An Exact Test for the Choice of the Combination of Changes and Relative Changes in Economic Forecast Models

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Abstract We have derived an exact test for the parameter vector of the Box-Cox difference transformation in linear regression models. By utilizing Taylor series approximations this reduces to a choice between two regression equations. The test statistic which has an exact *F*-distribution can be easily calculated from these two regressions by least squares algorithm. Monte Carlo results have demonstrated that our proposed procedure is generally more capable than the likelihood approach in stating the correct size of the test, yet it is equally powerful to the latter in rejecting false null hypotheses. It is therefore a simple and ready statistical procedure for assessing the suitable choice of the combination of the changes or relative changes in econometric forecast models, thereby allowing more flexible and appropriate economic relations be formulated and their validity be tested.

1. Introduction

Econometric models are often formulated in terms of some functional form of the variable which may be generalized by the Box-Cox transformation:

$$(1.1)\quad y_t^{(\lambda)} = \begin{cases} \dfrac{y_t^{\lambda}-1}{\lambda}, & \lambda \neq 0 \\ \ln y_t, & \lambda = 0 \end{cases}, \qquad t = 1, \ldots, T$$

where $\{y_t\}$ is a positive series of an economic variable and λ is a Box-Cox transformation parameter. Typically many econometric applications examine not the level of the variable, but its change per time period. Examples include the St. Louis equation due to Anderson et al. [1970], the money demand model by Hafer et al. [1980] and the consumer index of Colclough et al. [1982]. A discrete approximation to the time derivative of $y_t^{(\lambda)}$ is given by Layson et al. [1984] as

$$(1.2)\qquad \Delta y_t^{(\lambda)} = y_{t-1}^{\lambda-1}\Delta y_t, \quad t = 1, \ldots, T,$$

where $\Delta y_t = y_t - y_{t-1}$ denotes the change of first difference. The usefulness of (1.2) is that $\lambda=1$ yields the first difference while $\lambda=0$ gives the relative or percentage change. This is termed the Box-Cox difference transformation by Seaks et al. [1990]. Both the first difference and the percentage change of an economic variable may therefore be generalized by $\Delta y_t^{(\lambda)}$. Thus a regression model which utilizes both the first difference and the percentage change can be represented by

$$(1.3)\qquad \begin{aligned} \Delta y_t^{(\lambda_1)} &= \alpha_1 x_{1t} + \cdots + \alpha_p x_{pt} \\ &\quad + \Delta z_{2t}^{(\lambda_2)}\beta_2 + \cdots + \Delta z_{qt}^{(\lambda_q)}\beta_q + \varepsilon_t \end{aligned}$$

where λ_i ($i=1, \ldots, q$) are Box-Cox difference transformation parameters, x_{jt} ($j=1, 2, \ldots, p$) are observations on the p independent variables which are not transformed, $\Delta y_t^{(\lambda_1)}$ is defined as in (1.2), $\Delta z_{it}^{(\lambda_i)}$, ($i=2, \ldots, q$) are similarly defined, α_j ($j=1, \ldots, p$) and β_i ($i=2, \ldots, q$) are regression coefficients, and ε_t is the error term. We may regard (1.3) as a general model in which the transformed parameters permit more general forms than first differences or percentage changes though $\lambda=0$ or 1 has a straightforward and simple interpretation.

Unfortunately model selection is a very difficult task in practice. As Dhrymes et al. [1972, p294] have pointed out that economic theory typically provides little guidance as to the proper functional form appropriate to the specification of the economic relationships. The choice of suitable functional forms in econometric applications thus relies heavily upon established statistical procedures.

In this paper, we shall develop an Andrews [1971] type procedure for testing the choice of the set of Box-Cox difference transformation parameters in (1.3), allowing different transformation parameters for different variables. Its statistical methodology follows the line of Milliken et al. [1970] and is based on the *F*-distribution. Section 2 reviews existing single transformation parameter procedures. In section 3 we shall develop the testing procedure which reduces to a choice between two regression equations. The alternative regression, though of no economic interest, is easy to understand. Section 4 outlines the steps how the test statistic can be computed using a standard statistical package. In section 5 the new procedure is compared with the likelihood ratio approach by

Monte Carlo models. Section 6 presents the conclusions of this paper.

