

# An Estimating Model of Soil Erosion Rate Using $^{137}\text{Cs}$ in Soil Profile for Uncultivated Soil

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**Abstract** By using the mass balance model and by introducing an erosion constant and depth distribution functions of  $^{137}\text{Cs}$  in soil, a quantitative model of soil erosion rate using  $^{137}\text{Cs}$  loss was developed for uncultivated soils. Depth distribution pattern of  $^{137}\text{Cs}$  in the soil profile, sampling year and the amount of  $^{137}\text{Cs}$  fallout each year are considered to overcome some uncertainty due to depth distribution existing and  $^{137}\text{Cs}$  fallout difference each year in undisturbed soil profiles. The model shows that the estimate rate of soil erosion is mainly controlled by the distribution pattern of  $^{137}\text{Cs}$  in the soil profile. By inputting different depth distribution functions of  $^{137}\text{Cs}$ , the year of sampling and the different input fraction of total fallout value each year, several simulation results of soil loss were given. The results of numerical simulation proved that the relationship between the rate of soil loss and  $^{137}\text{Cs}$  depletion is neither linear nor logarithmic. They are depended on the distribution patterns of  $^{137}\text{Cs}$  in the soil profile, sampling year and input fraction of total fallout value.

## 1. INTRODUCTION

Soil degradation in the form of erosion has been recognized as a serious environmental problem in many parts of the world, but the quantitative evaluation of soil loss remains difficulty. The world-wide fallout of Caesium-137 ( $^{137}\text{Cs}$ ) associated with the nuclear weapon testing during the 1950s and 1960s has provided a valuable man-made tracer for studies of soil erosion and sediment delivery (e.g. Ritchie *et al.*, 1974). Generally, an input or reference value of  $^{137}\text{Cs}$  is determined and compared with soil inventories of  $^{137}\text{Cs}$  in erosion sites. Ritchie and McHenry (1990) reviewed the method in detail. In order to provide quantitative estimates of rates of erosion, it is necessary to establish a relationship between the amount of  $^{137}\text{Cs}$  lost from the soil profile (usually expressed as percentage of the local reference or input value) and the rate of erosion. There have been a lot of methods used to calculate soil loss rates from  $^{137}\text{Cs}$  measurements as Walling and Quine (1990) reviewed. These methods can be concluded to two types, empirical relationship (e.g. Ritchie and McHenry, 1975; Wilkin and Hebel, 1982; Campbell *et al.* 1986; Loughran *et al.*, 1990; Elliott *et al.*, 1990) and theoretical models (e.g. Mitchell *et al.*, 1980; Brown *et al.*, 1981a, 1981b; de Jong *et al.*, 1983, 1986; Kachanoski and de Jong, 1984; Fredericks and Perrens, 1988; Zhang *et al.*, 1990; Cao *et al.*, 1993; Garcia-Oliva *et al.*, 1995). However, all the models did not consider the  $^{137}\text{Cs}$  distribution pattern in the soil profile, although some researches had modified the proportional model by considering  $^{137}\text{Cs}$  in undisturbed soils generally decreased exponentially with depth (e.g. Garcia-Oliva *et al.*, 1995) as well as the amount of  $^{137}\text{Cs}$  fallout deposition every year. It is well known that  $^{137}\text{Cs}$  is not uniformly distributed with soil depth in undisturbed soil profiles. Many researchers have shown that even in cultivated field,  $^{137}\text{Cs}$  is not uniformly distributed in soil profile (e.g. Brown *et al.*, 1981a; Cooper *et al.*, 1987; Soileau *et al.*, 1990; Zhang *et al.* 1990; Bulygin *et al.*, 1993; Quine *et al.*, 1994). If  $^{137}\text{Cs}$  is concentrated near the soil surface, a small soil loss will result in a comparatively large loss in  $^{137}\text{Cs}$ . In other

words, since  $^{137}\text{Cs}$  distribution pattern differs between soil profile, soil loss may not be equal even if loss of  $^{137}\text{Cs}$  is same. Without considering depth distribution of  $^{137}\text{Cs}$ , it might overestimate or underestimate erosion rates, especially use proportional method for undisturbed soil which would overestimate net erosion as mentioned by Garcia-Oliva *et al.* (1995). To establish a quantitative model using amount of  $^{137}\text{Cs}$  to estimate rate of erosion from uncultivated soil, depth distribution pattern of  $^{137}\text{Cs}$  in soil profile must be considered. Sampling year also influences the relation between the amount of  $^{137}\text{Cs}$  lost from the soil profile and the rate of erosion due to  $^{137}\text{Cs}$  inventory is time-dependent. However, it is only used to calculate mean annual soil loss simply by arithmetic mean (divided by time period, e.g. de Jong *et al.*, 1986; Soileau *et al.*, 1990).

