Optimal Location of Tsunami Detectors

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Abstract The development of deep-sea detectors and communication technology has significantly enhanced the potential for tsunami detection, particularly in the Pacific Ocean. The detectors are located on the sea floor, sense long-wave, or tsunami, changes in water level, and communicate to wave-rider surface buoys. These buoys then communicate via satellite to a suitable receiving and management centre. The detectors and communications buoys are expensive to manufacture and maintain. NOAA has identified six suitable buoy sites which satisfy maintenance requirements. The objective of this study is to provide the optimal detector locations for up to six detectors. Two objective functions are developed, one for a Pacific-wide warning potential, and the second a more specific USA warning potential. The resulting non-linear 0-1 integer programming problems are relatively small and readily solved using enumeration techniques. The results indicate that three detectors in the Tsunami Warning System can achieve the maximum warning potential for both the Pacific region and also for the USA. The results also indicate the effects of response time by emergency services on the warning potential.

1. INTRODUCTION

All coastal regions in the Pacific have experienced growth in infrastructure and population density. Economic factors, recreational and aesthetic values are influencing this trend and are accelerating this development pattern

Unfortunately, the coastal zones of the Pacific Ocean are subject to tsunamigenic risk, both from near-field and far-field-generated tsunamis. In the past, tsunamis have caused considerable loss of life and destruction of property in the coastal areas (Satake [1992]). Various Tsunami Warning Systems have been developed and implemented to detect the generation of a tsunami and to warn of its approach toward coastal regions (Bernard [1997]). Currently operating Tsunami Warning Systems utilise seismographic observations to detect the occurrence of earthquakes and communication networks to communicate information rapidly

Not every earthquake generates a tsunami, a feature which can lead to false alarms which can be costly in dollar terms and also in terms of community reaction to "warnings" (Blackford and Kanamori [1995], Folger [1994]). The traditional instrument used to detect whether a tsunami has been generated is the tide gauge, usually located on a pier or wharf. While the tide gauge may be capable of detecting tsunamis, it was designed to measure tides. Their location is also in the worst part of the coastal zone where the tsunami is most energetic and infrastructure suffers most damage. Thus the tide gauges nearest to the generation zone of the tsunami are frequently quickly destroyed, and unable to detect and inform of the existence of the tsunami, let alone measure its characteristics.

Fortunately, tsunamis are far less damaging in the open ocean and may be detected by suitable sea-floor-mounted detectors (Eble and Gonzales [1991]). These detectors use acoustic coupling to communicate to the surface, to wave-rider buoys which can then communicate via satellite (Bernard [1997]). The detectors and wave-rider buoys are expensive to make and are currently limited in number. Some six possible sites have been selected, after consideration of regular NOAA ship passages and other maintenance and cost factors (Tsunami Hazard Mitigation Federal/State Working Group [1996], referred to as THM). The problem is to locate a small number of detectors at a selection of the possible communication-buoy locations, so as to give the maximum warning of the generation of a tsunami. This leads to an integer programming problem which can be tackled using standard enumeration techniques (Winston [1991], Braddock and Carmody [1999]). Once a warning has been issued by the Pacific Tsunami Warning Service, local emergency services need to react by activating warning procedures, getting the warning to the population, and in clearing people from the danger areas in time. The warning is only fully effective once this state has been attained.

The objective of this paper is to show how this optimal location problem can be formulated and solved. A small sample of the possible population centres at risk, and identified generation regions in the Pacific, have been selected. These are used to illustrate the methods. The paper seeks to optimise the population which can be warned - the objective function. The effects of response times by local emergency services are also estimated

