Comparison of models for estimation of groundwater recharge, using data from a deep weighing lysimeter

Tom Chapman
School of Civil and Environmental Engineering, The University of New South Wales
and
Robert Malone
USDA - Agricultural Research Service, Coshocton, Ohio

Abstract Most conceptual models of catchment hydrology include algorithms for estimation of recharge to groundwater, which is then routed to stream base flow and/or deep regional groundwater. In recent complex models, these algorithms take the form of solution of the Richards equation for vertical flow in the soil column above the water-table, while other models treat the transformation of excess soil water into base flow as a single process. However, the majority of models have an algorithm for simulation of the process of percolation to groundwater from an overlying soil water store, which may be conceived as a single store or a series combination of stores. The purpose of this study is to compare the performance of 11 of these simple models, selected from the literature, using an 11 year daily record from a weighing lysimeter 2.4 m deep at Coshocton, Ohio, supporting a pasture grass. The percolation data are measured directly, while the volume of water in the soil column is measured by weighing the monolith every 5 minutes. The quality of the data has been verified by comparing monthly measurements of soil water in the column with estimates from the daily water balance of the lysimeter. The data also provide an opportunity to assess the spatial variability of groundwater recharge, as the weighing lysimeter is located in a nest with three non-weighing lysimeters, in each of which the percolation is measured on a daily basis. The results of the study show that the recharge algorithm in the generalised SFB model is unequivocally superior to any of the other models tested, and that it achieves a satisfactory standard relative to the natural spatial variability of recharge. However, it does not succeed in matching large peaks in the recharge hydrograph.

1. INTRODUCTION

The development and performance of computer models of catchment hydrology has been widely reported in the literature in the last 30 years, with an ever-increasing complexity due to the rapidly expanding computer power available, as shown by the differences between two well known compilations [Fleming, 1975; Singh, 1995]. Most of these models can be regarded as conceptual, in that they reflect the modeller's view of how the hydrological processes in the real world can be reduced to a tractable set of algorithms.

While emphasis in these models is typically given to their ability to predict the hydrograph of surface runoff, most contain a simulation of the process of groundwater recharge, which may then be routed to base flow or regional groundwater having its discharge outside the catchment boundary. Recent models achieve this by solution of the Richards equation for vertical flow in the unsaturated zone, or by routing the flow through a large number of discrete layers. Both these approaches require measurement or estimation of a large number of parameters, and there appears to be little evidence that they attain better results than earlier models based on lumping the unsaturated zone into one or two storages. However, many algorithms have been used in these simple models, and the purpose of this paper is to compare their performance.

This can be achieved by making use of data from a weighing lysimeter, in which precipitation, surface runoff and percolation are measured directly, while actual evapotranspiration is determined by changes in weight of the soil mass. Thus the volume of water stored in the lysimeter can be calculated, usually on a daily basis, and used to study the algorithms which relate recharge to soil water storage or other measured variables.

2. DESCRIPTION OF MODELS

In the following definitions of the models, a common set of symbols has been adopted, but the authors' symbols have been retained where there is no conflict. The first 8 models to be described are based on the concept of a single soil water store of capacity SC, as illustrated in Figure 1. S is the current depth of soil water, and FC (for field capacity) is the depth which must be exceeded before any recharge R can occur. Thus each of these 8 models carries the condition

$$\text{If } S \le FC,\ R = 0 \qquad (1)$$

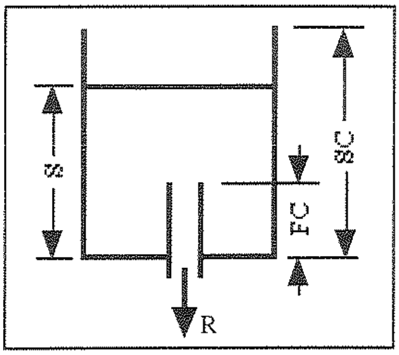

Figure 1. Definition diagram for Models 1 to 7.

2.1 Model 1 - simple threshold exceedance

This model was proposed by Dawdy and O'Donnell [1965] and in a description of the first Institute of Hydrology model [O'Connell et al., 1970]. The algorithm is

$$R = S - FC \qquad (2)$$

where FC is a parameter to be fitted.

2.2 Model 2 - fixed proportion of exceedance

While this model appears in the two-store Model 9, it does not appear to have been used in a one-store model. It is included here for completeness, and as a logical step between Models 1 and 3. The algorithm is

$$R = DPF * (S - FC) \qquad (3)$$

where DPF and FC are 2 parameters to be fitted.