2. Single Transformation Parameter

Existing procedures include the likelihood ratio (LR) test and variants of the Lagrange multiplier (LM) tests. They assume the same transformation parameter be used on both the dependent and independent variables. The procedure of Layson et al. [1984] is primarily a LR test. It requires a search for an optimal solution of the unrestricted model which in practice can be completed by a sequence of regressions on transformed data with various values of the transformation parameter over a grid. Though the first difference of the dependent variable and percentage change of the regressor are used in the poverty and growth model of Thornton et al. [1978], Layson et al. [1984] have only applied their procedure to test the latter choice but not the use of the first difference of the dependent variable. The search could be very expensive if not impossible when different parameters of transformation are allowed. Those of Coulson et al. [1985] and of Park [1991] are different variants of LM tests, constructed from rather complicated auxiliary regressions in which the regressors involve either the time derivatives of the transformed variables or the derivatives of the unrestricted log-likelihood function with respect to the individual variables. The regressions are artificially constructed to generate tests for misspecification and their relations to the hypothesis to be tested are not obvious. The computation involved is by no means simple or straightforward. Further, the LR and LM procedures are asymptotic and therefore are only approximate in small and moderate sample situations.

3. The Test Procedure

Using matrix notation we may express (1.3) as

(3.1) $$\Delta \mathbf{y}^{(\lambda_1)} = \mathbf{X}\alpha + \Delta \mathbf{Z}^{(\lambda_2)}\beta + \varepsilon$$

where $\lambda_2=(\lambda_2\ \lambda_3\ \dots\ \lambda_q)'$, $\Delta \mathbf{y}^{(\lambda_1)}$ is the Tx1 vector of transformed dependent variables, $\mathbf{X}$ is the Txp matrix of original independent variables, $\Delta \mathbf{Z}^{(\lambda_2)} = (\Delta \mathbf{z}_2^{(\lambda_2)}\ \Delta \mathbf{z}_3^{(\lambda_3)}\ \dots\ \Delta \mathbf{z}_q^{(\lambda_q)})$ is the $T\times(q\text{-}1)$ matrix of transformed independent variables, $\alpha=(\alpha_1\ \alpha_2\ \dots\ \alpha_p)'$ is a $p\times 1$ and $\beta=(\beta_2\ \beta_3\ \dots\ \beta_q)'$ is a $(q\text{-}1)\times 1$ vectors of coefficients, and ε is the $T\times 1$ vector of independently and normally distributed disturbances with zero mean and constant variance σ^2.

Expanding $\Delta \mathbf{y}^{(\lambda_1)}$ in Taylor series about a hypothetical value $\lambda_1^{(0)}$ yields

(3.2) $$\Delta \mathbf{y}^{(\lambda_1)} \approx \Delta \mathbf{y}^{(\lambda_1^{(0)})} + (\lambda_1 - \lambda_1^{(0)})\mathbf{w}(\lambda_1^{(0)})$$

where $\mathbf{w}(\lambda_1^{(0)})=\partial \Delta \mathbf{y}^{(\lambda_1)}/\partial\lambda_1$ evaluated at $\lambda_1=\lambda_1^{(0)}$.

It can be readily shown that $\mathbf{w}(\lambda_1^{(0)})$ has t-th element

(3.3) $$w_t(\lambda_1^{(0)}) = (\ln y_{t-1})\ \Delta y_t^{(\lambda_1^{(0)})}.$$

Similarly, we have

$$\Delta \mathbf{Z}^{(\lambda_2)} \approx \Delta \mathbf{Z}^{(\lambda_2^{(0)})} + \mathbf{V}(\lambda_2^{(0)})(\lambda_2 - \lambda_2^{(0)})$$

where $\lambda_2^{(0)} = (\lambda_2^{(0)}\ \lambda_3^{(0)}\ \cdots\ \lambda_q^{(0)})'$ is a hypothetical value of λ_2,