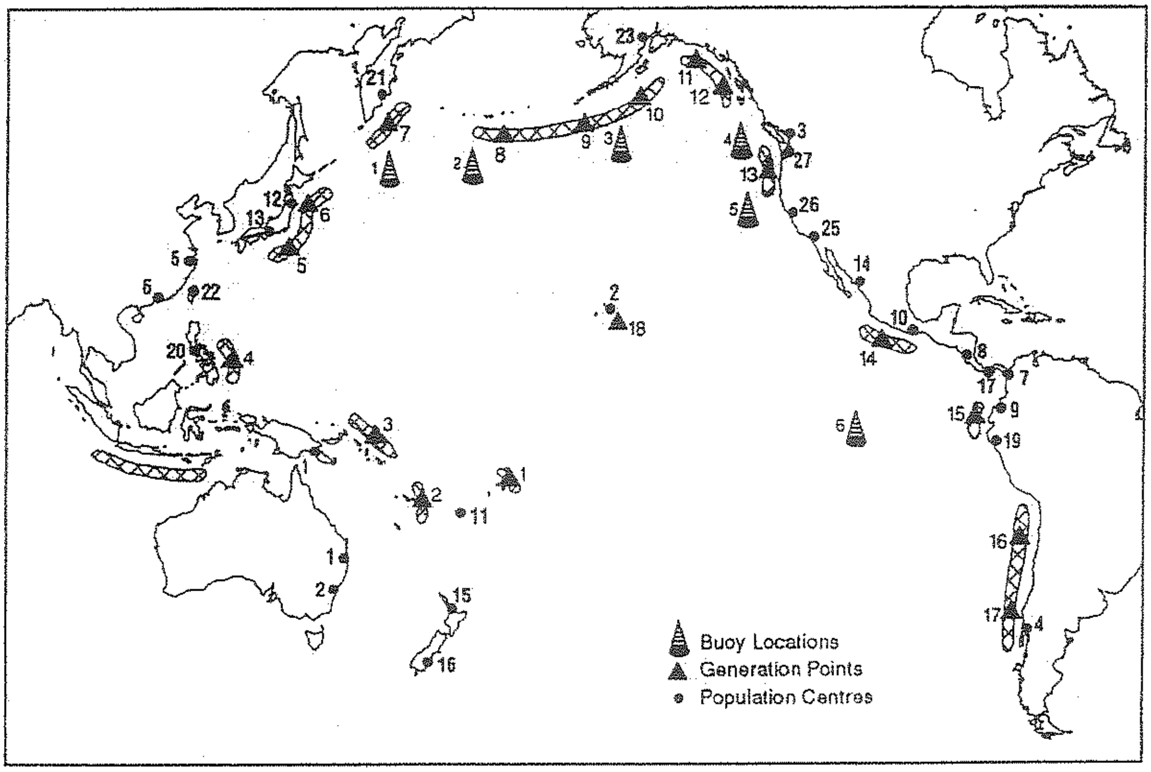

Figure 1. Population centres, generation points and buoy locations for the Pacific Ocean (after THM [1996])

2. THE TEST PROBLEM

The locations of the six NOAA wave-rider buoys are shown in Figure 1, after THM [1996]. The acoustic coupling from the detector to the communication buoy can operate reliably over distances of 40 km.

These positions are denoted by

$$\mathbf{B}_w = (\text{latitude, longitude})_w\,, \qquad w = 1, \ldots, W\,,$$

where, for this example, W = 6 and the sites are illustrated in Figure 1.

The common tsunamigenic regions of the Pacific Ocean are well known (Bernard [1997]) and are illustrated by the hatched areas on Figure 1. To reduce the computational burden of the integer-programming problem, representative point locations were selected to represent these generation regions, and these are given in Figure 1. These locations are denoted by

$$\mathbf{G}_u = (\text{latitude, longitude})_u\,, \qquad u = 1, \ldots, U\,,$$

where, for this example, U = 18. Some of the generation regions shown in Figure 1 are extensive and two generation points have been used to represent them. Thus $\mathbf{G}_{16}$ and $\mathbf{G}_{17}$ both lie in the tsunami generation region near Chile.