The objective of this article is to present a quantitative model which relates the amount of  $^{137}\text{Cs}$  lost from an uncultivated soil profile to the rate of soil erosion. According to mass balance model, we considered  $^{137}\text{Cs}$  distribution pattern in the soil profile, sampling year and the difference of the amount in  $^{137}\text{Cs}$  fallout every year.

## 2. QUANTITATIVE MODEL

### 2.1 Mass Balance Model

Roughly following Kachanoski and de Jong (1984), we adopt the follow mass balance model, the  $^{137}\text{Cs}$  inventory at the end of a given year can be expressed as:

$$S_t = S_{t-1} + F_t - E_t \quad (t=1,2, \dots, N) \quad (1)$$

where:  $S_t$  and  $S_{t-1}$  are total  $^{137}\text{Cs}$  in inventory profile at end of year  $t$  and  $t-1$ , respectively ( $\text{Bq m}^{-2}$ ),

$F_t$  is fallout deposition during year  $t$  ( $\text{Bq m}^{-2}$ ),

$E_t$  is the amount of  $^{137}\text{Cs}$  lost from the soil profile during year  $t$  ( $\text{Bq m}^{-2}$ ), and

$N = M - 1954$ ,  $M$  represent the year of sampling.

Here, Radioactive decay of  $^{137}\text{Cs}$  is ignored.

This basic model can be adapted to reflect local conditions and to estimate the amount of  $^{137}\text{Cs}$  remaining in a soil subject to a specified erosion rate during a specified period of time. We use this model to establish relationship between mean annual soil loss and percentage reduction in the reference  $^{137}\text{Cs}$  inventory.

## 2.2 The Amount of $^{137}\text{Cs}$ of Fallout Deposition during Year t

The amount of  $^{137}\text{Cs}$  of fallout deposition during a given year ( $F_t$ ) can be expressed as:

$$F_t = r_t C_T \quad (t=1,2,3 \dots, N) \quad (2)$$

where:  $F_t$  is the amount of  $^{137}\text{Cs}$  deposited during a given year t ( $\text{Bq m}^{-2}$ ),

$$C_T = \sum_{t=1}^N F_t \quad (3)$$

is the total  $^{137}\text{Cs}$  deposited in given research area ( $\text{Bq m}^{-2}$ ) in N years,

$r_t = F_t / \sum F_t = F_t / C_T$  is input fraction of the total  $^{137}\text{Cs}$  deposited during a given year t.

Since it is difficult to determine the total input  $^{137}\text{Cs}$  in given research area ( $C_T$ ), we may use reference  $^{137}\text{Cs}$  inventory in the research area ( $C_R$ ) instead of  $C_T$  to calculate the amount of  $^{137}\text{Cs}$  fallout deposition during a given year t:

$$F_t = r_t C_R \quad (t= 1, 2, \dots, N) \quad (4)$$

where  $C_R$  is reference  $^{137}\text{Cs}$  inventory ( $\text{Bq m}^{-2}$ ) of sampling year.

Based on radioactive fallout measurements (Wise, 1980 ; Longmore ,1982; Cambray et al. 1985), fallout deposition of  $^{137}\text{Cs}$  mainly occurred from 1954 to 1982. However, it is difficult to know the amount of  $^{137}\text{Cs}$  of fallout deposition during an individual year for a research area. Although amount of total fallout is different in different area, we may assume that every input fraction of the total  $^{137}\text{Cs}$  fallout deposition during a given year t (i.e.  $R_t$ ,  $t=1,2,3, \dots, N$ ) is the same for Northern Hemisphere. Based on Figure 2 of Walling and Quine (1990) we get  $^{137}\text{Cs}$  input fractions in the Northern Hemisphere (Table 1). Although, in some areas of the world an additional short-term input was received in 1986 as a result of the Chernobyl accident, it is not considered in our calculation.

