# Microeconometric Models of Choice under Uncertainty

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**Abstract** We incorporate a stochastic element, in various ways, into the theory of choice under uncertainty, in the context of pairwise choice problems. By splitting the decision-making process into three distinct stages, it is demonstrated that there are three alternative approaches to incorporating the stochastic element. These are (1) the Random Preference model [Loomes and Sugden, 1997]; (2) The Hey-Orme Model [Hey and Orme, 1994]; and (3) the Harless-Camerer Model [Harless and Camerer, 1994]. We estimate various combinations of the three models by maximum likelihood using experimental data from a sample of 92 subjects who were each presented with 90 pairwise choice problems. A core theory of simple expected utility (EU) is considered first. As a generalisation of EU, we then adopt "Rank Dependent" Expected Utility theory [Quiggin, 1982] as the core theory and we see a significant improvement in explanatory power over simple EU. An obvious feature of the data is that subjects' choices tend to change in particular ways over time, and these important effects are captured by allowing certain key parameters to *decay* over time. It is thus established that observed choices correspond more closely to simple EU, the more accustomed subjects become to solving pairwise choice problems.

## 1. Introduction

The mainstream decision theory literature has tended to revolve around deterministic models which make no explicit allowance for any stochastic element in people's decision behaviour. However, it has frequently been observed that when a participant in a decision experiment is presented with exactly the same problem twice in the course of an experiment, s/he often responds differently on each of the two occasions. Given the prevalence of such "reversals", and the desire to conduct soundly-based statistical tests to discriminate between competing models, there is a clear need to incorporate an appropriate stochastic specification into modern decision theory.

Basic notation is established in section 2. Section 3 then sets out the various stochastic versions of EU and describes how each can be econometrically estimated. We also show how we have allowed for any "experience" effect to be picked up. Section 4 introduces Rank Dependent Expected Utility Theory (henceforth RD) as an alternative core theory and shows how its parameters can be estimated. Section 5 describes the data used in estimation. Section 6 reports and discusses the results. Section 7 concludes.

## 2. Basic notation

Participants in the experiment were presented with a series of choices between pairs of lotteries which involved no more than three possible payoffs. To formalise this, we specify a payoff vector $\underline{x} = (x_0\ x_1\ x_2)'$ where the x's are amounts of money, with $x_2 > x_1 > x_0 = 0$. We also specify two probability vectors $\underline{p} = (p_0\ p_1\ p_2)'$ and $\underline{q} = (q_0\ q_1\ q_2)'$ whose elements correspond to the elements of the payoff vector. If the choice does not involve dominance, $\underline{p}$ can be described as the "riskier" option, and $\underline{q}$ as the "safer" option. As a consequence, for non-dominance questions, it is always the case that $p_0 > q_0$, $p_1 < q_1$ and $p_2 > q_2$. For questions where one option dominates the other, the terms "riskier" and "safer" are not appropriate, so we assign $\underline{p}$ to the dominating option, and q to the dominated one. In such cases, it is always the case that $p_2 \geq q_2$ and $p_0 \leq q_0$, but the inequality involving $p_1$ and $q_1$ can be of either direction.

Let U(x) be a von Neumann-Morgenstern utility function, normalised so that U(0) = 0; $U(x_1) = 1$; and $U(x_2) = u$, with $u>1$. The value u is a measure of the subject's attitude-to-risk: the higher u is, the more risk-loving the subject. We find the expected utility of each option:

$$EU(\underline{p}) = p_1 + p_2 u \qquad (1)$$

$$EU(\underline{q}) = q_1 + q_2 u \qquad (2)$$

Using (1) and (2) we see that the condition for choosing the riskier option $\underline{p}$ can be written as:

$$d_1 + d_2 u > 0 \qquad (3)$$

where $d_1 = p_1 - q_1$ and $d_2 = p_2 - q_2$. Clearly the condition (3), as it stands, cannot be used for our purposes, because it is deterministic. However, it provides the basis for the stochastic models developed in section 3.

## 3. Stochastic versions of EU theory

In what follows, we shall assume that the sample consists of n subjects, each of whom has been asked the same set of T questions. Let $y_{it} = 1$ if subject i chooses p in response to question t, and zero otherwise. Further, let $d_{1t}$ and $d_{2t}$ be the probability differences, as in (3) above, pertaining to question t.

