# Testing the Life-Cycle Permanent Income Hypothesis Using Intra-year Data for Germany

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**Abstract** It has been found that real non-durable consumption expenditure for many countries exhibits substantial seasonality. Osborn (1988) introduced a seasonally-varying utility function for consumption, in which Hall's (1978) consumption function implies a periodically integrated model of order 1 for real non-durable consumption. Using quarterly seasonally unadjusted consumption data for Germany, formal tests for periodic integration are used to examine the modified rational expectations life-cycle permanent income hypothesis. Seasonal habit persistence is introduced in the form of a periodic seasonal ARIMA model, its empirical adequacy is examined and it is found to be significant.

## 1. Introduction

Consumption is, in many ways, a significant economic variable. It is not surprising, therefore, that the broad topic of consumer behaviour has attracted considerable theoretical and empirical research at both the microeconomic and macroeconomic levels over extended periods.

One of the most influential recent contributions to consumption has been the Euler equation approach of Hall (1978), in which the life-cycle permanent income hypothesis (LC-PIH) is used to model the real consumption of non-durables under rational expectations. Hall's result that consumption expenditure follows a random walk has important policy implications, not only for consumption, but also for national saving.

One aspect of consumption behaviour that has largely been ignored in the literature to date is that of seasonality. If intra-year observed data are considered, consumption is typically found to exhibit substantial seasonal fluctuations. Since consumption expenditure is highly seasonal, the inherent seasonality should be accommodated in any serious empirical modelling of consumption expenditure. Osborn (1988) extended Hall's (1978) approach to accommodate seasonality, and derived a periodic analogue of the random walk model for real non-durable consumption.

In this paper, quarterly seasonally unadjusted consumption data for Germany are used to test the validity of the seasonal LC-PIH. The purpose in examining a seasonal extension of Hall's model is to examine the extent to which intra-year data and seasonality can affect the analysis. In this paper, a formal test for periodic integration is used to test the LC-PIH. Failure of the random walk model to model consumption expenditure adequately may be due to the presence of habit persistence, so a seasonal habit persistence model is also applied to the consumption series.

The plan of the paper is as follows. Section 2 discusses various sources of endogenous and exogenous seasonality prevalent in intra-year observed consumption expenditure. Osborn's (1988) extension of Hall's model is presented in Section 3. Periodic autoregressive models and unit roots are discussed in Section 4, and this new approach to testing Osborn's hypothesis is considered in Section 5. Section 6 introduces seasonal habit persistence, which is tested and evaluated using periodic seasonal ARIMA models, namely, ARIMA models with periodically changing coefficients and a seasonal lag structure. Concluding remarks follow in Section 7.

## 2. Seasonality and Consumption

To test the various hypotheses, quarterly seasonally unadjusted real non-durable consumption expenditure of Germany (1960Q1-

1967Q4) are used in this paper. In considering quarterly data, consumption exhibits marked seasonality.

Traditionally, the approach to econometric modelling of seasonality has been to remove or seasonally adjust such fluctuations in the belief that seasonality contaminates the data by adding irrelevant information.

More recently, serious questions have been raised regarding the validity of the seasonal adjustment approach. An argument against seasonal adjustment is as follows. In the traditional approach to time series analysis, time series are a function of four independent components, namely a trend, cycle, seasonal and an irregular component. A the crucial assumption underlying seasonal adjustment procedures is that it is possible to disentangle seasonality from a time series. This orthogonality assumption has been shown to be highly questionable. Hence, when seasonally adjusted data are used, some trend and cyclical variations, may also be removed.

Given the above considerations, it is important to model seasonality explicitly in consumption. There are various sources of seasonality in consumption, some of which are endogenous, while others are exogenous.

Examples of exogenous seasonality in consumption include the fact that consumption of energy is often highest in winter, and expenditure on alcohol increases in the fourth quarter with Christmas and the New Year. Such seasonality is also likely to be deterministic, as Christmas and New Year always occur in the fourth quarter.