A precise estimate was not made of the number of potentially endangered people in each country bordering the Pacific Ocean. The population at risk depends on the magnitude of the tsunami and also on the relative positions of generation point and population. Instead, major representative population centres in the tsunami risk zones were selected to provide a general estimate of the population at risk. This data is designated by

$$\mathbb{P}_v = (\text{latitude, longitude, population})_v\,, \quad \text{for } v = 1, \ldots, V\,,$$

where, for this example, V = 27 and the location and population data are given in Table 1. Also let

$$p_v = \text{population numbers in } \mathbb{P}_v\,.$$

3. THE MODEL

Now introduce the integer variables y_w, $w = 1, \ldots, W$, where

$$y_w = \begin{cases} 0 \text{ if buoy location } B_W \text{ is not occupied by a detector} \\ 1 \text{ if buoy location } B_W \text{ is occupied by a detector} \end{cases}$$

The vector **y** of 0's and 1's then represents a particular deployment of detectors. The total number of detectors available, i.e. X, may be limited by capital costs or by maintenance costs. Thus

$$\sum_{w=1}^{W} y_w \leq X, \qquad (1)$$

expresses the constraint on the number of available detectors.

The travel time from a particular earthquake generation point, $\mathbf{G}_u$, to the nearest occupied buoy and detector is

$$t_u^* = \underset{w}{\text{Min}} \text{ (tsunami travel time from } G_u \text{ to } B_w \text{, with } y_w = 1), \qquad (2)$$

for $u = 1, \ldots, U$, where the minimum is taken over the deployment of detectors. Then t_u^* can be calculated for each generation point for a given deployment.

Each detector and communication buoy requires a time, t_d, for processing and signalling to confirm the generation and detection of the tsunami. For the purposes of this study, $t_d = 30$ minutes [Bernard, personal communication). Thus the population, $\mathbf{P}_v$, can be warned after the elapsed time

$$t_u^* + t_d + r_v, \qquad (3)$$

after the earthquake, where r_v, $v = 1, \ldots, V$ is the reaction warning time or period at population centre v. Let **r** be the vector of these emergency reaction times. This reaction time is the period required after receipt of the warning for emergency services to actually warn the population and move them to safety. This length of time is significant, and of the order of 1 to 4 hours, or even longer. Let

$$t_{u,v} = \text{tsunami travel time from } \mathbf{G}_u \text{ to } \mathbf{P}_v, \quad u = 1, \ldots, U, \quad v = 1, \ldots, V$$

Table 1. Co-ordinates and populations of major population centres for countries in the tsunami risk zones

	City/Country	Population (* 10^3)
1.	Brisbane, Australia	1 284*
2.	Sydney, Australia	3 828*
3.	Vancouver, Canada	1 380*
4.	Santiago, Chile	5 509*
5.	Fuzhou, China	2 498
6.	Shanghai, China	12 162*
7.	Buenaventura, Colombia	160
8.	San Jose, Costa Rica	670*
9.	Guayaquil, Ecuador	1 540
10.	San Salvador, El Salvador	920*
11.	Suva, Fiji	141*
12.	Hachinohe, Japan	1 588
13.	Tokyo, Japan	39 687
14.	Los Mochis, Mexico	1 175
15.	Auckland, New Zealand	850*
16.	Christchurch, New Zealand	482
17.	Panama, Panama	770*
18.	Port Moresby, Papua New Guinea	193*
19.	Lima, Peru	4 608
20.	Davao, Philippines	1 103
21.	Petropavlovsk-Kamchatskiy, Russian Federation	743
22.	Taipei, Taiwan	6 130
23.	Alaska, United States	560*
24.	Hawaii, United States	1 133*
25.	Los Angeles, United States	15 643*
26.	San Francisco, United States	6 253*
27.	Seattle, United States	2 559*