There are various possible ways of introducing a stochastic element into the deterministic model described in section 2. To introduce these, it is helpful to imagine the choice process being split into three stages. In the "preference selection" stage, the subject identifies her utility function at the present moment in time. In the "computation" stage, she uses that function to weigh the two options. In the "action" stage, she "presses the button" corresponding to the option judged to have the higher expected utility.

Within our framework, there are three different ways of introducing a stochastic element, each corresponding to one of the three stages outlined above. First, there is the possibility of introducing a degree of randomness at the preference selection stage, by allowing u to be a random variable, with range $(1, \infty)$. This effectively implements the random preference (RP) model, originally due to Becker et al [1963], and recently reconsidered by Loomes and Sugden [1997].

Our version of the RP model assumes the following distribution for u:

$$\ln(u-1) \sim N\left(\ln(m-1), \sigma_u^2\right)$$
$$\sigma_u = \exp(\sigma_1 + \sigma_2 m)\,. \qquad (4)$$

The parameter m is the median of u. The higher a subject's value of m, the further to the right the distribution of u is, so the less risk-averse, on average, that subject is.

We allow subjects to differ by assuming that they all have their own median attitude to risk, $m_i$ i=1,...,n. The probability of subject i choosing p on question t is:

$$P(y_{it}=1) = \Phi\left( \frac{\ln\left(-\dfrac{d_{2t}}{d_{1t}+d_{2t}}\right) + \ln(m_i - 1)}{\exp(\sigma_1 + \sigma_2 m_i)} \right) \qquad (5)$$

The log-likelihood function for estimating the n+2 parameters can be constructed from (5). A problem with the RP model is that it is incapable of explaining violations of dominance, and we shall return to this point.

The second possibility is to introduce randomness at the computation stage. This approach is similar to that of Hey and Orme [1994], so we shall refer to this model as the Hey-Orme (HO) model. Here, u is a fixed parameter, which we allow to differ between subjects. Let $u_i$ be subject i's value of u. We add a stochastic term to the difference of expected utilities, so that the subject i chooses the riskier option p on question t if:

$$d_{1t} + d_{2t}u_i + \varepsilon_{it} > 0 \quad \varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$$
$$\sigma_\varepsilon = \sigma_1 (u_i - 1) + \sigma_2 (u_i - 1)^2 \qquad (6)$$

where $\varepsilon_{it}$ represents computational error. This is assumed to have mean zero, implying that mental computations are correct on average. The purpose of the variance restriction in the second line of (6) is to guarantee that $u_i$ is always greater than 1.

According to the HO model, the probability of subject i choosing p on question t is:

$$P(y_{it}=1) = \Phi\left( \frac{d_{1t} + d_{2t}u_i}{\sigma_1(u_i - 1) + \sigma_2(u_i - 1)^2} \right) \qquad (7)$$

The log-likelihood function for estimating the n+2 parameters can be constructed from (7). Note that since (7) is valid for both non-dominance and dominance questions, this model can be estimated using the complete sample.

The third possibility is to introduce a degree of randomness at the action stage. We can assume that there is a small probability, $\omega$ say, that the choice between the two options will be made as if at random, whatever the outcome of the computation stage. This parameter $\omega$ may be interpreted as the probability that the subject does not fully understand the question, or alternatively as the probability that she is not concentrating at the time of answering the question. This assumption leads to a model similar in spirit to that of Harless and Camerer [1994]. Accordingly, we shall henceforth refer to this model as the Harless-Camerer (HC) model. This model is not useful in its own right, but is very useful when combined with the other models. It is to these combinations that we now turn.

It was pointed out earlier that the RP model as specified in (5) cannot explain violations of dominance. To deal with this problem, we combine the RP model with the HC model. We shall refer to the combined model as RP-HC. Under this model, the probability of p being chosen for the non-dominance questions is:

$$P(y_{it}=1)=$$
$$(1-\omega)\Phi\left(\frac{\ln\left(-\frac{d_{2t}}{d_{1t}+d_{2t}}\right)+\ln(m_i-1)}{\exp(\sigma_1+\sigma_2 m_i)}\right)+\frac{\omega}{2}\cdot \quad (8)$$

For the dominance questions, the probability of p being chosen is simply:

$$P(y_{it}=1)=1-\frac{\omega}{2} \quad (9)$$

The log-likelihood function for RP-HC can be constructed from (8) and (9). This model can be estimated using the complete sample.