## 2.3 Introducing Depth Function and Erosion Constant

If the  $^{137}\text{Cs}$  depth distribution in soil profile in the reference inventory can be described by following function:

$$Cs = f(z) \quad (5)$$

where  $Cs$  is concentration of  $^{137}\text{Cs}$  at a given depth ( $\text{Bq kg}^{-2}$ ),  $f(z)$  represents a regressive function,  $z$  is depth (m). Thus,

$$C_R = \int_0^H Df(z)dz \quad (6)$$

where  $C_R$  is reference  $^{137}\text{Cs}$  inventory ( $\text{Bq m}^{-2}$ ),  $D$  is bulk density of soil ( $\text{kg m}^{-3}$ ) and  $H$  is the thickness in which  $^{137}\text{Cs}$  can be detected. Usually,  $H$  is less than 50cm.

**Table 1** The typical annual value of  $^{137}\text{Cs}$  fallout ( $F_t$ ) and its fraction ( $R_t$ ) for a site in the Northern Hemisphere (Based on Walling and Quine, 1990)

Year	t	$F_t^*$ ( $\text{mBq cm}^{-2}$ )	$R_t$ (%)**
1954	1	5.9	1.18
1955	2	15.3	3.05
1956	3	17.1	3.41
1957	4	19.8	3.95
1958	5	32.4	6.47
1959	6	36.2	7.23
1960	7	10.0	2.00
1961	8	14.2	2.83
1962	9	54.4	10.86
1963	10	125.0	24.96
1964	11	68.2	13.62
1965	12	30.0	5.99
1966	13	18.0	3.59
1967	14	6.6	1.32
1968	15	6.6	1.32
1969	16	4.4	0.88
1970	17	6.0	1.20
1971	18	6.0	1.20
1972	19	4.2	0.84
1973	20	2.0	0.40
1974	21	4.2	0.84
1975	22	1.8	0.36
1976	23	1.7	0.34
1977	24	3.3	0.66
1978	25	3.7	0.74
1979	26	1.4	0.28
1980	27	0.4	0.08
1981	28	1.8	0.36
1982	29	0.3	0.06
After 1982 (M)	M-1953	0	0
Total	-	500.90	100

\* Based on Figure 2 of Walling and Quine (1990);

\*\*  $R_t = F_t / \sum F_t * 100\%$

If we take mean annual thickness of soil loss as  $h$  and assume the soil loss occurred only at soil surface, then mean annual relative loss of  $^{137}\text{Cs}$  as a fraction of the total present in the profile could be expressed approximately as follow:

$$\text{Mean annual relative loss of } ^{137}\text{Cs} = \frac{\int_0^h Df(z)dz - \int_h^H Df(z)dz}{\int_0^H Df(z)dz} = \frac{\int_0^h Df(z)dz}{C_R} \quad (7)$$

Here we introduce an erosion constant ( $\lambda$ ) defined as mean annual relative loss of  $^{137}\text{Cs}$  as a fraction of the total  $^{137}\text{Cs}$  present in the profile. Thus,  $\lambda$  is expressed as:

$$\lambda = \frac{\int_0^h Df(z)dz}{C_R} \quad (7)$$

## 2.4 The Amount of $^{137}\text{Cs}$ Lost of a Given Year

Let  $\lambda_t$  as the fraction of  $^{137}\text{Cs}$  loss of year t over last year's amount and fallout of the same year in the profile, that is,

$$\lambda_t = E_t / (S_{t-1} + F_t) \quad (1 \geq \lambda \geq 0) \quad (8)$$

We assume that  $^{137}\text{Cs}$  depth distribution pattern of each year is similar to each other, as well as similar to the reference  $^{137}\text{Cs}$  inventory. That is, if the  $^{137}\text{Cs}$  shows an exponential decrease with soil depth between 0 - 20cm in year t, it would show a similar exponential distribution between 0 - 20cm in another year. And in the reference place the  $^{137}\text{Cs}$  also shows a similar exponential decrease with soil depth between 0 - 20cm in the sampling year. The several years of observations by Rogowski and Tamura (1970) and Filipovic-Vincekovic *et al.* (1991) had shown some evidence proving the assumption. Then, we can conjecture, at least in the first approximation, that:

$$\lambda_t = \lambda = \text{Constant} \quad (t = 1, 2, \dots, N; 1 \geq \lambda \geq 0) \quad (9)$$