There are also policy-related influences on consumption expenditure, such as broad-based consumption taxes and short-term deposit interest rates. In many macroeconomic models, variables such as taxes and interest rates are endogenous. Hence, the seasonality that arises due to the periodic changes in these variables is endogenous. The behaviour of economic agents to such variables may change over time. Therefore, seasonality may be stochastic in nature.

To obtain an indication of the amount of deterministic seasonality in German consumption, the growth rate of consumption is regressed on a set of seasonal dummy variables using the following regression:

$$\Delta \ln c_t = \delta_1 D_{1t} + \delta_2 D_{2t} + \delta_3 D_{3t} + \delta_4 D_{4t} + \upsilon_t \qquad (1)$$

where $D_{st}$ (s = 1,2,3,4) is a seasonal dummy variable, and $\upsilon_t$ is assumed to be a stationary and invertible ARMA process. The $\overline{R}^2$ value is 0.956, which indicate that, after the removal of the stochastic trend, seasonality accounts for well over 90% of the variation in the growth rate of consumption.

Seasonal patterns in consumption can also change over time. Slowly changing seasonal patterns can reflect the presence of seasonal unit roots, or stochastic seasonality. The most commonly used test for stochastic seasonality is the HEGY test of Hylleberg *et al.* (1990), which is based on the following regression:

$$\begin{aligned} a_t = {} & \gamma_1 D_{1t} + \gamma_2 D_{2t} + \gamma_3 D_{3t} + \kappa t + \gamma_4 D_{4t} \\ & + \pi_1 b_t + \pi_2 c_t + \pi_3 d_t + \pi_4 e_t + \varepsilon_t \end{aligned} \qquad (2)$$

where t is a deterministic time trend, and

$a_t = (1 - L^4)y_t$;
$b_t = (1 + L + L^2 + L^3)y_{t-1}$;
$c_t = -(1 - L + L^2 - L^3)y_{t-1}$;
$d_t = -(1-L^2)y_{t-2}$;
$e_t = -(1-L^2)y_{t-1}$;
$\varepsilon_t$ is an iid $(0, \sigma_\varepsilon^2)$ error term.

The HEGY hypotheses are formalised as follows:

1) $H_0$: $\pi_1 = 0$, $H_1$: $\pi_1 < 0$;
2) $H_0$: $\pi_2 = 0$, $H_1$: $\pi_2 < 0$;
3) $H_0$: $\pi_3 = \pi_4 = 0$, $H_1$: $\pi_3 \neq 0$ and/or $\pi_4 \neq 0$.

Standard t-tests are used for the first two hypotheses, while an F-test is used for the third hypothesis. If the first hypothesis is not rejected, there is a zero frequency unit root in the series. In the second case, the null hypothesis is consistent with a semi-annual unit root, implying that any shocks to the variable will lead to permanent changes in the seasonal pattern of the variable at the semi-annual level. A non-rejection of the third null hypothesis implies that the series have at least one of the two unit roots at the annual frequency. With annual unit roots, a shock to the variable will change permanently the seasonal pattern of the variable at the annual level.

Using 8 lags in the HEGY regression, with diagnostic tests revealing no serial correlation ($F_{AR,1-4}$ = 0.782), the estimated test statistics for are $t(\pi_1)$ = -0.983, $t(\pi_2)$ = -1.169, and $F(\pi_3, \pi_4)$ = 2.325. At the 5 percent significance level, all the three null hypotheses are not rejected, indicating that all three consumption variables are seasonally integrated.

## 3. The Seasonal Extension to the Life-Cycle Permanent Income Hypothesis

To derive Hall's random walk result, Muellbauer (1983) uses the following utility function:

$$u_t = \frac{c_t^{1-\beta}}{1-\beta}, \quad 0 < \beta < 1 \tag{3}$$

where $\beta$ is the level of risk aversion.