and an effective warning at $\mathbf{P}_v$ can be issued, provided that

$$T_{u,v} = t_u^* + t_d + r_v - t_{uv} \leq 0 \qquad (4)$$

for all u, v. Then let

$$e_{u,v}(y, r_v) = \begin{cases} 0, & T_{uv} > 0 \\ p_v, & T_{uv} \leq 0 \end{cases} \qquad (5)$$

for all u, v and $e_{u,v}(y, r_v)$ is a function of the deployment vector **y**. The total population across the Pacific region which can be effectively warned is then

$$E_u(y, r) = \sum_{v=1}^{V} e_{u,v}(y, r_v) \qquad (6)$$

for earthquakes generated at $\mathbf{G}_u$. Summing over the generation points and normalising, then the total warning potential is

$$E(\mathbf{y},\mathbf{r})=\left[\sum_{u=1}^{U}\sum_{v=1}^{V}e_{u,v}(\mathbf{y},r_v)\right]\bigg/\left(U\sum_{v=1}^{V}p_v\right). \quad (7)$$

The problem is to maximise the warning potential with respect to the deployment of the detectors, but subject to the constraint on availability. Formally,

$$\begin{array}{ll} \text{Maximise} & E(\mathbf{y},\mathbf{r}) \\ \text{subject to} & \displaystyle\sum_{w=1}^{W} y_w \le X, \\ & y_w = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}. \end{array} \quad (8)$$

4. METHODS OF SOLUTION

Formally, (8) constitutes a non-linear 0-1 integer program which depends heavily on the computation of travel times for the tsunami. Here, the problem is relatively small in that there are only six possible sites for the detectors and the numbers of population centres, (i.e. 27) and generation points (i.e. 18) are relatively small. The major computational burden in the solution lies in the estimation of the travel time from each generation point, $\mathbf{G}_u$, to each detector location, $\mathbf{B}_w$, and from each $\mathbf{G}_w$ to each population centre $\mathbf{P}_v$. In this study, Great Circle paths (Fox [1963]) were used to estimate distances between appropriate points, and an average speed of 600 km per hour was used to estimate travel times. Generally, the Pacific Ocean is relatively homogeneous in depth, and this averaging technique is sufficiently accurate. It will obviously be less accurate near coasts or areas with complicated bathymetry.

Using this averaging technique, two travel-time matrices, $D_{18,6}$ and $A_{18,27}$, were generated. The elements of $D_{18,6}$ are the travel times from each of the 18 generation points to each of the six buoy locations, while $A_{18,27}$ contains the travel times from the generation points to the population centres, i.e. these elements are $t_{u,v}$. These large matrices are not displayed here. The matrices D and A are then used to solve (8), given a value of X. With only six buoy locations to be occupied, (8) admits to only 64 possible deployments, and these are located at the corners of a six-dimensional cube. The optimal solutions were obtained by enumeration of all possible options (Winston [1991]).

The reaction or effective warning times of the emergency services was studied by varying $r_v = r^*$ (for all v), with $r^* = 0, 1, 2, 3$ and 4 hours. This effective warning time r^* was assumed to be constant across all population centres.

5. RESULTS

Solutions to (8) were obtained for $\mathbf{r} = 0$ for values of X = 1, 2, ..., 6. Thus, with X = 1, there is only one available detector which can be placed at any one of the six buoy sites. The enumeration method also produced warning potentials for all of the detector deployments, including the worst. The results are shown in Table 2, where the range relates to the range of values of the warning potentials from the lowest to the highest. The mean warning potential is the average across all configurations for the deployment of that particular number of detectors. The last column shows the deployment of the detectors, or values of $\mathbf{y}$, which maximise the warning potential for that particular number of detectors. In many cases, there are several deployments where the values of the warning potential are very similar. Small changes in the initial data will produce different optimal solutions.

As expected, a deployment of one detector, X = 1, provides the largest range of values of the warning potential and the smallest maximum warning potential. The mean warning potential for one detector is 0.618, which is an inadequate level of warning from a risk perspective (Morgan [1993]). The best deployment is $\mathbf{y}$ = (001000), or to locate the single detector at Site 3 (see Figure 1). Employing a deployment of two detectors substantially improved the range, mean and maximum values of the warning potential. Sites 3 and 6 were the sites which yielded the maximum warning potential.