The other combination we shall consider is HO and HC, whose likelihood function is similarly constructed. We shall refer to this combination as HO-HC.

We have used the subscript t to indicate question number t. However, the question numbers, t=1,...,T, have nothing to do with the actual order in which the T questions were asked. Every subject was asked the questions in a different order, which was determined randomly. We therefore require an additional variable, $t'$, which indicates the actual position of the question in the ordering. $t'_{it}$ is the time when individual i was asked question t.

Knowledge of $t'$ is necessary in order to investigate the way in which parameters change with experience. For example, if, with experience, subjects become less likely to misunderstand questions, the HC term, $\omega$, would tend to fall as time progresses. To model this effect, we replace $\omega$ by,

$$\omega_{it}=\frac{\exp(\omega_0+\omega_1 t'_{it})}{1+\exp(\omega_0+\omega_1 t'_{it})}. \quad (10)$$

A negative value of the parameter $\omega_1$ would indicate that the probability of misunderstanding decays as subjects become more familiar with the choice task.

## 4. Rank dependent (RD) EU theory

Loomes and Sugden [1997], analysing the present dataset in a rather different way, find that apparent violations of EU occur in a particular pattern. When q involves a zero probability of the highest payoff, subjects tend to be disproportionately likely to choose p (which necessarily involves a positive probability of the highest pay-off, unless the question involves dominance). This phenomenon has come to be known as the "bottom-edge" effect, since an option with a zero probability of the highest pay-off is located on the bottom edge of the Machina triangle [see Loomes and Sugden, 1997]. One way of allowing for this effect is by using a "rank dependent" generalisation of EU - henceforth RD - originally proposed by Quiggin [1982].

The extent of the bottom edge effect is represented by a parameter b, where $0\leq b<1$. When b=0, there is no bottom-edge effect, and we have straightforward EU. Let us define $BE_t$ to be a binary variable taking the value one if question t is a bottom-edge question, zero otherwise. The condition for choosing p on question t can then be written [see Loomes et al., 1997, for further details]:

$$b(u-1)BE_t + (1-b)(d_{1t} + d_{2t}u) > 0. \quad (11)$$

In section 3, we discussed the possible effect of time/experience, and described a method for estimating any such effect on the parameter $\omega$. We now extend this method to the parameter b, by replacing it with:

$$b_{it}=b_0\exp(b_1 t'_{it}) \qquad b_0\geq 0\,;\, b_1<0. \quad (12)$$

## 5. Data

The models developed in sections 3 and 4 have been estimated using pairwise choice data from a sample of 92 subjects, randomised between two subsamples of 46. Subjects 1 through 46 were presented with 90 pairwise choices involving payoffs £0, £10 and £30, while subjects 47 through 92 were presented with 90 pairwise choices involving payoffs £0, £10 and £20. Details of the experimental design and procedure are given in Loomes and Sugden [1997].

Of the 90 questions, 80 were non-dominance and 10 involved dominance. The number of times each subject chose p for non-dominance questions are shown in parentheses in the leftmost column of table 1.

## 6. Results

Three stochastic versions of EU have been econometrically estimated, using the MAXLIK routine in GAUSS. We always use the BHHH algorithm [Berndt et al., 1974], with analytic first derivatives. The three stochastic models we have estimated are straight Hey-Orme (HO), and two combinations: HO-HC and RP-HC. The three stochastic models have been applied to both EU and RD. EU and RD will be used as subscripts to the acronyms of the various stochastic specifications. So, for example, RP-$HC_{RD}$ represents the random preference model with a Harless-Camerer non-concentration term, applied to rank-dependent expected utility theory.

The results are shown in table 1, although due to space constraints they are incomplete. Rows 1 through 46 of the results contain estimates of u for subjects 47 through 92, or, in the case of RP models, estimates of the median of u. (The corresponding estimates for subjects 1 through 46 are not shown). This is an estimate of U(£20). These estimates seem sensible. They are all greater than one, in accordance with monotonicity of the utility function, which has been imposed in estimation. The estimates are less than 2 for nearly all subjects, implying risk aversion. In fact, most are close to one, implying a high degree of risk aversion for most subjects. We further note that the estimated standard errors of these estimates are usually small, indicating a high degree of precision in estimation. The missing rows correspond to the subjects who chose q every time, or nearly every time, for whom there is insufficient variation of response to allow estimation of attitude to risk.