(3.4) $$\mathbf{V}(\lambda_2^{(0)}) = [\mathbf{v}_2(\lambda_2^{(0)})\ \mathbf{v}_3(\lambda_3^{(0)})\ \cdots\ \mathbf{v}_q(\lambda_q^{(0)})]$$

and $\mathbf{v}_i(\lambda_i^{(0)})=\partial \Delta \mathbf{z}^{(\lambda_i)}/\partial\lambda_i$ being evaluated at $\lambda_i=\lambda_i^{(0)}$, i=2, ..., q. The individual elements of $\mathbf{v}_i(\lambda_i^{(0)})$ are given similarly to (3.3). Substituting (3.2) and (3.4) into (3.1) and writing $\lambda_0 = (\lambda_1^{(0)}\ \lambda_2^{(0)}\ \cdots\ \lambda_q^{(0)})'$, $\gamma=\lambda-\lambda_0$ and $\mathbf{U}(\lambda_0, \beta)=[-\mathbf{w}(\lambda_1^{(0)})\vdots\mathbf{V}(\lambda_2^{(0)})\beta]$, we shall obtain

(3.5) $$\Delta \mathbf{y}^{(\lambda_1^{(0)})} = \mathbf{X}\alpha + \Delta \mathbf{Z}^{(\lambda_2^{(0)})}\beta + \mathbf{U}(\lambda_0,\beta)\gamma + \varepsilon.$$

Box [1980] calls the new explanatory variable $\mathbf{U}$ a *constructed variable*. The testing of the null hypothesis H_0: $\lambda=\lambda_0$ against the alternative H_1: $\lambda\neq\lambda_0$ has become a choice between (3.5) and the null model

(3.6) $$\Delta \mathbf{y}^{(\lambda_1^{(0)})} = \mathbf{X}\alpha + \Delta \mathbf{Z}^{(\lambda_2^{(0)})}\beta + \varepsilon.$$

Equivalently, we are testing H_0: $\gamma=\mathbf{0}$ against H_1: $\gamma\neq\mathbf{0}$ in model (3.5). To eliminate the dependence of $\mathbf{U}$ on ε in (3.5) the former will be replaced by its least squares estimate from the null model. Let $\hat{\alpha}$ and $\hat{\beta}$ be the least squares estimates of α and β in (3.6), the fitted values $\Delta\, \hat{y}_t^{(\lambda_1^{(0)})}$ of $\Delta\, y_t^{(\lambda_1^{(0)})}$ can be computed from (3.6) after replacing the regression coefficients by their least squares estimates, and the fitted values of $y_t^{(\lambda_1^{(0)})}$ can thus be determined recursively by

(3.7) $$\hat{y}_t^{(\lambda_1^{(0)})} = \Delta\, \hat{y}_t^{(\lambda_1^{(0)})} + y_{t-1}^{(\lambda_1^{(0)})},\quad t = 1, .., T.$$

The fitted value $\hat{y}_t$ of y_t can then be calculated using the inverse of the Box-Cox formula (1.1). Substituting $\Delta\, \hat{y}_t^{(\lambda_1^{(0)})}$ and $\ln \hat{y}_{t-1}$ into (3.3) we shall get $\hat{w}_t(\lambda_1^{(0)})$ and hence $\hat{\mathbf{w}}(\lambda_1^{(0)})$, the least squares estimate of $\mathbf{w}(\lambda_1^{(0)})$, and finally

(3.8) $$\hat{\mathbf{U}} = [-\hat{\mathbf{w}}(\lambda_1^{(0)})\vdots\mathbf{V}(\lambda_2^{(0)})\hat{\beta}]$$

upon substituting $\hat{\beta}$ in $\mathbf{U}(\lambda_0, \beta)$. Replacing $\mathbf{U}$ in (3.5) by $\hat{\mathbf{U}}$ gives

$$(3.9) \qquad \Delta \mathbf{y}^{(\lambda_1^{(0)})} = \mathbf{X}\alpha + \Delta \mathbf{Z}^{(\lambda_2^{(0)})}\beta + \hat{\mathbf{U}}\gamma + \varepsilon$$