Introducing Equation 9 into Equation 8, we get

$$E_t = \lambda (S_{t-1} + F_t) \quad (10)$$

## 2.5 Establish Soil Erosion Equation

Introducing Equations 4 and 10 into Equation 1 we get:

$$S_t = (S_{t-1} + r_t C_R) (1 - \lambda) \quad (t=1, 2, \dots, N) \quad (11)$$

Let  $t=N$ , we get:

$$S_N = r_1 C_R (1 - \lambda)^N + r_2 C_R (1 - \lambda)^{N-1} + r_3 C_R (1 - \lambda)^{N-2} + \dots + r_N C_R (1 - \lambda), \quad (12)$$

In fact, where  $S_N$  is the  $^{137}\text{Cs}$  amount in the eroded soil profile at sampling year ( $C_E$ ), that is  $S_N = C_E$ . Then, Equation 12 can be changed as:

$$(C_R - C_E) / C_R = 1 - [r_1 (1 - \lambda)^N + r_2 (1 - \lambda)^{N-1} + r_3 (1 - \lambda)^{N-2} + \dots + r_N] (1 - \lambda) \quad (13)$$

Let

$$Y = (C_R - C_E) / C_R \quad 100 \quad (\%) \quad (14)$$

$$R_t = 100 r_t \quad (15)$$

Where Y is the percentage loss in total  $^{137}\text{Cs}$  of the sampling year,  $R_t$  is percentage of total  $^{137}\text{Cs}$  fallout deposition during a given year t. If ignoring  $^{137}\text{Cs}$  fallout after 1982 as shown in Table 1 (e.g.  $R_t = 0$ , when  $t > 29$ ) and introducing Y and  $R_t$  into Equation 13, we get:

$$Y = 100 - [R_1 (1 - \lambda)^{29} + R_2 (1 - \lambda)^{28} + R_3 (1 - \lambda)^{27} + \dots + R_{29}] (1 - \lambda)^{M-1982} \quad (M \geq 1983) \quad (16)$$

where M represent the year of sampling. The right of Equation 16 is increased progressively, so Equation. 16 has only one solution ( $1 \geq \lambda \geq 0$ ) for a given Y ( $100 \geq Y \geq 0$ ). If, the net loss of  $^{137}\text{Cs}$  at sampling area relative to the reference place (Y) is decided through measurement,  $\lambda$  can be solved using numerical solution or graphic methods. Figure 1 shows the relationship between erosion constant

( $\lambda$ ), total  $^{137}\text{Cs}$  loss (%) and sampling years by using Equation 16 and  $R_t$  of Table 1. If  $R_t$  is assumed to be the same as Table 1,  $\lambda$  can be got from Figure 1.

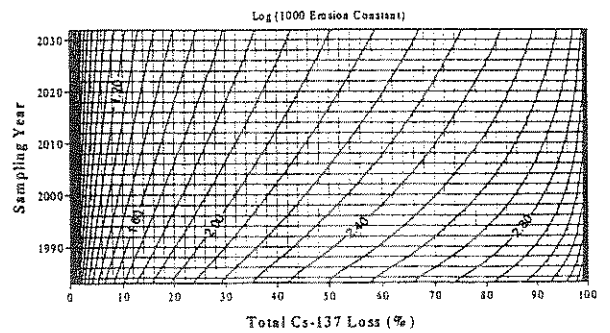


Figure 1: Relationship between erosion constant ( $\lambda$ ), total  $^{137}\text{Cs}$  loss (%) and sampling years deduced using Equation 16 and  $R_t$  of Table 1.

## 2.6 Mean annual soil loss

As described above, If detailed reference inventory data is obtained, we can know the depth function ( $f(z)$ ). If relative  $^{137}\text{Cs}$  loss of an eroded uncultivated field (Y) is obtained, we can know the erosion constant ( $\lambda$ ) by introducing fallout fraction of total fallout amount from Table 1 or other sources to Equation 16 or by Figure 1. After getting  $f(z)$  and  $\lambda$ , we can get mean annual erosion thickness (h) by Equation 7 as shown later. After we get a value of h we may get mean annual soil loss using following equation,

$$E_R = 10000 D h, \quad (17)$$

where  $E_R$  is mean annual soil loss ( $\text{kg ha}^{-1} \text{y}^{-1}$ ),  
h, mean annual thickness of soil loss (m), and  
D= bulk density of soil ( $\text{kg m}^{-3}$ ).