The most dramatic feature of the results is the effect of time/experience on the Harless-Camerer probability of non-concentration/mis-understanding, embodied in the parameter $\omega_1$. For reasons given later, the last column of results, from RP-$HC_{RD}$, is the column which should be taken the most seriously, so we use these results here. We see from these results that this relationship is:

$$\hat{\omega}_{it} = \frac{\exp(-2.327 - 0.016t'_{it})}{1 + \exp(-2.327 - 0.016t'_{it})}. \quad (13)$$

(13) implies that at the outset, when no questions have been asked, the probability of misunderstanding is around 0.09. After 90 questions have been asked, however, this probability has fallen to 0.02. The asymptotic t-ratio associated with this time effect is around -2.7, indicating strong significance.

We next consider the parameters of RD. Once again focusing on the last column of results, we see that the bottom-edge parameter b decays over time according to:

$$\hat{b}_{it} = 0.165 \exp(-0.006t'_{it}). \quad (14)$$

(14) implies that at time zero, parameter b is 0.165, indicating a strong bottom-edge effect. However, after 90 questions have been asked, the parameter b has decayed to 0.096. Moreover, the effect of time has overwhelming significance, with an asymptotic t-ratio of -6.0.

Finally, we address the question of which of the models we have estimated is best at explaining the observed data. We can go some way towards answering this question by adopting the straightforward criterion of the maximised log-likelihood. Where one model is nested within another, we can apply a straightforward likelihood ratio (LR) test. In figure 1, single-pointed arrows represent nested hypotheses, and the LR statistics are shown. Each test is of two restrictions, so we see that all of the LR tests conducted result in overwhelming rejections of the restrictions under test, and therefore overwhelming evidence in favour of the nesting model in each case. This means that both of the generalisations to simple EU that we have considered, the HC error probability and the bottom-edge effect, have given rise to significant increases in explanatory power. There is therefore no doubt that the most preferred model will be one that incorporates both of these generalisations.

The question remains as to which is preferred out of HO-$HC_{RD}$ and RP-$HC_{RD}$. For the purpose of this comparison, we make use of Vuong's [1989] non-nested likelihood ratio test [see Loomes et al., 1997, for details of the application of this procedure]. This statistic, Z, is standard normal under the null hypothesis that the two models are equivalent. The way we have constructed the statistic is such that a significantly positive value of Z provides evidence that the model based on RP is closer to the true data generating process than the one based on HO.

