Time-Varying CAPM Betas: Kalman Filter Estimates and Their Relationship to Macroeconomic Variables

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Abstract. It is well known that the CAPM beta is not stable over time. We investigate the nature of the time-variation in betas using monthly Australian data from 1979-1994 for 23 sectors. We estimate betas for sub-periods. We test the market model used to estimate the betas for statistical adequacy. We estimate time-varying betas using the Kalman Filter. We find considerable time-variation in the estimated betas and find that many are non-stationary. Finally we relate the estimated time-varying betas to macroeconomic variables and find that the return to the market portfolio is the main explanatory variable.

1. INTRODUCTION

The "beta" of the Capital Asset Pricing Model (CAPM) is the slope coefficient in a regression of an asset's return on the return to a market portfolio (the "market model") and is a central measure of risk in finance. The CAPM predicts that the variation in expected returns across assets is related linearly to variation in βs.

An important assumption underlying the standard CAPM is that asset returns are stationary so that their distributions have time-invariant moments. This implies that an asset's β does not change over time and is implicit in tests of the CAPM where the mean return to an asset is measured by the sample mean over some period of time and the asset's β is estimated as the (constant) slope parameter in the market model. The time-invariance assumption is also implied in the practice of using a β estimated over a given period to make inferences (about asset value, say) in some different (normally future) time period.

It has long been recognised, though, that asset returns may not be stationary in practice; see, e.g., Bos and Newbold (1984) for the US and Faff, Lee and Fry (1992) and Brooks, Faff and Lee (1992, 1994) for evidence using Australian data. This paper provides further evidence on beta instability for Australia; it then goes on to report estimates of time-varying betas based on the use of the Kalman Filter, examines the time-series properties of the estimated betas and finally explores the relationship of the estimated betas to a set of macroeconomic variables.

We find evidence of widespread β instability for a set of 23 Australian industry portfolios. Approximately half the industries were found to have non-stationary betas. Much of the variability of the βs over the sample is explained by a time trend and a break both in intercept and trend at October 1987. Once the non-stationarity of the betas was taken into account (by specifying them in first-difference form), an analysis of their relationship to macroeconomic variables found that only the return to the market portfolio has any explanatory power and this relationship appears to have been stronger after the 1987 Crash.

2. THE DATA AND PRELIMINARY EVIDENCE

The results are based on monthly returns calculated from 23 industry share-price indexes obtained from the Australian Stock Exchange for the period December 1979 to February 1994. The return to the market portfolio was based on the All Ordinaries Index. Summary statistics for the 23 industry return series indicate widespread departure from normality although these departures are less marked when the observation for October 1987 is omitted. These features are not unusual for financial data.

Preliminary investigation of beta stability was carried out in various ways. The first was to estimate the betas over sub-periods and compare the estimated betas for these shorter samples with each other and with the full-sample betas. Considerable variability of betas over time was found.

Secondly, more formal diagnostic tests of the market model were carried out on the basis of the argument that if the betas are time-varying, a model which assumes them to be constant will show evidence of mis-specification. The Durbin-Watson statistic, tests based on the recursive residuals (Harvey's t and F tests) and the RESET test of functional form all provide little evidence of mis-specification – at most 5 of the 23 sectors appear to have problems. A more direct test of parameter stability is the Chow test which, however, requires a prior specification of the break-point. If October 1987 is chosen as a break-point, stability of beta is rejected for 5 of the 23 sectors. If a break-point is permitted at the end of 1984 and again at the end of

1989 to match the five-year sub-samples used earlier, a dummy variable interacting with beta is significant in 13 of the 23 equations suggesting considerable instability in the betas over shorter periods.

3. TIME-VARYING PARAMETER ESTIMATES

3.1 Recursive and Rolling Betas

Before presenting the time-varying betas estimated using the Kalman Filter, we briefly discuss estimates based on recursive and rolling regressions. The recursive betas were obtained from a recursive application of OLS to the market model while the rolling betas were obtained from the application of OLS for a fixed 50-period sample period which was progressively moved from the beginning to the end of the full sample period.