Therefore, Equations. 16 or 13 (if sampling year earlier than 1983), 7 and 17 form a quantitative model of soil erosion rate using  $^{137}\text{Cs}$  for uncultivated soil. From the equations of the model it is clear that this model relates relative  $^{137}\text{Cs}$  loss, depth function of  $^{137}\text{Cs}$  of reference inventory, sampling year and  $^{137}\text{Cs}$  fallout fraction of total fallout amount to soil erosion.

## 2.7 Concrete Models for Typical $^{137}\text{Cs}$ Depth Distributions

According to lots of published literatures on  $^{137}\text{Cs}$ , the depth distribution pattern of  $^{137}\text{Cs}$  in undisturbed soil profile could be divided into three types (Du *et al.*, 1997). The following are the regressive functions for the three types, respectively:

$$C_s = a e^{-bz} \quad (a > 0, b > 0), \quad (18)$$

$$C_s = a \left[ 1 - \left( k - \frac{z}{H} \right)^b \right] \left( k - \frac{z}{H} \right)^{b-1}, \quad (19)$$

( $a > 0, b > 0$  and  $0 < k \leq 1$ )

$$C_s = a \left( 1 - \frac{z}{H} \right)^b \quad (a > 0, b > 0), \quad (20)$$

where  $C_s$  is concentration of  $^{137}\text{Cs}$  at a given depth ( $\text{Bq m}^{-2}$ ),

$z$  represents given depth in soil profile (m),

$a$ ,  $b$ , and  $k$  are coefficient constants, respectively.

We call these three types as exponential type (Equation 18), peak type (Equation 19) and decreasing type (Equation 20), respectively. Examples of the three types of depth distribution pattern are shown in Figure 2. For a real case for using the model produced here, the coefficients, as well as the depth function type can be obtained from observation data though detailed measurement method such as described in Wallbrink and Murray (1996).

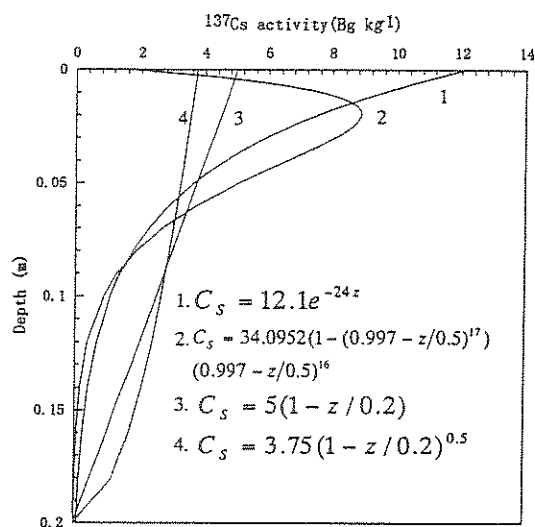


Figure 2: Typical types of  $^{137}\text{Cs}$  depth distribution patterns and their equations for uncultivated field.

By introducing Equations 18 to 20 into Equations 6, 7 and 17, respectively, we can get mean annual thickness of soil loss ( $h$ ) and then the relationship between mean annual soil loss ( $E_R$ ) and the erosion constant ( $\lambda$ ) as follows:

For type  $C_s = ae^{-bz}$ ,

$$E_R = -10000D \ln(1-\lambda)/b \quad (21)$$

For type  $C_s = a(1 - (k - z/H)^b)(k - z/H)^{b-1}$ ,

$$E_R = 10000DH \left\{ k - \left[ 1 - \sqrt{\lambda \left[ (1 - (1-k)^b)^2 + (1-\lambda)(1-k^b)^2 \right]} \right]^{1/b} \right\} \quad (22)$$

For type  $C_s = a(1 - z/H)^b$ ,

$$E_R = 10000DH \left[ 1 - (1-\lambda)^{\frac{1}{b+1}} \right] \quad (23)$$

Equations 21 to 23 are the concrete equations for the three typical  $^{137}\text{Cs}$  depth distribution patterns.

### 3. NUMERICAL SIMULATION AND DISCUSSIONS

By using different values of relative  $^{137}\text{Cs}$  loss, depth function of  $^{137}\text{Cs}$  of reference inventory, sampling year and  $^{137}\text{Cs}$  fallout fraction of total fallout amount to the model described, some numerical simulation has been carried out for discussing the effect of individual element on soil erosion.