Table 1: Results from stochastic versions of simple EU and Rank Dependent EU

| | *Simple EU* | | | *Rank Dependent EU* | | |
|---|---|---|---|---|---|---|
| *Subj.* | *HO* | *HO-HC* | *RP-HC* | *HO* | *HO-HC* | *RP-HC* |
| 47 (21) | 1.28(0.04) | 1.31(0.04) | 1.32(0.05) | 1.23(0.03) | 1.25(0.02) | 1.27(0.03) |
| 48 (18) | 1.26(0.03) | 1.29(0.03) | 1.28(0.05) | 1.21(0.02) | 1.23(0.02) | 1.24(0.03) |
| 49 (15) | 1.23(0.03) | 1.24(0.03) | 1.23(0.05) | 1.19(0.02) | 1.20(0.02) | 1.22(0.03) |
| 50 (25) | 1.32(0.05) | 1.36(0.04) | 1.40(0.06) | 1.26(0.03) | 1.28(0.03) | 1.31(0.03) |
| 51 (42) | 1.70(0.09) | 1.65(0.07) | 1.74(0.09) | 1.50(0.06) | 1.48(0.04) | 1.51(0.06) |
| 52 (30) | 1.37(0.05) | 1.41(0.05) | 1.46(0.07) | 1.30(0.03) | 1.32(0.04) | 1.35(0.04) |
| 53 (9) | 1.20(0.02) | 1.21(0.03) | 1.15(0.04) | 1.17(0.02) | 1.17(0.02) | 1.15(0.03) |
| 54 (15) | 1.25(0.03) | 1.25(0.03) | 1.23(0.05) | 1.20(0.02) | 1.20(0.02) | 1.21(0.03) |
| 55 (0) | - | - | - | - | - | - |
| 56 (28) | 1.41(0.06) | 1.44(0.05) | 1.46(0.07) | 1.31(0.04) | 1.33(0.03) | 1.34(0.04) |
| 57 (2) | - | - | - | - | - | - |
| 58 (12) | 1.28(0.03) | 1.20(0.03) | 1.19(0.04) | 1.24(0.02) | 1.17(0.02) | 1.18(0.03) |
| 59 (4) | - | - | - | - | - | - |
| 60 (0) | - | - | - | - | - | - |
| 61 (6) | - | - | - | - | - | - |
| 62 (7) | 1.16(0.02) | 1.17(0.03) | 1.11(0.04) | 1.13(0.02) | 1.15(0.02) | 1.14(0.03) |
| 63 (9) | 1.19(0.02) | 1.21(0.03) | 1.15(0.04) | 1.15(0.02) | 1.18(0.02) | 1.16(0.03) |
| 64 (0) | - | - | - | - | - | - |
| 65 (34) | 1.40(0.05) | 1.43(0.05) | 1.54(0.07) | 1.32(0.04) | 1.35(0.04) | 1.40(0.04) |
| 66 (25) | 1.40(0.06) | 1.42(0.05) | 1.40(0.06) | 1.30(0.04) | 1.29(0.03) | 1.30(0.03) |
| 67 (14) | 1.22(0.03) | 1.23(0.03) | 1.21(0.04) | 1.18(0.02) | 1.19(0.02) | 1.20(0.03) |
| 68 (39) | 1.68(0.09) | 1.65(0.07) | 1.68(0.09) | 1.48(0.06) | 1.46(0.04) | 1.47(0.05) |
| 69 (27) | 1.39(0.05) | 1.39(0.05) | 1.44(0.07) | 1.30(0.04) | 1.29(0.03) | 1.33(0.04) |
| 70 (15) | 1.36(0.04) | 1.24(0.03) | 1.21(0.05) | 1.29(0.03) | 1.20(0.02) | 1.20(0.03) |
| 71 (0) | - | - | - | - | - | - |
| 72 (18) | 1.26(0.03) | 1.29(0.03) | 1.28(0.05) | 1.21(0.02) | 1.23(0.02) | 1.24(0.03) |
| 73 (28) | 1.34(0.04) | 1.37(0.04) | 1.42(0.06) | 1.27(0.03) | 1.30(0.03) | 1.34(0.04) |
| 74 (8) | 1.17(0.02) | 1.18(0.02) | 1.15(0.04) | 1.14(0.02) | 1.15(0.02) | 1.14(0.03) |
| 75 (24) | 1.39(0.04) | 1.32(0.04) | 1.37(0.06) | 1.32(0.03) | 1.25(0.02) | 1.29(0.03) |
| 76 (30) | 1.50(0.05) | 1.42(0.05) | 1.45(0.07) | 1.40(0.04) | 1.32(0.03) | 1.35(0.04) |
| 77 (59) | 1.94(0.06) | 2.01(0.14) | 2.15(0.13) | 1.77(0.06) | 1.78(0.08) | 1.91(0.12) |
| 78 (48) | 2.01(0.07) | 1.92(0.09) | 1.92(0.11) | 1.67(0.07) | 1.61(0.06) | 1.61(0.07) |
| 79 (45) | 1.73(0.08) | 1.69(0.07) | 1.82(0.09) | 1.55(0.06) | 1.50(0.05) | 1.56(0.06) |
| 80 (24) | 1.33(0.04) | 1.35(0.04) | 1.37(0.06) | 1.26(0.03) | 1.28(0.03) | 1.30(0.03) |
| 81 (14) | 1.21(0.03) | 1.23(0.03) | 1.22(0.04) | 1.17(0.02) | 1.19(0.02) | 1.21(0.03) |
| 82 (30) | 1.47(0.05) | 1.39(0.04) | 1.45(0.06) | 1.38(0.04) | 1.31(0.03) | 1.35(0.04) |
| 83 (28) | 1.45(0.05) | 1.39(0.06) | 1.43(0.06) | 1.35(0.04) | 1.29(0.03) | 1.33(0.04) |
| 84 (10) | 1.18(0.02) | 1.20(0.03) | 1.16(0.04) | 1.15(0.02) | 1.18(0.02) | 1.17(0.03) |
| 85 (11) | 1.20(0.02) | 1.22(0.03) | 1.18(0.04) | 1.16(0.02) | 1.19(0.02) | 1.18(0.03) |
| 86 (0) | - | - | - | - | - | - |
| 87 (1) | - | - | - | - | - | - |
| 88 (0) | - | - | - | - | - | - |
| 89 (39) | 1.75(0.10) | 1.69(0.07) | 1.68(0.09) | 1.50(0.06) | 1.48(0.04) | 1.46(0.05) |
| 90 (3) | - | - | - | - | - | - |
| 91 (21) | 1.30(0.04) | 1.31(0.04) | 1.31(0.05) | 1.24(0.03) | 1.23(0.03) | 1.26(0.03) |
| 92 (21) | 1.28(0.04) | 1.32(0.04) | 1.33(0.06) | 1.23(0.03) | 1.25(0.02) | 1.27(0.03) |
| $\sigma_1$ | 0.387(0.023) | 0.247(0.017) | 0.390(0.236) | 0.350(0.019) | 0.183(0.013) | -0.572(0.285) |
| $\sigma_2$ | -0.283(0.034) | -0.159(0.023) | -0.412(0.148) | -0.284(0.036) | -0.107(0.022) | 0.039(0.191) |
| $\omega_0$ | | -1.629(0.220) | -1.768(0.282) | | -1.881(0.207) | -2.327(0.278) |
| $\omega_1$ | | -0.018(0.005) | -0.031(0.010) | | -0.010(0.004) | -0.016(0.006) |
| $b_0$ | | | | 0.185(0.014) | 0.179(0.012) | 0.165(0.012) |
| $b_1$ | | | | -0.007(0.001) | -0.005(0.001) | -0.006(0.001) |
| nobs | 6660 | 6660 | 6660 | 6660 | 6660 | 6660 |
| LogL | -3071.6 | -2989.8 | -2925.5 | -2920.6 | -2784.9 | -2678.3 |