A selection of the estimated βs is pictured in Figure 1. There is clearly considerable variability in the recursive and rolling βs although less in the Kalman Filter βs. In many cases there is a distinct break at the date of the October 1987 Crash (observation 40), usually in level but often also in trend. The way in which the recursive and rolling βs vary over the sample period parallels the sub-period OLS results discussed in the previous section.

After estimating the betas we proceeded in each case to first analyse the time-series properties of the betas by testing them for stationarity and regressing them against a time trend (including a break at October 1987) and then regressing them against a selection of macroeconomic variables.

The βs for all sectors are non-stationary, whether estimated by the recursive or the rolling procedure. Thus the βs not only change over time but have no tendency to vary about a fixed mean. Most of the time variation in the βs is explained by a trend term and a break both in level and trend, with the break occurring at October 1987. The regression results coincide with the graphs presented in Figure 1

The second stage of our analysis of the recursive and rolling βs involved an investigation of their relationship to a variety of macroeconomic variables. Our choice of macro variables was based on the hypothesis that returns are influenced by three classes of factors - real domestic activity, nominal domestic factors and foreign variables. Changes in any of these variables may conceivably influence agents' risk perceptions and therefore the βs. Of all the macro variables experimented with only the market return (Rm) proved to have any appreciable explanatory power for most sectors. Hence we proceeded only with Rm and added a trend, and breaks at October 1987. Not surprisingly, the trend term was generally insignificant given the first-difference form of the dependent variable. The October 1987 dummy variable was also generally not significant alone but did prove significant when interacted with Rm. The estimated equations exhibit surprisingly high explanatory power, especially for the recursive betas, given that the dependent variable is a first difference. There is strong evidence of a break in the slope coefficient at October 1987 with the RmD87 interaction term being significant for 19 of the 23 sectors. The Rm term is significant in about half of the sectors. It is noteworthy that in most cases the coefficients of Rm and RmD87 are of the opposite sign.

3.2 Kalman Filter Betas

The use of the Kalman Filter proceeds as follows. Consider the linear regression model with time-varying coefficients:

$$(1) \qquad y_t = x_t'\beta_t + \varepsilon_t$$

where x_t and β_t are k-component vectors and ε_t is a random error term with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t^2) = n_t$. In the Kalman Filter model equation (1) is the measurement equation. The evolution of the time-varying parameter vector, β_t, is given by the state equation which in the model used in this paper has the AR (1) form:

$$(2) \qquad \beta_t = \rho\beta_{t-1} + (1-\rho)\overline{\beta} + \eta_t$$

where ρ is the AR (1) parameter, $\overline{\beta}$ is a constant and η_t is a vector of random variables each of which is uncorrelated with ε_t, $E(\eta_t) = 0$ and $E(\eta_t\eta_t') = M_t$.

The Kalman Filter estimates β_t conditional on y_t, x_t, ρ, $\overline{\beta}$, $\hat{\beta}_{t-1}$ and $\hat{\Sigma}_{t-1}$, the estimated covariance matrix of $\hat{\beta}_{t-1}$. The estimation was carried out in RATS. To apply the Kalman Filter to the estimation of β_t in (1) we need starting values for $\hat{\beta}$ and $\hat{\Sigma}$, values for ρ and $\overline{\beta}$ and for the entire time series for n_t and M_t. It was assumed that n and M are constant over the sample and that only the slope coefficient of the market model is time-varying so that only one element of M needs to be obtained. The value of ρ was chosen so as to maximise the quasi-likelihood function which RATS evaluates for each iteration of the Kalman Filter. Further details on starting values can be found in Groenewold and Fraser (1997). The optimal value of ρ for each sector is reported in Table 1 together with the time-series characteristics of the Kalman βs.

The first column of figures in the table gives the optimal value for ρ, the AR (1) parameter in the transition equations for the βs. Approximately half the sectors have βs based on an optimal ρ in the range 0.8-1.0 and the next two column show that in most cases the two unit-root tests used are unable to reject a (false) null of non-stationarity. There is one sector with a value of ρ in the 0.7-0.8 range and here the tests provide mixed evidence of a unit root – the ADF test indicated non-stationarity while the PP test rejects the non-stationary null hypothesis. Even the value of ρ of 0.39 for the Other Metals sector produces an ADF statistic which point to non-stationarity. The six sectors which have optimal ρ values of approximately zero are all found to have stationary βs with the exception of the Banks sector which fails to reject non-stationarity with the ADF test. Finally, there is one sector with a value of ρ which is substantially negative.