2.3 Model 3 - quadratic function of storage

This was put forward in the SMAP model [Lopes et al., 1982]. The algorithm is

$$R = CREC * (S - FC) * S / SC \qquad (4)$$

where CREC and FC are 2 parameters to be fitted.

2.4 Model 4 - exponential function of storage

This was developed from measurements on a weighing lysimeter in Wisconsin [Black et al., 1969]. The algorithm is

$$R = C ** \{b\,(S - FC)\} \qquad (5)$$

where C, b and FC are 3 parameters to be fitted.

2.5 Model 5 - cubic function of storage

This model, reported in Fleming [1975, p.155], is due to Huggins and Monke [1967]. The algorithm is

$$R = I_c * \{1 - (SC - S)/(SC - FC)\} * * 3 \qquad (6)$$

where I_c and FC are 2 parameters to be fitted.

2.6 Model 6 - power function of storage

This model was proposed by Aston and Dunin [1977] on the basis of data from a weighing lysimeter containing a podzolic sandy loam. The algorithm is

$$R = C\,(S - FC) * * b \qquad (7)$$

where C, FC and b are 3 parameters to be fitted. Here FC is regarded as the capacity of the B-horizon (deeper soil) rather than the field capacity.

2.7 Model 7 - Proportion of net input to storage

This model is used in MODHYDROLOG [Chiew et al., 1993]. The algorithm is

$$R = CRAK * (P - Q - E) * S / SC \qquad (8)$$

where P, Q, and E are the precipitation, runoff and evapotranspiration respectively, and CRAK is a fitted parameter.

2.8 Model 8 - IHACRES model

The IHACRES model [Jakeman and Hornberger, 1993; Ye et al., 1997] does not model recharge as such, but uses a storage index, based on an exponentially decreasing weighting of precipitation and evapotranspiration, to predict the rainfall excess, which is then partitioned into quickflow and slowflow runoff. For this study, the storage index has been replaced by the ratio of soil water to soil water capacity, and the recharge calculated as the difference between the rainfall excess and the measured runoff Q. Thus

$$R = P * \{(S - FC)\} * * C - Q \qquad (9)$$

with 2 parameters FC and C, which correspond to the symbols 'l' and 'p' in Ye et al [1997].

The next two models are based on a subdivision of the soil water store into two storages, as shown in Figures 2 and 3.

2.9 Model 9 - Generalised SFB model (GSFB)

This model, described by Ye et al [1997], is a modified version of the SFB model [Boughton, 1984]. Using the definitions shown in Figure 2, the algorithm for infiltration from the upper to the lower store is

$$\begin{aligned} INF &= \min\{F, (US - FC)\} \text{ for } US > FC \\ &= 0 \text{ otherwise} \end{aligned} \qquad (10)$$

where F is the maximum infiltration rate.

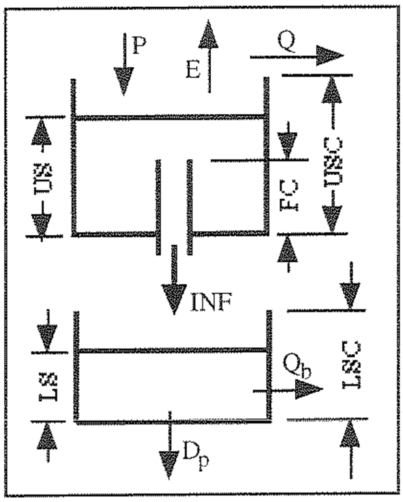

Figure 2. Definition diagram for Model 9.

Outflow from the lower store is divided into base flow Q_b and deep percolation D_p which are here added to obtain the total recharge as

$$R = DPF * (LS - BXSDR) \qquad (11)$$

There are 4 parameters: F, FC, DPF and BXSDR.

The water balance equation is used to adjust the soil water in each store after the calculation of INF and R.

Model 10 - ARBM model

This model was developed for analysis of catchment data from the Australian Representative Basins Program [AWRC, 1969; Chapman, 1970]. For a uniform soil, the redistribution RD (Figure 3) between the upper and lower soil stores is

$$RD = A_0 (US - A_1 * LS) \qquad (12)$$

while the recharge from the lower zone is

$$\begin{aligned} R &= LS - LSC \quad \text{for } LS > LSC \\ &= LDF * LS / LSC \text{ otherwise} \qquad (13) \end{aligned}$$

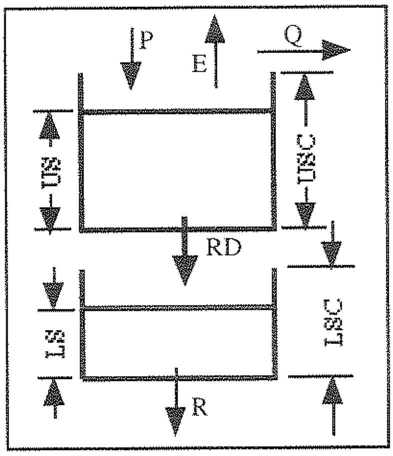

Figure 3. Definition diagram for Model 10.