The three-detector deployments again significantly improved the performance of the system. The most important result for the three-detector deployment is that it can achieve a warning potential of 0.805, and this is within 0.12% of the maximum possible warning potential where all six sites are occupied.

Table 2. Warning potentials for deploying X detectors, with $\mathbf{r}$ = 0.

Number of Detectors X	Warning Potential E(y,r*)		Optimal Deployment
	Range	Mean	
1	0.453-0.704	0.618	(001000)
2	0.636-0.776	0.719	(001001)
3	0.711-0.805	0.746	(100101)
4	0.725-0.807	0.778	(110101)
5	0.745-0.807	0.794	(111101)
6	0.808	0.808	(111111)

The four-, five- and six-detector deployments continue to improve the range and mean values of the warning potential. Overall, the trend in warning capacity for the Pacific region follows the 'economic law of diminishing returns'. The most significant increases in warning potential occur as the number of detectors increases from one to three. Thereafter, the maximum warning potential increases only marginally, while the range and mean continue to increase.

The range, mean and optimal deployment y for X = 3 (deploying three detectors) are given in Table 3 for various values of **r**, where $r_v = r^*$ for all v. Both the maximum and mean warning potentials decrease at about the same rate as r* increases. The corresponding optimal deployment shows considerable variation as the value of r* changes. The results indicate that Site 6 should be occupied for all values of r* which have been considered. Apart from r* = 0, or 1, where the optimal deployments are identical, the optimal deployment depends heavily on the value of r*, and alters as the value of r* is varied. Note that the range of values of the warning potential increases as r* increases.

Table 3. Warning potentials E(y,r*) for X = 3

r*	Warning Potential E(y,r*)		Optimal Deployment
	Range	Mean	
0	0.711-0.805	0.746	(100101)
1	0.647-0.749	0.704	(100101)
2	0.546-0.718	0.660	(010101)
3	0.481-0.644	0.581	(110001)
4	0.457-0.616	0.551	(010011)

Figure 2 shows the warning potentials as a function of the emergency services reaction time r* for X = 1, 2, ..., 6. The graphs all reflect the behaviour of the maximum warning potential in Table 3. All decrease markedly as r* increases, and at about the same rate.