The remainder of Table 1 reports the results of regressing the βs on a trend, a dummy variable for October 1987 and an interaction term formed from these two variables. There is clearly a great deal of variation in the extent to which β could be explained, judging from the R^2s. The results may be roughly divided into two groups; the first has high estimated ρ, non-stationary β, high R^2, low Durbin-Watson statistic and significant breaks both in level and trend and October 1987. The other group has generally stationary β, low ρ, low R^2 and often insignificant regressors in the equations explaining the estimated βs.

We turn, finally, to the relationship between the Kalman βs and macroeconomic variables. We began by regressing the βs (in first-difference form to remove the effects of non-stationarity) on each of the macro variables in turn. Only the market return, Rm, proved to have significant explanatory power. We then added a dummy variable for October 1987 both alone and in interaction with Rm. Only the latter proved significant. The results are reported in Table 2. The equations achieve only modest explanatory power although it must be recalled that the dependent variable is in first-difference form. The Durbin-Watson statistic is relatively large for several of the sectors; these are all sectors with a low estimated ρ and stationary β and it is likely that the high Durbin-Watson statistics are the result of negative autocorrelation induced by differencing already stationary series. The return to the market, Rm, is significant in only a small number of sectors but with interaction with the October 1987 dummy variable it is significant for the majority of sectors. Since D87 takes on the value of 1 after October 1987, these results indicate a more consistent relationship between $\Delta\beta$ and Rm after the Crash. For most of the cases reported in Table 2, the coefficients of Rm and Rm D87 have opposite signs indicating a tendency for the pre-Crash effect to be offset after the Crash.

4. CONCLUSIONS

We have used monthly data on 23 Australian industry share-price indexes for the period 1979(12)-1994(2) to test the specification of the constant-beta market model and to estimate and examine time-varying betas. Standard tests of the market model fail to detect widespread specification problems but significant structural breaks were found when dummy variables were used at approximately five-year intervals for over one half of the sectors in the sample.

Time-varying betas were estimated using Kalman Filter. Many of the resulting betas were found to be non-stationary but generally well explained by a time trend and a dummy variable at October 1987 both alone and in interaction with the trend. When the estimated betas were regressed in first-difference form on a range of macroeconomic variables, only the return to the market portfolio, both alone and in interaction with an October 1987 dummy variable, proved to be a significant explanatory variable. Thus betas vary systematically over the cycle defined by the return to the market as a whole but the relationship changed at October 1987.