#### 3.1 Effect of Different $^{137}\text{Cs}$ Depth Distribution Patterns

Equations 21 to 23 were used for numerical simulation. In order to simulation and comparison easily, the coefficients of the three types ( $a$ ,  $b$ ,  $k$ ) and the  $^{137}\text{Cs}$  exiting thickness ( $H$ ) were given by adjusting the total inventory amount to be a same value (about  $650 \text{ Bq m}^{-2}$ ) within same depth ( $H=0.2\text{m}$ ) as shown in Figure 2. Figure 3 gives a numerical result of the relationship between erosion rate and total  $^{137}\text{Cs}$  loss sampling in 1996 for different types of  $^{137}\text{Cs}$  depth distribution pattern. Curves 1-4 represent equations of depth distribution pattern in Figure 2, and curve 5 based on proportional method by de Jong et al. (1983). It is obviously that different depth distribution patterns results in different estimating rates of soil loss although the reference inventory and relative loss of  $^{137}\text{Cs}$  are the same. Relationship between soil erosion rate and  $^{137}\text{Cs}$  depth distribution has following characteristics:

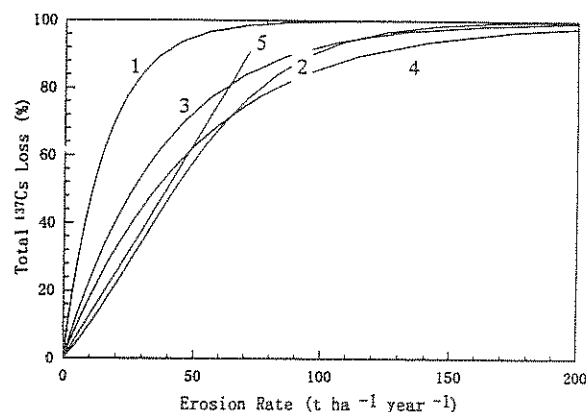


Figure 3: Relationship between erosion rate and total  $^{137}\text{Cs}$  loss for different types of  $^{137}\text{Cs}$  profile distribution pattern. Curves 1-4 represent equations of profile distribution pattern in Figure 2, and curve 5 based on proportional method by de Jong et al. (1983).

(1) Soil erosion rate is directly related with the fraction of  $^{137}\text{Cs}$  content near the soil surface. When total  $^{137}\text{Cs}$  loss is the same, the more  $^{137}\text{Cs}$  is concentrated to the surface, the less soil erosion will be. Difference of soil erosion rate between different depth distribution types will be over two times.

(2) Relation between soil loss and  $^{137}\text{Cs}$  loss is near linearly as proportional method when the percentage reduction in total  $^{137}\text{Cs}$  content is less (when  $Y < 40\%$ ), but their slope is different for different  $^{137}\text{Cs}$  depth distribution pattern. When amount of  $^{137}\text{Cs}$  loss is more striking ( $Y > 60\%$ ), a small soil loss will result in a comparatively large loss in  $^{137}\text{Cs}$  for all the three types depth distribution, presenting a striking contrast to the proportional method. Difference of soil erosion rate between different depth

distribution types will become greater as total  $^{137}\text{Cs}$  loss increases.

(3) From Figures 1 and 3, it can be seen that, using our model,  $^{137}\text{Cs}$  loss can not be used for estimating soil erosion when total  $^{137}\text{Cs}$  loss is extremely large (>95% in 1996). But, soil loss can be estimated when total loss of  $^{137}\text{Cs}$  reaches to 100% by using proportional method.

### 3.2 Effect of Sampling Year

As described above, according to mass balance model, mean annual relative loss of  $^{137}\text{Cs}$  or erosion constant ( $\lambda$ ) should not be deduced simply by arithmetic mean (divided by time period) as many researchers done (e.g. de Jong *et al.*, 1986; Soileau *et al.*, 1990). It should be obtained by Equations 16 or 13 (if sampling year earlier than 1983) or by Figure 1. Thus, same total loss of  $^{137}\text{Cs}$  will deduce different soil loss for different sampling year. Figure 4 gives numerical results of the relationship between erosion rate and total  $^{137}\text{Cs}$  loss for different sampling years using exponential depth distribution pattern as type 1 of Figure 2.

As shown in Figure 4, when total  $^{137}\text{Cs}$  loss is the same, the earlier sampling year is, the more soil loss will be. But, soil loss rate dose not changes lineally with time due to using Equation 17.