which satisfies the standard conditions of ordinary least squares. The use of $\hat{\mathbf{U}}$ to replace the unobservable $\mathbf{U}$ has been put forward by Milliken et al. [1970] for two obvious reasons, namely, $\hat{\mathbf{U}}$ can be computed from the data and is independent of the disturbance. The same tactic has also been used in Andrews [1971]. Let S_0 and S_1 be the residual sums of squares by least squares on the regression equations (3.6) and (3.9) respectively. It follows from the results of Milliken et al. [1970] that the quantity

$$(3.10) \qquad F = \frac{(S_0 - S_1)/q}{S_1/(T-p-2q+1)}$$

will follow a F-distribution with q and T-p-$2q$+1 degrees of freedom. Hence F is a test of the hypothesis $\gamma=\mathbf{0}$ or $\lambda=\lambda_0$.

A detailed development of the above results can be found in Milliken et al. [1970] who have shown that F has a F-distribution when H_0: $\gamma = 0$ is true. The test is 'exact' in the sense that an 'exact' significance will be obtained from which 'exact' confidence limits may be calculated when H_0 is true. However, little is known about F when $\gamma \neq 0$. See Ward et al. [1952]. Andrews [1971] has pointed out that the precision in (3.9) may affect the efficiency of the test but it will not affect the 'exactness' of the distribution of the test statistic. The proposed procedure is therefore more capable of capturing the correct size of the test than any other asymptotic ones. For more discussion of other advantages of the proposed procedure over asymptotic ones; see Andrews [1971].

4. Computing Procedures

This section briefly outlines procedures for computing the test statistic F. The procedures described here assume the use of a statistical package. The steps are as follows:

1. Transform $\mathbf{y}$ and $\mathbf{Z}$ as in (1.2) to get $\Delta \mathbf{y}^{(\lambda_1^{(0)})}$ and $\Delta \mathbf{Z}^{(\lambda_2^{(0)})}$ and to form the regression equation (3.6).
2. Estimate equation (3.6) by least squares to obtain coefficient estimates $\hat{\alpha}$ and $\hat{\beta}$, and the error variance estimate s_0^2, say.
3. Compute $\Delta \mathbf{y}^{(\lambda_1^{(0)})} = \mathbf{X}\hat{\alpha} + \Delta \mathbf{Z}^{(\lambda_2^{(0)})}\hat{\beta}$, from which calculate $\hat{y}_t^{(\lambda_1^{(0)})}$ using the recursive relation (3.7) and $\hat{y}_t$ by the inverse Box-Cox formula.
4. Compute $\hat{w}_t(\lambda_1^{(0)})$ by (3.3) using computed values from step 3 and form the vector $\hat{\mathbf{w}}(\lambda_1^{(0)})$. Compute $\mathbf{V}(\lambda_2^{(0)})$ using a formula similar to (3.3). Augment - $\hat{\mathbf{w}}(\lambda_1^{(0)})$ and $\mathbf{V}(\lambda_2^{(0)})\hat{\beta}$ to form $\hat{\mathbf{U}}$ given in (3.8).
5. Form the regression (3.9) and estimate it by least squares to obtain error variance estimate s_1^2, say.
6. Finally calculate the test statistic F in (3.10) by putting S_0=(T-p-q+1) s_0^2 and S_1=(T-p-$2q$+1) s_1^2.

When the package such as RATS used has built-in testing procedure for the regression coefficient vector, Steps 5 and 6 would be combined to one of testing $\gamma=\mathbf{0}$ in the regression equation (3.9). The null model is only used to compute the fitted values $\hat{\mathbf{U}}$ of the alternative model.