With $\alpha$ representing the subjective discount rate, total utility is given by:

$$U_t = \sum_{i=0}^{T} \alpha^i u_{t+i} = \sum_{i=0}^{T} \alpha^i \left( \frac{c_t^{1-\beta}}{1-\beta} \right), \quad 0 < \alpha \leq 1. \tag{4}$$

The consumer's optimisation problem is to maximise (4) subject to the budget constraint:

$$c_t = y_t - a_t + a_{t-1}(1 + r_{t-1}) \tag{5}$$

where c is the volume of consumption, y is real labour and transfer income, a is the real asset level, and r is the real interest rate.

By forming the Lagrangian function for the optimisation problem, and treating the a's as the decision variables, the following random walk model for consumption is derived:

$$\ln c_t = \gamma + \ln c_{t-1} + \varepsilon_t . \tag{6}$$

The results of the previous section indicates that consumption exhibits substantial seasonality. In Osborn (1988), it is noted that $\alpha$ and $\beta$ are the most obvious parameters that can change with the seasons. Hence, equation (3) can be replaced by the following seasonal utility function:

$$u_t = \frac{c_t^{1-\beta_s}}{1-\beta_s}, \quad 0 < \beta_s < 1. \tag{7}$$

By maximising total utility, defined as:

$$U_t = \sum_{i=0}^{T} \alpha_s^i u_{t+i} = \sum_{i=0}^{T} \alpha_s^i \left( \frac{c_t^{1-\beta_s}}{1-\beta_s} \right), \quad 0 < \alpha \leq 1 \tag{8}$$

subject to the budget constraint, the marginal condition becomes:

$$\frac{c_{t+1}^{-\beta_{s+1}}}{c_t^{-\beta_s}} = \frac{1}{\alpha_{s+1}(1+r)} . \tag{9}$$

Equation (9) can be interpreted as the Euler equation when the utility function and the subjective discount rate vary seasonally.

To capture the inherent randomness in consumption, a disturbance term $\varepsilon_{t+1}$ is added to (9). Taking natural logarithms leads to:

$$\beta_{s+1} \ln c_{t+1} = \beta_s \ln c_t - \ln\left[ \frac{1}{\alpha_{s+1}(1+r)} + \varepsilon_{t+1} \right]. \tag{10}$$

If expectations are rational, $\varepsilon_{t+1}$ represents the response of consumers to the arrival of new information in period t+1. Therefore, under rational expectations, the disturbance term has mean zero and is serially uncorrelated.

In Osborn (1988), if the stochastic term in equation (10) is assumed to be normal and identically distributed, namely

$$\ln\left[ \frac{1}{\alpha_{s+1}(1+r)} + \varepsilon_{t+1} \right] \sim N(\mu_{s+1}, \sigma^2) \tag{11}$$

a property of the log-normal distribution suggests the following conditional expectation (with $I_t$ being the information set at period t):

$$E\left[ \frac{1}{\alpha_{s+1}(1+r)} + \varepsilon_{t+1} \middle| I_t \right] = \exp\left( \mu_{s+1} + \frac{1}{2}\sigma^2 \right). \tag{12}$$

Evaluating the conditional expectation gives:

$$\frac{1}{\alpha_{s+1}(1+r)} = \exp\left(\mu_{s+1} + \frac{1}{2}\sigma^2\right) \quad (13)$$

which, after taking logarithms and rearranging, gives:

$$\mu_{s+1} = \ln\left[\frac{1}{\alpha_{s+1}(1+r)}\right] - \frac{1}{2}\sigma^2 . \quad (14)$$

Equation (10) can be approximated by:

$$\ln c_{t+1} = \gamma_{s+1} + \phi_{s+1} \ln c_t + u_{t+1} \quad (15)$$

where $\gamma_{s+1} = \frac{1}{\beta_{s+1}}\left[\ln \alpha_{s+1} + \ln(1+r) + \frac{1}{2}\sigma^2\right]$.