There are 3 parameters: A_0 , A_1 and DPF.

3. DATA

3.1 The weighing lysimeters at Coshocton

The monolith lysimeters constructed in the late 1930's at the North Appalachian Experimental Watershed at Coshocton, Ohio, were an important part of a research plan designed to uncover basic laws related to agricultural hydrology [Harrold and Dreibelbis, 1958; Harrold and Dreibelbis, 1967]. Eleven lysimeters, four at each of two sites and three at a third site, were constructed around undisturbed soil blocks 4.3 m long and 1.9 m wide (measured horizontally), and 2.4 m deep. One lysimeter at each site was equipped with an automatic weighing device which records weight changes in the 65 t mass to an accuracy of 5 kg, equivalent to about 0.25 mm of water over its surface area. Although precipitation is measured on a nearby standard gauge, it is preferably estimated by weight changes, as on average the lysimeter surface catches about 10% more precipitation than the rain gauge [Harrold and Dreibelbis, 1967, pp. 14-16]. All lysimeters are equipped for daily measurement of surface runoff and percolation.

At the site selected for this study (Y101), the soil is a well drained Dekalb silt loam, and the agronomic practice has been to maintain a permanent grassland, though with some variations in species and fertiliser between lysimeters and over time.

The data available for this study were daily measurements of precipitation, runoff and percolation for the period 1987-1997, and calculated values of daily evapotranspiration for the weighing lysimeter (Y101D). Additional data used for checking the water balance were approximately monthly measurements of soil moisture. For the first 180 mm, these were determined gravimetrically, and for 150 mm intervals below that depth by means of a neutron probe, which was replaced by a dielectric sensor at the start of 1994.

3.2 Data quality

The soil moisture measurements at each depth were integrated to obtain a total depth of water in the profile at each date of measurement. Using the measured value on 6/1/87 as a starting point, the water balance was used to calculate the depth of water in the lysimeter for each day of the record. Figure 4 shows the comparison between the two estimates, for each day on which the soil moisture was measured directly. It will be noted that the agreement is excellent until 1994, after which there is an increasing discrepancy between the two sets. The reason for this has not been finally elucidated, and it was decided that only data from 1987 to 1993 should be used in this study. This illustrates the need for an independent check on the water balance, such as has been carried out on each lysimeter at Coshocton since their construction.

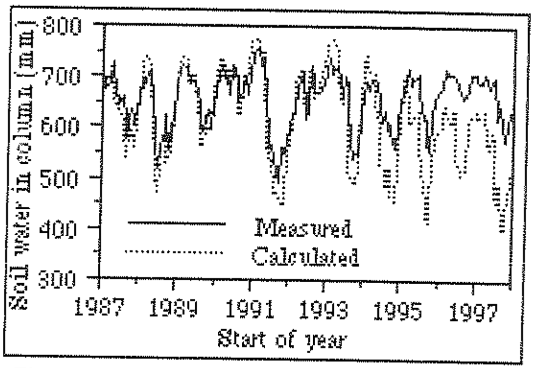

Figure 4. Total soil water in lysimeter Y101D, from measurements and the water balance.

3.3 Spatial variability of percolation

The presence of four adjacent lysimeters in a 'nest' at site Y101 provides for an illustration of the spatial variability of percolation. Figures 5 and 6 show the percolation from each lysimeter in 1990 (the wettest year in the study period - 1440 mm rainfall) and 1992 (a dry year - 950 mm - following the driest year - 754 mm). It will be seen that in the wet year the hydrographs all peak together and follow similar recessions, but in the dry year lysimeter D has no percolation until after day 200, while lysimeter C has none until the following year. These differences can be attributed to a much higher measured plant yield from C and D, implying correspondingly higher evapotranspiration. The contrast in scales between Figures 5 and 6 should also be noted: lysimeter D has a total of 507 mm percolation in 1990 and only 61 mm in 1992.