TABLE 1
TIME-SERIES PROPERTIES OF THE KALMAN BETAS

Sector	$\hat{\rho}$	Stationarity ADF	Stationarity PP	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	R^2	DW
				$\beta_t = \gamma_0 + \gamma_1 t + \gamma_2 D87 + \gamma_3 D87t + \varepsilon_t$				
Alcohol & Tobacco	0.81	7.52	7.62	0.0001 (2.14)	0.0203 (4.81)	-0.0002 (3.86)	0.2283	0.51
Banks	-0.00	4.28	65.95	-0.0000 (0.52)	-0.0006 (0.86)	0.0000 (0.88)	0.0244	2.17
Building Materials	0.90	2.70	3.20	0.0001 (3.22)	0.0048 (1.24)	-0.0001 (2.47)	0.1802	0.31
Chemicals	0.86	5.21	4.79	0.0002 (1.53)	0.0116 (1.26)	-0.0001 (1.32)	0.0312	0.32
Developers & Contractors	0.94	2.52	2.51	0.0010 (5.60)	0.2147 (12.82)	-0.0019 (9.51)	0.7098	0.17
Diversified Industrials	0.92	1.56	5.89	0.0002 (3.51)	0.0528 (9.64)	-0.0004 (6.62)	0.6382	0.61
Diversified Resources	0.00	11.53	56.15	-0.0000 (1.34)	-0.0049 (2.02)	0.0001 (1.89)	0.0458	2.08
Engineering	0.82	2.57	4.68	0.0000 (0.13)	-0.0113 (5.08)	0.0001 (2.70)	0.4410	0.40
Entrepreneurial	0.35	11.71	31.86	-0.0000 (1.06)	0.0025 (0.58)	0.0000 (0.15)	0.0628	1.40
Finance & Investment Services	0.95	2.50	2.49	0.0030 (13.56)	0.5330 (26.91)	-0.0047 (19.99)	0.9247	0.32
Food & Household Goods	0.95	3.48	6.51	0.0008 (16.37)	0.1018 (22.34)	-0.0010 (18.79)	0.8837	0.28
Gold	0.00	8.79	59.49	0.0001 (1.00)	-0.0001 (0.18)	-0.0000 (0.42)	0.0387	2.04
Insurance	0.97	4.42	6.88	0.0048 (25.23)	0.5505 (31.65)	-0.0056 (27.38)	0.9407	0.23
Media	0.94	2.67	5.16	0.0012 (13.15)	0.0908 (10.94)	-0.0011 (11.07)	0.7634	0.19
Misc Industrials	0.00	6.72	51.83	-0.0000 (0.21)	0.0010 (0.76)	-0.0000 (0.33)	0.0239	1.88
Misc Services	-0.74	13.80	685.53	-0.0001 (0.83)	-0.0077 (0.60)	0.0001 (0.75)	0.0068	3.66
Oil and Gas	0.98	2.82	3.28	-0.0080 (19.43)	-0.8459 (22.67)	0.0078 (17.72)	0.9589	0.32
Other Metals	0.39	1.88	35.38	0.0001 (1.82)	0.0070 (2.26)	-0.0001 (2.27)	0.0541	1.59
Paper & Packaging	-0.00	7.40	65.17	-0.0000 (1.60)	-0.0017 (2.06)	0.0000 (2.14)	0.0709	2.24
Property Trusts	0.00	14.18	52.39	-0.0000 (0.12)	0.0004 (0.61)	-0.0000 (0.24)	0.0172	1.89
Retail	0.95	3.62	3.71	0.0010 (6.63)	0.1506 (11.68)	-0.0013 (8.68)	0.7281	0.27
Solid Fuels	0.96	4.35	4.45	-0.0029 (28.23)	-0.3178 (33.75)	0.0033 (29.42)	0.9529	0.30
Transport	0.73	4.62	8.76	0.0000 (1.66)	0.0026 (2.39)	-0.0000 (2.41)	0.1008	0.60

Notes: 10% critical value for ADF and PP is 5.34. Figures in parentheses are t-ratios.

TABLE 2
KALMAN BETAS AND MACROECONOMIC VARIABLES

$$\Delta\beta_t = \gamma_0 + \gamma_1 R_{mt} + \gamma_2 D87 R_m + \varepsilon_t$$