Note that, from equations (10) and (15), $\phi_{s+1} = \frac{\beta_s}{\beta_{s+1}}$ for s = 1,2,3,4, which directly implies that $\phi_1\phi_2\phi_3\phi_4 = 1$. Hence, equation (15) is a periodically integrated autoregressive process of order 1.

The next section presents the theory underlying periodic autoregressive models with a unit root.

## 4. Periodic Autoregressive Models With a Unit Root

A periodic autoregressive model of order 1, PAR(1), is given by:

$$y_t = \mu_s + \phi_s y_{t-1} + \varepsilon_t , \ s = 1,2,3,4 \quad (16)$$

where the coefficients differ for each season for quarterly observed data.

For an analysis of non-stationarity, a multivariate representation is used. The multivariate representation involves rewriting the model so that the coefficients are invariant to the seasons. By stacking the observations into vectors of annual observations, the (4x1) vector $Y_T$ is formed as follows:

$$Y_T = (Y_{1,T}, Y_{2,T}, Y_{3,T}, Y_{4,T})'$$

where $Y_{s,T}$ is the observation in season s (s = 1,2,3,4) in year T, where the annual index T = 1,2,...,N (with N = n/4). Using this notation, the univariate PAR(1) model can be represented by the following multivariate form:

$$\Phi_0 Y_T = \mu + \Phi_1 Y_{T-1} + \varepsilon_T \quad (17)$$

where $\mu = (\mu_1, \mu_2, \mu_3, \mu_4)'$ is the intercept vector and the $\varepsilon_T = (\varepsilon_{1,T}, \varepsilon_{2,T}, \varepsilon_{3,T}, \varepsilon_{4,T})'$, where $\varepsilon_{s,T}$ is the observation of the error process in season s of year T. The $\Phi$ parameters are (4x4) matrices containing the periodically-varying coefficients in the univariate PAR model.

The vector process $Y_T$ is stationary if the root of the characteristic equation:

$$|\Phi_0 - \Phi_1\lambda| = (1 - \phi_1\phi_2\phi_3\phi_4\lambda) = 0$$

is outside the unit circle. Hence, $Y_T$ is stationary if $|\phi_1\phi_2\phi_3\phi_4| < 1$. If $\phi_1\phi_2\phi_3\phi_4 = 1$, $Y_T$ and $y_t$ contain a unit root.

Formally, the null and alternative hypotheses of whether a PAR(1) process $y_t$ is periodically integrated of order 1 are:

$H_0$: $\phi_1\phi_2\phi_3\phi_4 = 1$
and
$H_1$: $|\phi_1\phi_2\phi_3\phi_4| < 1$.

The unrestricted PAR(1) model can be written as:

$$y_t = \sum_{s=1}^{4} \mu_s D_{st} + \sum_{s=1}^{4} \phi_s D_{st} y_{t-1} + \varepsilon_t . \quad (18)$$

Under the assumption of normality of the error process, and with fixed starting values, the maximum likelihood estimator of the parameters is the OLS estimator of the PAR(1) model.

Under the unit root hypothesis, the restricted model is:

$$y_t = \sum_{s=1}^{4} \mu_s D_{st} + \phi_1 D_{1t} y_{t-1} + \phi_2 D_{2t} y_{t-1} + \phi_3 D_{3t} y_{t-1} + (\phi_1\phi_2\phi_3)^{-1} D_{4t} y_{t-1} + \varepsilon_t \quad (19)$$

which can be estimated using non-linear least squares.

In order to test the unit root hypothesis, Boswijk and Franses (1997) consider the likelihood ratio statistic:

$$LR = n.\ln\left(\frac{RRSS}{URSS}\right)$$

where RRSS and URSS denote the restricted and unrestricted residual sums of squares, respectively. As the alternative hypothesis is one-sided, a studentized version of the LR statistic is given by:

$$LR_{\tau} = [\text{sign}(\hat{\phi}_1\hat{\phi}_2\hat{\phi}_3\hat{\phi}_4 - 1)].\sqrt{LR}$$

where the estimated coefficients are the OLS estimates of the parameters of the unrestricted model. Under the null hypothesis, the asymptotic distribution of the test is the same as that tabulated by Fuller (1976) for non-periodic autoregressive models.