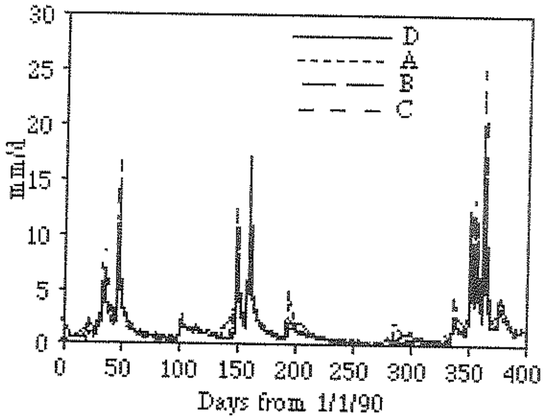

Figure 5. Percolation from Y101 lysimeters in 1990 (D is the weighing lysimeter)

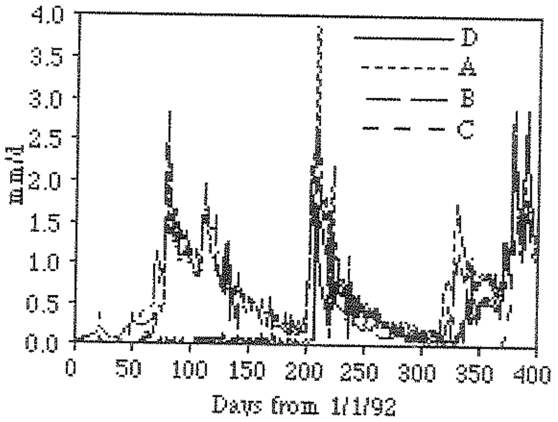

Figure 6. Percolation from Y101 in 1992

4. METHODOLOGY

4.1 Calibration and validation periods

The 5 year period of calendar years 1987 to 1991 was used as the calibration period for determining the best fit between modelled recharge and observed percolation, for each of the models. Each model was then tested, using the calibrated parameter values, over the 2 year period 1992-1993. The total storage capacity SC was taken as the maximum observed value in the record period, which was 760 mm. The initial storage US in the upper zone, required for Models 9 and 10, was taken as the measured storage in the top 1 m of the soil profile at the start of the relevant period.

4.2 Objective functions

Each model was calibrated by minimising an objective function OF, defined by

$$OF = \Sigma (R_i^{\,a} - \hat{R}_i^{\,a})^2 \qquad (14)$$

where R_i and $\hat{R}_i$ are the observed and modelled values of recharge. A value of 1 for 'a' defines the least squares criterion, which gives emphasis to matching peak values, while lower values of 'a' place more emphasis on matching lower flows. Following Chiew et al. [1993] and Ye et al. [1997], calibrations were made with a =1, 0.5 and 0.2. The simplex method [Nelder and Mead, 1965] was used to minimise the objective function.

4.3 Assessment criteria

The performance of each model was measured by three statistics A, B and E [Ye et al., 1997].

The mean absolute deviation A is defined by

$$A = \{ \Sigma \,| R_i - \hat{R}_i | \} / N \qquad (15)$$

where N is the number of observations in the period.

The bias B is defined by

$$B = \{\Sigma (R_i - \hat{R}_i)\}/N \qquad (16)$$

and the efficiency E by

$$E = 1 - \{\Sigma(R_i - \hat{R}_i)^2\} / \{\Sigma(R_i - \bar{R})^2\} \qquad (17)$$

where $\bar{R}$ is the mean recharge.

5. RESULTS

5.1 The calibration period

Results for the calibration period are given in Table 1 for a=1 and a=0.2; the results for a=0.5 were consistently intermediate between these.

Table 1. Assessment criteria for the calibration period 1987-1991 (4 dots indicate negative efficiency)

Model	a = 1			a = 0.2		
No.	A	B	E	A	B	E
1	0.82	0.81	....	5.01	-4.34	....
2	0.49	0.14	0.49	0.50	0.40	0.22
3	0.48	0.12	0.50	0.51	0.41	0.21
4	0.45	0.07	0.57	0.48	0.20	0.53
5	0.46	0.10	0.54	0.49	0.40	0.29
6	0.48	0.11	0.52	0.52	0.44	0.21
7	0.82	0.66	....	0.81	0.77	....
8	0.79	0.75	....	0.78	0.64	....
9	0.36	-0.01	0.60	0.35	-0.01	0.60
10	0.48	-0.02	0.57	0.50	-0.05	0.42
11	0.83	0.77	....	0.83	0.78	....

5.2 The validation period

Table 2 shows the results for the validation period in a similar format.

