	76.6	19.1	95.8
$K(h)$	96.3	3.7	99.9	96.3	3.7	99.9
$\theta\|K$	98.9	1.1	100	98.9	1.1	100
5						
$\theta(h)$	75.1	15.4	90.6	84.1	10.6	94.7
$K(h)$	92.6	7.2	99.8	92.6	7.2	99.8
$\theta\|K$	95.4	4.5	99.8	95.4	4.5	99.8
6						
$\theta(h)$	83.2	7.9	91.2	90.0	6.0	96.0
$K(h)$	94.4	5.1	99.4	94.4	5.1	99.4
$\theta\|K$	95.1	4.5	99.6	95.1	4.5	99.6
7						
$\theta(h)$	76.2	12.7	88.9	88.5	8.3	96.8
$K(h)$	93.9	6.0	99.9	93.9	6.0	99.9
$\theta\|K$	95.5	4.4	99.9	95.5	4.4	99.9
8						
$\theta(h)$	67.5	23.5	90.9	75.7	19.8	95.5
$K(h)$	98.8	1.2	100	98.8	1.2	100
$\theta\|K$	99.1	0.8	100	99.1	0.8	100
9						
$\theta(h)$	81.7	7.8	89.5	86.9	8.1	95.0
$K(h)$	98.0	2.0	100	98.0	2.0	100
$\theta\|K$	98.2	1.7	100	98.2	1.7	100

table 2 (New-Mexico)

Wendroth		New-Mexico	
type of soil	korr(λ_1, λ_2)	horizon	corr(λ_1, λ_2)
Braunerde	0.8494	1	0.5920
Pelosol	0.6870	5	0.2556
Pseudogley	0.9380	6	0.5942
Gley-Pseudogley	0.9065	7	0.4156
Parabraunerde	0.9003	8	0.4504
		9	0.5089

table 3: correlation of the scaling parameters

A result of 90% or more in the 1^{st} principle component reveals that for both data sets 90% or more of the variability can be removed by one scaling factor. For the data from WENDROTH the variabililty of $\theta(h)$ from the Pseudogley and Parabraunerde cannot be adequatily represented by one scaling factor only. For the New Mexico dataset this is the case for $\theta(h)$ from all horizons. However, introducing the 2^{nd} principle component yields for all datasets a result better than 94%. Although the 2^{nd} principle component does not allow for a physical interpretation it might be concluded that the introduction of a second independend similarity parameter still improves the description of similar soils significantly.

For a further investigation of the similarity hypothesis we calculated similarity factors (λ_1 and λ_2) from both models $h(\theta)$ as well as for $K(\theta)$ and analysed afterwards the correlation between λ_1 and λ_2. The results are shown in table 3.

If the Miller assumption of geometric similarity was sufficient to explain the differences between soils a high correlation between the two factors should be obtained. This is for the New Mexico dataset not the case. Therefore, it can be concluded that one scaling factor is not sufficient to describe the heterogeneity incorporated within this dataset.

5 Experimental Setup

The experimental strategy aims towards evaluation of data, which allow a unique characterisation of soil hydraulic properties.

5.1 Determination of a characteristic standard volume (CSV)

Any similarity approach firstly requires the definition of a reference soil. Commonly, this is performed using soil hydraulic properties obtained from small samples. This procedure ignores the existence of soil dependent different correlation lengths and seems not suitable for defining a reference soil. To our understanding a reference soil should comprise the typical behaviour which incorporates flow in macropores an preferential flow paths. Therefore, our similarity concept employs a reference soil with a finite volume, the Characteristic Standard Volume (CSV) which extends the REV definition (BEAR [1972]) with respect to similarity considerations. Contrary to the REV, the CSV has a soil specific volume which has to be determined by an adequate experiment. For this purpose we conduct dye tracer experiments on a scale of 10m to 10m to make the hydraulically relevant structures of the soil visable. Counting of visable macropores and preferential flow paths allows for the statistical estimation of the characteristic standard volume. The CSV is the sample volume which incorporates for one class of similar soils the characteristic internal hydraulically relevant heterogeneity which can be represented by effective parameters. The CSV will be different for each class of similar soils.

5.2 Laboratory experiments

In the second step we take an undisturbed sample with the volume of the CSV (up to several m^3) and conduct inflow- outflow as well as multistep outflow and evaporation experiments with the same sample. Tensiometer and TDR measurements at different heights during these experiments are recorded. The results are datasets of different stationary as well as instationary flow regimes due to the different boundary conditions. Because changing flow regimes are more the rule than the exeption under natural conditions the different datasets allow for the determination of representative model parameters (step 3).

5.3 Identification of soil models and parameter estimation

In the third step we conduct numerical experiments for the identification of a suitable soil model. The main criterion is the identificability of unambigeous model parameters.

Employing a physically based model together with a multiobjective optimization procedure allows for the inverse determination of the set of model parameters which fits best the outcome of the different experiments.

5.4 Identification of similarity parameters

In the last step we estimate similarity parameters and soil hydraulic properties for the whole model domain using results from soil survey and soil maps. Soil maps and soil survey data allow for the estimation of the porosity and the bulk density for various points within the relevant domain of similarity. The soil hydraulic functions for these data points can than be derived using our similarity model and the soil hydraulic properties derived in step two and three. Finally we get a whole set of similarity parameters which describe the hydraulically relevant heterogenity for the model domain and which serve as input parameters for the stochastic soil water transport model.

6 Summary and conclusions

A new physically based concept of soil hydraulic similarity was presented. It describes the hydraulic variability of similar soils by the stochastic variation of two independent, physical parameters, thus providing a more flexible alternative to the concept of strict geometrical similarity of MILLER and MILLER [1956] which only uses one similarity parameter. The proposed, rather easily measurable parameters, namely the porosity and the density of the soils, were introduced by the means of a sound, physically consistent development originating from the widely approved model of a bundle of cylindrical tubes with different diameters (BURDINE[1957], BROOKS & COREY[1966], MUALEM[1976]). Common soil models (e.g. VAN GENUCHTEN/ MUALEM) can thus be used for characterising the soil hydraulic properties. Introducing a new definition for the reference soil with a finite volume, the Characteristic Standard Volume (CSV), was shown to extend the REV definition with respect to similarity considerations. It thus takes into account soil typical phenomena like flow in macropores and preferential flow paths, avoiding the commonly used small samples which ignore the existence of soil dependent different correlation lengths. Contrary to the REV, the CSV has a soil specific volume, determined by a dye tracer experiment via visualisation of hydraulically relevant structures of the soil and the statistical analysis of visible macropores and preferential flow paths. A statistical analysis of two comprehensive data sets confirmed the adequacy of employing two stochastically independent similarity parameters. Due the rather easily measurable similarity parameters, our methodology can provide a basis for upscaling soil hydraulic properties using data of various origin, e.g. lab measurements together with soil survey data.

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