## 5. Testing the Seasonal Life-Cycle Permanent Income Hypothesis

In this section, the Boswijk and Franses approach to testing for a periodic unit root is used to test the hypothesis for Germany.

The unrestricted PAR(1) model is estimated as follows (with standard errors in parentheses):

$$\begin{aligned} \ln\hat{c}_t &= \underset{(0.082)}{-0.899D_{1t}} + \underset{(0.068)}{0.525D_{2t}} + \underset{(0.073)}{0.136D_{3t}} \\ &+ \underset{(0.074)}{0.435D_{4t}} + \underset{(0.010)}{1.095D_{1t}\ln c_{t-1}} + \underset{(0.009)}{0.941D_{2t}\ln c_{t-1}} \\ &+ \underset{(0.009)}{0.984D_{3t}\ln c_{t-1}} + \underset{(0.009)}{0.958D_{4t}\ln c_{t-1}} \end{aligned}$$

<u>Diagnostics (with prob. values in brackets):</u>
$F_{AR,1\text{-}4} = 2.203$ [0.074]
$F_{ARCH,1\text{-}4} = 2.140$ [0.081]
LM(N) = 5.265 [0.072]
URSS = 0.01607.

With no first-to-fourth-order serial correlation ($F_{AR,1\text{-}4}$), no ARCH effects ($F_{ARCH,1\text{-}4}$) and normality of the residuals (LM(N)), a first-order PAR process is empirically adequate for German non-durable consumption.

In this case, the seasonal LC-PIH is empirically testable. Imposing the unit root restriction, the estimated model is (with standard errors in parentheses):

$$\begin{aligned} \ln\hat{c}_t &= \underset{(0.072)}{-0.963D_{1t}} + \underset{(0.060)}{0.472D_{2t}} + \underset{(0.064)}{0.079D_{3t}} \\ &+ \underset{(0.064)}{0.374D_{4t}} + \underset{(0.009)}{1.103D_{1t}\ln c_{t-1}} + \underset{(0.008)}{0.947D_{2t}\ln c_{t-1}} \\ &+ \underset{(0.008)}{0.991D_{3t}\ln c_{t-1}} + \underset{(0.008)}{0.965D_{4t}\ln c_{t-1}}. \end{aligned}$$

<u>Diagnostics (with prob. values in brackets):</u>
$F_{AR,1\text{-}4} = 2.215$ [0.073]
$F_{ARCH,1\text{-}4} = 3.101$ [0.019]
LM(N) = 4.085 [0.130]
RRSS = 0.01646.

The one-sided likelihood ratio test statistic is $LR_{\tau} = -1.632$, and the 5% critical value is -2.89. Thus, the null hypothesis of a periodic unit root cannot be rejected for German real consumption expenditure. With the absence of serial correlation, a PAR(1) model is empirically adequate for German consumption.

In Osborn (1988), seasonal habit persistence is introduced and tests are conducted to evaluate the empirical adequacies of the periodically integrated model against the seasonal habit persistence model. The next section applies a test for seasonal habit persistence to the German consumption series.

## 6. Seasonal Habit Persistence and Consumption

The intuition behind habit persistence is that consumption is dictated partly by habits. In times of high economic growth andhigh income, consumption expenditure will be above its trend level, and the desire for a high level of consumption is likely to remain in the future. As mentioned in Osborn (1988), the fact that the utility function is seasonal implies that consumers regard expenditure in each of the quarters as separate commodities. Thus, habit persistence is expected to be seasonal.