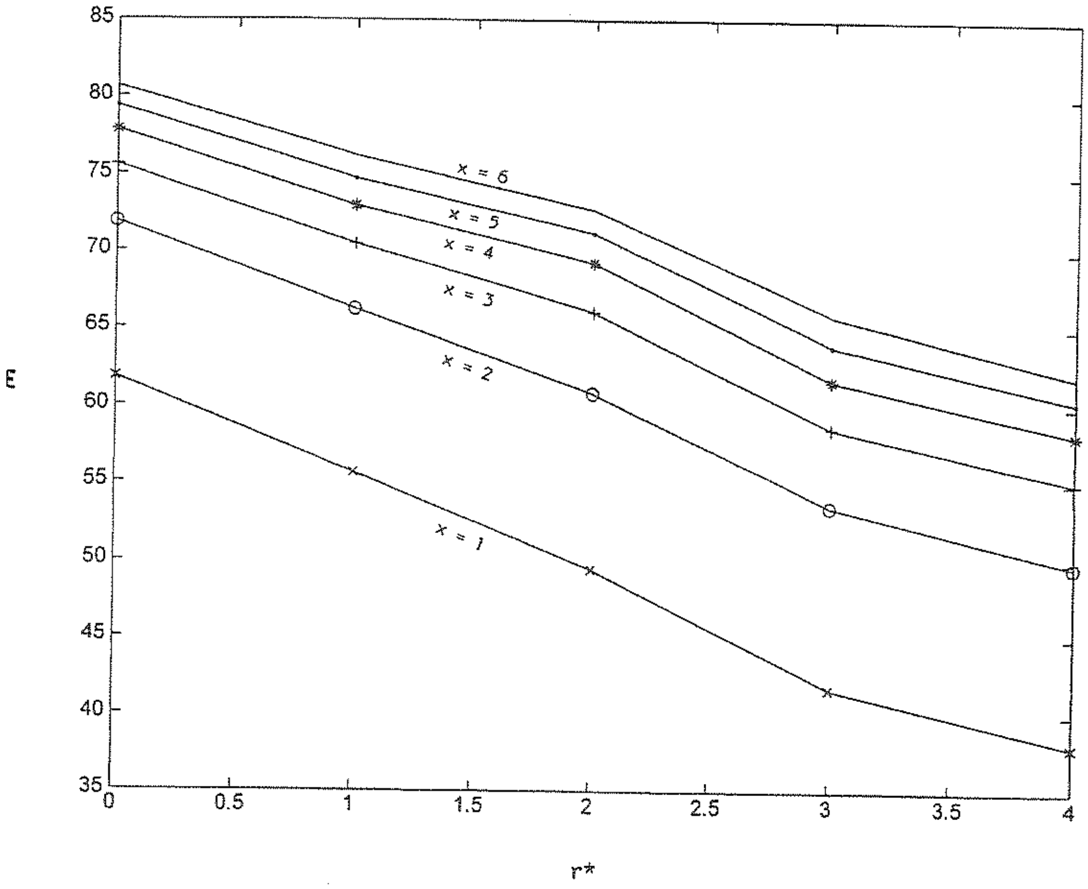

Figure 2. The optimal warning potential as a function of r* for the deployment of X detectors

6. CONCLUSIONS

Tsunami warning systems need to be equipped with reliable and secure tsunami detectors and communications systems. The latest technology is able to provide suitable deep-sea detectors which need to be located optimally to maximise the warning potential. This problem can be formulated as a 0-1 non-linear integer programme and can be solved using enumeration techniques.

The solutions to the NOAA problem indicate that the maximum possible Pacific-wide warning potential can be effectively achieved with just three detectors, and the results also follow the 'economic law of diminishing returns'. The results indicate that NOAA Sites 1, 4 and 6 should be utilised. The inclusion of a reaction time r* for the emergency services indicates that the optimal warning potential degrades as r* increases. The disturbing feature is that the optimal deployment also depends on the value of r*. Similar results apply to the USA warning potential.

The research at this stage has been limited by the availability of data. These deficiencies do not detract from the methods used, and the methods can be applied to more complete data sets and computational inputs. Indeed, the model has been presented in general terms and can be easily modified to handle more or less of generation points, population centres and communication sites. To this end, the variables $u = 1, \ldots, U$ etc. have been specified in general terms and are readily applied to different data configurations. The data can be improved by

(a) identification of more or less sites for communication buoys;
(b) identification of more or less generation points;
(c) identification of more or less population centres, and particularly to the tighter identification of the population numbers at risk.

The objective function can also be recast in terms of the value of the infrastructure at risk. This can be done by replacing the population, p_v, at P_v by a cost or value, c_v, at each location. The generation areas can also be weighted in importance, depending on the frequency and severity of the tsunamis which are generated. The determination of the values of c_v was beyond the scope of this paper.

In the computations, the calculation of the travel times could be improved by using suitable bathymetric data or travel-time charts. This would improve the precision of the technique. However, the reaction warning period, r_v, will have a large bearing on the success of a particular warning event. The warning will be less than effective if emergency services do not have time to act. The use of $r_v = 0$, $v = 1, \ldots, v$ is obviously unrealistic. There are also difficulties in determining the size of r_v, although a value of three hours seems appropriate. The uncertainties in the value of r_v far outweigh errors which may occur in the calculation of the appropriate travel times.

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