5. The Monte Carlo

To evaluate our proposed F-test and to compare its performance with that of the LR procedure Monte Carlo models, coded in RATS, with combinations of first differences and percentage changes are performed. Sample sizes of 20, 30, 40, 60 and 100 are selected so that both the small sample and asymptotic properties can be studied. Each experiment involves 2000 replications. The four models studied are:

M00: $\%\Delta y_t = 0.01 + 0.9\ \%\Delta z_t + \varepsilon_t$, $\%\Delta z_t \sim U(0, 0.07)$, $\varepsilon_t \sim N(0, 0.015^2)$, $z_0 = 1.0$, $y_0 = 5.0$

M01: $\%\Delta y_t = 0.01 + 0.03\ \Delta z_t + \varepsilon_t$, $\Delta z_t \sim U(-0.5, 1.5)$, $\varepsilon_t \sim N(0, 0.015^2)$, $z_0 = 5.0$, $y_0 = 15.0$

M10: $\Delta y_t = 100 + 55\ \%\Delta z_t + \varepsilon_t$, $\%\Delta z_t \sim U(0, 0.07)$, $\varepsilon_t \sim N(0, 1)$, $z_0 = 10$, $y_0 = 100$

M11: $\Delta y_t = 10 + 2\ \Delta z_t + \varepsilon_t$, $\Delta z_t \sim U(0, 2)$, $\varepsilon_t \sim N(0, 1)$, $z_0 = 10$, $y_0 = 100$

The population R^2 are .593, .571, .553 and .571 respectively. M00 and M11 have been used by Seaks et al. [1990]. The models are tested under each of the null hypotheses: H_0: (λ_1=0, λ_2=0), (λ_1=0,λ_2=1), (λ_1=1,λ_2=0) and (λ_1=1,λ_2=1) in turn, at 1%, 5% and 10% significance levels. The empirical significance level is recorded, when the null hypothesis is true this is the Type I error and when the null is false this will be the power of the relevant test.

A slight modification of Layson et al. [1984] result gives the log likelihood function under normality:

$$l = -\frac{T}{2}[\ln(2\pi) + \ln\sigma^2] - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\varepsilon_t$$

$$+(\lambda_1 - 1)\sum_{t=1}^{T}\ln y_{t-1}.$$

The RATS procedure MAXIMIZE is used to compute the maximum likelihood estimate $\hat{\lambda}$ of λ and to calculate the LR test statistic

$$\chi^2 = 2[l(\hat{\lambda}) - l(\hat{\lambda}_0)] \sim \chi_q^2.$$

H_0 λ_1 λ_2		Sample Size 20	30	40	60	100
0 0	F	0.8	1.0	0.9	0.8	0.9
	LR	2.1	1.4	1.1	0.9	0.8
0 1	F	7.0	42.6	89.6	100	100
	LR	14.3	58.7	96.2	100	100
1 0	F	34.1	95.2	100	100	100
	LR	31.4	95.5	100	100	100
1 1	F	3.0	10.8	23.4	41.0	65.1
	LR	5.4	29.8	79.2	100	100

Table 1a: Size/Power at 1% Significance Level When True Model is (λ_1=0, λ_2=0)

H_0 λ_1 λ_2		Sample Size 20	30	40	60	100
0 0	F	5.0	5.6	4.9	5.0	5.0
	LR	4.8	4.3	3.8	3.7	4.5
0 1	F	23.3	70.6	97.5	100	100
	LR	28.1	76.0	99.0	100	100
1 0	F	60.7	98.7	100	100	100
	LR	52.0	98.7	100	100	100
1 1	F	12.4	27.0	44.1	64.6	79.6
	LR	12.9	50.3	91.2	100	100

Table 1b: Size/Power at 5% Significance Level When True Model is (λ_1=0, λ_2=0)

H_0 λ_1 λ_2		Sample Size 20	30	40	60	100
0 0	F	9.4	10.5	9.9	10.0	9.6
	LR	7.9	7.3	7.2	7.9	8.9
0 1	F	35.3	81.5	98.8	100	100
	LR	36.6	84.1	99.5	100	100
1 0	F	72.5	99.5	100	100	100
	LR	63.8	99.4	100	100	100
1 1	F	21.1	38.0	57.0	74.2	85.3
	LR	20.0	60.7	94.7	100	100

Table 1c: Size/Power at 10% Significance Level When True Model is (λ_1=0, λ_2=0)

While our F procedure took few seconds the LR procedure took several minutes to few hours to complete 2000 replications even though the true parameter values were used to start with. When initial values other than the parameter values were used as initial values the latter often located the optimal solution after several hundred iterations.