A generalisation of the periodically integrated model of order one to incorporate habit persistence is given by the following periodic seasonal ARIMA model:

$$(1-\phi_s L)(1-\theta_s L^4)\ln c_t = \gamma_s + \varepsilon_t. \quad \textbf{(20)}$$

Estimation of the seasonal habit persistence model involves imposing the periodic unit root restriction, given by:

$$\begin{aligned} \ln c_t &= \gamma_1 D_{1t} + \gamma_2 D_{2t} + \gamma_3 D_{3t} + \gamma_4 D_{4t} + \phi_1 D_{1t}\ln c_{t-1} \\ &+ \phi_2 D_{2t}\ln c_{t-1} + \phi_3 D_{3t}\ln c_{t-1} + (\phi_1\phi_2\phi_3)^{-1} D_{4t}\ln c_{t-1} \\ &+ \theta_1 D_{1t}\ln c_{t-4} + \theta_2 D_{2t}\ln c_{t-4} + \theta_3 D_{3t}\ln c_{t-4} \\ &+ \theta_4 D_{4t}\ln c_{t-4} - \phi_1\theta_4 D_{1t}\ln c_{t-5} - \phi_2\theta_1 D_{2t}\ln c_{t-5} \\ &- \phi_3\theta_2 D_{3t}\ln c_{t-5} - (\phi_1\phi_2\phi_3)^{-1}\theta_3 D_{4t}\ln c_{t-5} + \varepsilon_t. \end{aligned} \quad \textbf{(21)}$$

Table 1 presents the non-linear least squares estimation results for equation (21). The $F_{AR,1-4}$ test suggests that the estimated seasonal habit persistence model is empirically adequate for real non-durable consumption. The test for ARCH effects is rejected, indicating that there may be potential biases in the estimated standard errors of the seasonal habit persistence models. However, the rejection is marginal.

To test the PIAR(1) model against the seasonal habit persistence model, a test for the restriction $\theta_1 = \theta_2 = \theta_3 = \theta_4 = 0$, where $\theta_s$ is the coefficient of the seasonal lag (s = 1,2,3,4) is used. The statistic is 1.499 [0.209], which implies that the restriction is retained. This result is consistent with the test results in Section 5, whereby German consumption was found to have a periodic unit root. Since the restriction is retained, the most appropriate model for German real non-durable consumption is the periodically integrated model.

## 7. Concluding Remarks

In this paper, the sources of endogenous and exogenous seasonality in real non-durable consumption for Germany have been discussed, and deterministic and stochastic seasonality have been found to be significant. The rational expectations life-cycle permanent income hypothesis has been augmented with seasonality. Using formal tests for a periodic random walk, it is shown that the German consumption series are consistent with the seasonal extension of the life-cycle permanent income hypothesis. Seasonal habit persistence is introduced in the form of a restricted periodic seasonal ARIMA model, and is also found be empirically adequate for Germany. However, test for restrictions indicate that the seasonal life-cycle permanent income hypothesis is applicable for Germany. The empirical findings in the paper suggest that seasonality plays an important role in consumption and should not be omitted in the process of empirical modelling.


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**Table 1: Estimates of Seasonal Habit Persistence Model for Germany**

| Parameters | Estimates |
|---|---|
| $\gamma_1$ | -0.816* (0.128) |
| $\gamma_2$ | 0.455* (0.083) |
| $\gamma_3$ | 0.100 (0.070) |
| $\gamma_4$ | 0.315* (0.087) |
| $\phi_1$ | 0.981* (0.110) |
| $\phi_2$ | 1.068* (0.128) |
| $\phi_3$ | 0.870* (0.102) |
| $\phi_4 = (\phi_1\phi_2\phi_3)^{-1}$ | 1.096* (0.123) |
| $\theta_1$ | 0.153 (0.097) |
| $\theta_2$ | 0.044 (0.130) |
| $\theta_3$ | 0.156 (0.120) |
| $\theta_4$ | 0.047 (0.133) |

Note: * indicates significant at 5 percent. Standard errors are in parentheses.