In leftmost column, parentheses contain the number of times p was chosen by each subject on non-dominance questions (maximum 80).
In other columns, parentheses contain asymptotic standard errors.

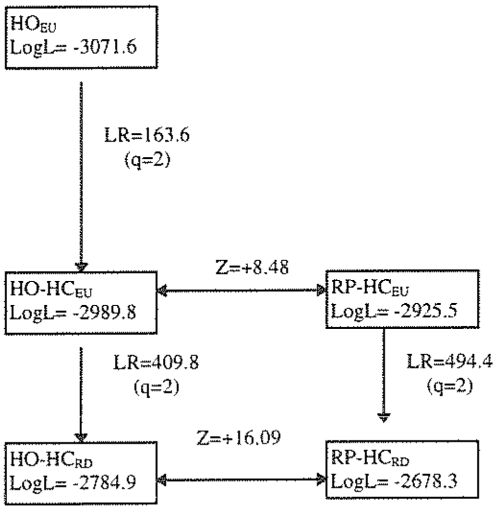


**Figure 1:** Nested and non-nested hypothesis tests

In figure 1, double-pointed arrows indicate non-nested tests. The important test is the one that results in a Z-statistic of +16.09. This provides overwhelming evidence that RP-HC$_{RD}$ is closer to the true data generating process than HO-HC$_{RD}$.

## 7. Conclusion

This paper has produced two insights in particular which we think may be especially useful and thought-provoking. Firstly, we have found convincing evidence that the random preference framework provides the best approach to forming a stochastic model for pairwise choice. Although the random preference model in its simplest form cannot be estimated in the presence of dominance violations, a generalisation of it which allows a small probability of "misunderstanding" has been seen to perform considerably better than a model which explains variation in terms of computational error. This result has been endorsed by a formal non-nested hypothesis test.

The second main result concerns the effect of time/experience. We found that the HC misunderstanding term and the bottom-edge parameter both decayed dramatically as subjects gained experience. These results imply that as they became more familiar with the task, many subjects' behaviour began to correspond much more closely with conventional EU Theory.

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