Tables 1a-1c give the size and power of the tests for model M00. The F-test appears generally to capture the correct size while the LR test overstates the Type I error at the 1% level in small samples and understate it at the 10% level for all sample sizes. It does not appear to give the correct size even for sample sizes as large as 100. The two procedures perform equally well under H_0: (λ_1=0, λ_2=1) and H_0: (λ_1=1, λ_2=0). They both lead to correct decision by rejecting the incorrect nulls H_0: (λ_1=0, λ_2=1) when T≥60 and H_0: (λ_1=1, λ_2=0) when T≥40 in all cases. When testing under H_0: (λ_1=1, λ_2=1) F appears less powerful than LR in rejecting the incorrect null.

H_0 λ_1 λ_2		Sample Size 20	30	40	60	100
0 0	F	10.8	52.2	85.8	98.9	100
	LR	12.4	55.9	99.2	100	100
0 1	F	0.8	1.0	1.0	0.9	0.9
	LR	1.6	1.3	1.6	0.8	1.0
1 0	F	29.2	94.0	100	100	100
	LR	34.8	96.0	100	100	100
1 1	F	3.1	26.0	67.3	98.2	100
	LR	3.6	38.6	100	100	100

Table 2a: Size/Power at 1% Significance Level When True Model is (λ_1=0, λ_2=1)

H_0 λ_1 λ_2		Sample Size 20	30	40	60	100
0 0	F	28.1	77.1	96.0	99.8	100
	LR	25.1	76.7	100	100	100
0 1	F	4.9	4.7	5.0	4.2	4.7
	LR	2.8	3.8	4.7	4.9	4.1
1 0	F	57.1	99.4	100	100	100
	LR	54.4	98.9	100	100	100
1 1	F	11.9	48.4	84.7	100	100
	LR	11.0	47.6	100	100	100

Table 2b: Size/Power at 5% Significance Level When True Model is (λ_1=0, λ_2=1)

H_0 λ_1 λ_2		Sample Size 20	30	40	60	100
0 0	F	42.1	86.9	98.6	99.9	100
	LR	34.9	98.4	100	100	100
0 1	F	9.5	9.9	9.7	9.0	9.9
	LR	7.9	7.3	8.0	6.9	7.8
1 0	F	71.1	99.8	100	100	100
	LR	66.7	99.4	100	100	100
1 1	F	20.7	61.3	91.3	100	100
	LR	18.1	58.6	100	100	100

Table 2c: Size/Power at 10% Significance Level When True Model is (λ_1=0, λ_2=1)

The size and power for model M01 are given in Tables 2a-2c. The F procedure is seen again to state the correct nominal size. On the other hand, LR tends to overstate the size at the 1% level in small samples and overstate it at the 5 and 10% levels for all sample sizes considered. The Type I error of LR by no means appear to converge to the nominal one even when the sample size is 100. When testing under incorrect hypotheses, both test procedures seem to perform equally well in rejecting the incorrect null. In some cases, a sample size of 40 is sufficiently large to lead to rejection of wrong nulls in 100% of the times.

H_0		$\lambda_1=1, \lambda_2=0$		$\lambda_1=1, \lambda_2=1$	
Test		F	LR	F	LR
	20	0.7	1.9	3.7	4.4
Sample	30	0.8	1.3	21.7	33.5
	40	1.0	1.6	61.8	78.5
Size	60	0.9	1.1	99.5	100
	100	0.9	1.0	100	100

Table 3a: Size/Size/Power at 1% Significance Level When True Model is ($\lambda_1=1$, $\lambda_2=0$)

H_0		$\lambda_1=1, \lambda_2=0$		$\lambda_1=1, \lambda_2=1$	
Test		F	LR	F	LR
	20	4.6	4.8	15.3	10.5
Sample	30	4.7	3.8	46.1	52.0
	40	4.7	4.8	83.7	90.4
Size	60	5.2	4.6	100	100
	100	5.1	4.5	100	100