Sector	$\hat{\gamma}_1$	$\hat{\gamma}_2$	R^2	DW
Alcohol & Tobacco	0.0093 (1.56)	-0.0377 (5.77)	0.4995	2.07
Banks	-0.0011 (0.39)	0.0038 (1.22)	0.0387	3.10
Building Materials	-0.0007 (0.13)	0.0009 (0.15)	0.0002	2.15
Chemicals	0.0214 (1.60)	-0.0072 (0.49)	0.0664	2.30
Developers & Contractors	0.0241 (1.18)	-0.0976 (4.36)	0.3623	2.05
Diversified Industrials	-0.0054 (0.50)	-0.0351 (2.99)	0.3807	1.93
Diversified Resources	-0.0105 (1.33)	0.0329 (3.81)	0.2672	2.62
Engineering	-0.0022 (0.58)	0.0127 (3.08)	0.2538	2.17
Entrepreneurial	0.0027 (0.24)	-0.0431 (3.57)	0.3665	2.18
Finance & Investment Services	0.0205 (0.60)	-0.2154 (5.74)	0.5825	2.10
Food & Household Goods	0.0078 (1.21)	-0.0249 (3.52)	0.2392	1.91
Gold	0.0029 (1.03)	0.0023 (0.73)	0.1332	2.77
Insurance	0.0925 (4.45)	-0.1709 (7.49)	0.4391	1.91
Media	0.0143 (1.79)	-0.0258 (2.95)	0.1053	2.00
Misc Industrials	-0.0000 (0.00)	-0.0110 (2.46)	0.2391	2.56
Misc Services	-0.0542 (0.86)	-0.0337 (0.49)	0.0823	3.57
Oil and Gas	-0.2603 (5.02)	0.3982 (6.99)	0.3434	1.87
Other Metals	0.0043 (0.42)	0.0088 (0.79)	0.0681	2.73
Paper & Packaging	-0.0015 (0.47)	0.0045 (1.29)	0.0384	3.15
Property Trusts	-0.0009 (0.43)	-0.0058 (2.38)	0.2854	2.43
Retail	-0.0084 (0.44)	-0.0436 (2.09)	0.2436	1.94
Solid Fuels	-0.0326 (2.31)	0.0466 (3.01)	0.0819	2.07
Transport	0.0045 (2.22)	-0.0084 (3.81)	0.1723	2.20

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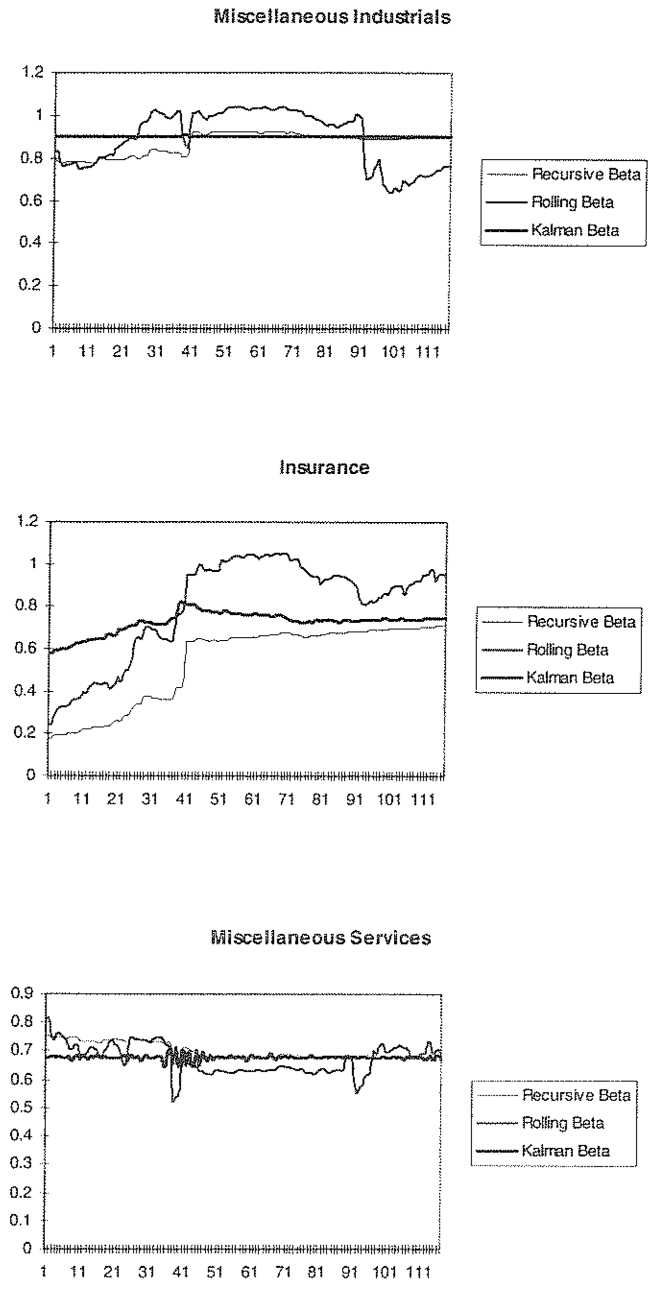

Figure 1: A selection of industry betas