Table 2. Assessment criteria for the validation period 1992 - 1993

Model	a = 1			a = 0.2		
No.	A	B	E	A	B	E
1	0.63	0.25	....	9.12	-8.77	....
2	0.63	-0.37	....	0.40	0.03	0.42
3	0.63	-0.39	....	0.40	0.04	0.44
4	0.74	-0.61	....	0.78	-0.56	....
5	0.66	-0.49	....	0.36	0.00	0.59
6	0.65	-0.43	....	0.38	0.07	0.49
7	0.60	0.39	....	0.55	0.49	....
8	0.53	0.45	....	0.58	0.34	....
9	0.32	-0.03	0.63	0.32	-0.04	0.63
10	0.48	-0.18	0.23	0.67	-0.27	....
11	0.55	0.52	....	0.55	0.53	....

5.3 Assessment of a reasonable target of fit

The data from the non-weighing lysimeters have been used to suggest what might be reasonable targets for the assessment criteria, taking account of spatial variability of recharge. If the data for each non-weighing lysimeter are regarded as an outcome of a hypothetical model, comparison with the data from the weighing lysimeter provides for calculation of the same performance statistics, as shown in Table 3.

Table 3. Performance statistics for comparisons of recharge from each non-weighing lysimeter with the weighing lysimeter (D).

Lysi-	Calibration			Validation		
meter	A	B	E	A	B	E
A	0.41	-0.26	0.53	0.34	-0.25	0.55
B	0.31	0.00	0.79	0.39	-0.09	0.50
C	0.32	-0.11	0.62	0.27	0.23	0.73

6. DISCUSSION

It must be noted first that the procedure used in this study assumes equivalence between the hydrograph of percolation leaving the bottom of the lysimeter and the hydrograph of recharge to the water-table, i.e. that the water-table is roughly at the level of the bottom of the lysimeter (2.4 m). For a deeper water-table, there would be further time lags and longer durations of recharge events, but it seems likely that the relative performance of the algorithms would be unaffected by this, although the parameters would be changed.

Tables 1 and 2 show that Model 9 (GSFB) is unequivocally the best on all criteria, for both the calibration and validation periods. Of the single store models, Models 2 - 6 behave similarly in the calibration period, with a consistent positive bias and lower efficiency in predicting low flows (a = 0.2) than high flows. In contrast, these models fitted the low flows better in the validation period. Model 10 (ARBM) performed reasonably well in the calibration period, but failed in the validation period. All other models (1, 7, 8 and 11) failed to achieve positive efficiencies in both periods.

Figures 7 and 8 compare the performance of Model 9 for two selected periods. It will be noted that the model does not succeed in matching high peaks of the hydrograph.

An additional assessment criterion which may be considered is the stability of the values of the fitted parameters under different forms of the objective function. On this basis, Model 9 also performed better than any of the other models.

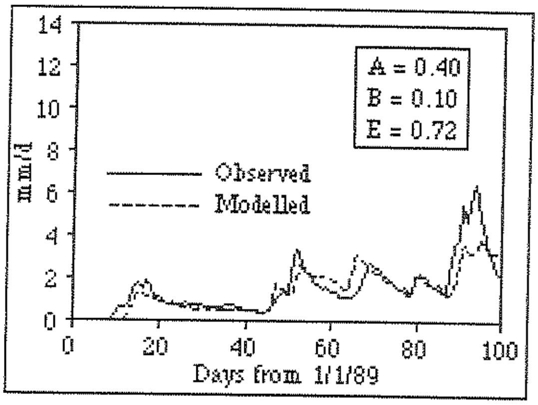

Figure 7. Performance of Model 9 in 1989.

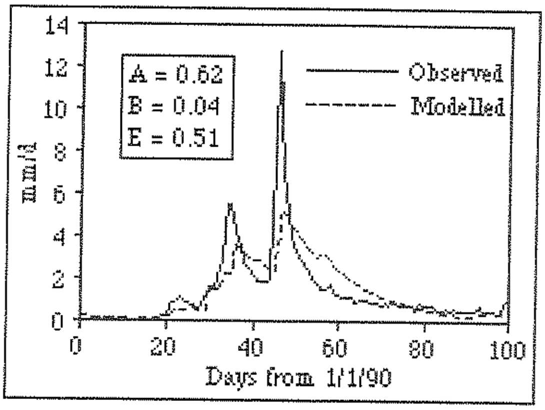

Figure 8. Performance of Model 9 in 1990.

Comparison of the assessment criteria for Model 9 with those shown in Table 3 suggest that the performance of this model is adequate relative to the natural spatial variability of recharge.

7. CONCLUSIONS

This study has shown that, for the data set used, the generalised SFB model, with 4 fitted parameters, performed better in predicting a recharge hydrograph than any of the other models considered. Taking into account the natural spatial variability of recharge, the assessment criteria met by this model reach a satisfactory target. However, there is scope for improvement in its ability to match sharp peaks in recharge.

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