Table 3b: Size/Power at 5% Significance Level When True Model is ($\lambda_1=1$, $\lambda_2=0$)

H_0		$\lambda_1=1, \lambda_2=0$		$\lambda_1=1, \lambda_2=1$	
Test		F	LR	F	LR
	20	10.2	7.8	25.5	16.5
Sample	30	9.6	7.3	58.8	61.3
	40	9.5	8.7	91.3	94.0
Size	60	10.3	7.9	100	100
	100	9.9	9.0	100	100

Table 3c: Size/Power at 10% Significance Level When True Model is ($\lambda_1=1$, $\lambda_2=0$)

For model M10 the two tests are equally powerful when in fact the null were false. When the null is H_0: ($\lambda_1=0$, $\lambda_2=0$) or H_0: ($\lambda_1=0$, $\lambda_2=1$) both of them reject the incorrect null in all cases and therefore results are not presented here. As visible from Tables 3a-3c they do not appear to outperform the other under the incorrect null H_0: ($\lambda_1=1$, $\lambda_2=1$). They only differ in stating the size under the true null. F generally gives correct sizes at all levels while LR overstates or understates it in most cases.

From Tables 4a-4c it can be seen that F yields Type I errors that are very close to the nominal ones. In contrast LR only produces close to stated sizes in large samples. For example, its type I error is at least two times the nominal value when $T \leq 40$ at the 1 % level. When the null is H_0: ($\lambda_1=1$, $\lambda_2=0$) both tests do not appear different in their ability of rejecting the null. When the null is H_0: ($\lambda_1=0$, $\lambda_2=0$) or H_0: ($\lambda_1=0$, $\lambda_2=1$), both approaches reject the wrong null in all of the cases.

H_0		$\lambda_1=1, \lambda_2=0$		$\lambda_1=1, \lambda_2=1$	
Test		F	LR	F	LR
	20	19.2	28.8	1.3	2.5
Sample	30	83.6	88.8	1.1	3.4
	40	97.0	98.4	1.1	2.0
Size	60	99.9	99.9	1.1	1.1
	100	100	100	1.0	1.0

Table 4a: Size/Power at 1% Significance Level When True Model is ($\lambda_1=1$, $\lambda_2=1$)

H_0		$\lambda_1=1, \lambda_2=0$		$\lambda_1=1, \lambda_2=1$	
Test		F	LR	F	LR
	20	42.7	44.7	5.7	6.9
Sample	30	95.8	96.3	5.3	7.7
	40	99.6	99.8	4.7	5.7
Size	60	99.9	100	5.2	5.2
	100	100	100	5.1	5.0

Table 4b: Size/Power at 5% Significance Level When True Model is ($\lambda_1=1$, $\lambda_2=1$)

H_0		$\lambda_1=1, \lambda_2=0$		$\lambda_1=1, \lambda_2=1$	
Test		F	LR	F	LR
	20	56.1	57.7	10.9	10.5
Sample	30	98.2	98.2	10.2	10.9
	40	99.9	99.9	10.9	9.3
Size	60	100	100	11.1	9.9
	100	100	100	10.1	9.4

Table 4c: Size/Power at 10% Significance Level When True Model is ($\lambda_1=1$, $\lambda_2=1$)

6. Conclusion

We have derived an exact test for the parameter vector of transformation in linear models. By utilizing Taylor series approximations this reduces to a choice between two regression equations. The foregoing analysis need not specifically assume the transformation parameters to be 0 or 1. Our proposed test is thus applicable to any other transformation parameter values though interpretation is straightforward when they are equal to 1 or 0. The test statistic which has an exact F-distribution can be easily calculated from these two regression equations by least squares algorithm. Monte Carlo results have demonstrated that our proposed procedure is generally more capable than the likelihood approach in stating the correct size of the test, yet it is equally powerful to the latter in rejecting false null hypotheses. It is therefore a simple and ready alternative to the likelihood ratio test for assessing the suitable choice of the combination of first differences and percentage changes in econometric forecast models, thereby allowing more flexible and appropriate economic relations be formulated and their validity be tested.

6. References

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