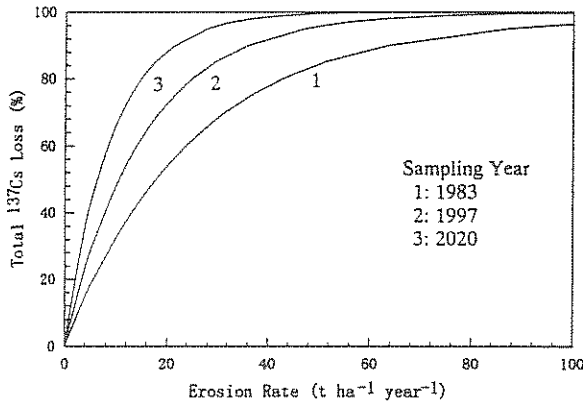


Figure 4: Relationship between erosion rate and total  $^{137}\text{Cs}$  loss for the different sampling years.  $^{137}\text{Cs}$  depth distribution pattern is type 1 in Figure 2.

### 3.3 Effect of Input Fraction

As described above,  $^{137}\text{Cs}$  input fraction of total  $^{137}\text{Cs}$  fallout input was used considering  $^{137}\text{Cs}$  input lasted 29 years and the difference of  $^{137}\text{Cs}$  input each year. Thus, same total loss of  $^{137}\text{Cs}$  will deduce different soil loss for different  $^{137}\text{Cs}$  input. Following three kinds of  $^{137}\text{Cs}$  input were considered.

- (1) The  $^{137}\text{Cs}$  input fraction is as Table 1.
- (2) Assuming all input occurred in 1963 (a year of the maximum rate of the  $^{137}\text{Cs}$  fallout) as many researchers done (e.g. Zhang *et al.*, 1990) and use the fraction as 100% in 1963 ( $R_{10}=100$ ). Thus, according to Equation 16

$$\lambda = 1 - (Y/100)^{1/(M-1963)} \quad (24)$$

- (3) Assuming all input occurred in 1963 and annual relative  $^{137}\text{Cs}$  loss can be get by arithmetic mean (divided total  $^{137}\text{Cs}$  loss by time period as many researchers done (e.g. de Jong *et al.*, 1986). that is

$$\lambda = Y/100/(M-1963) \quad (25)$$

Figure 5 gives a numerical result of the relationship between erosion rate and total  $^{137}\text{Cs}$  loss for the different  $^{137}\text{Cs}$  input using exponential depth distribution pattern as type 1 of Figure 2 and sampling year as 1983.

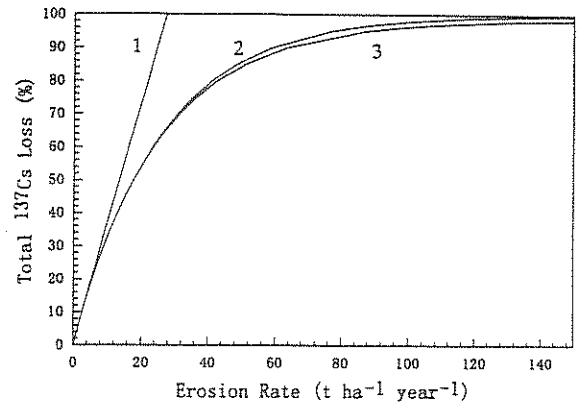


Figure 5: Relationship between erosion rate and total  $^{137}\text{Cs}$  loss for different fallout input 1) assuming all input occurred in 1963 and total  $^{137}\text{Cs}$  loss divide by 20 years (1983-1963); 2) assuming all input occurred in 1963 and input fraction is  $R_{10}=100$ ; 3) input fraction as shown in Table 1.  $^{137}\text{Cs}$  depth distribution pattern is type 1 in Figure 2 and sampling year is 1983.

As show in Figure 5, the result of numerical simulation of the different input fraction show that the input fraction may influence the amount of soil loss, but the extent of its influence is not large. However, if annual relative  $^{137}\text{Cs}$  loss was get by arithmetic mean (divided total  $^{137}\text{Cs}$  loss by time period, that is  $\lambda=Y/100/(M-1963)$ ) as many researchers done, the soil erosion rate will be extremely deferent when total  $^{137}\text{Cs}$  loss is over 30%.

## 4. CONCLUSIONS

Considering depth distribution patterns of  $^{137}\text{Cs}$  in soil for uncultivated soils, sampling year and  $^{137}\text{Cs}$  input fraction, a quantitative model of soil erosion rate using  $^{137}\text{Cs}$  loss was deduced (Equations 7, 16 and 17) by introducing an erosion constant (annual relative  $^{137}\text{Cs}$  loss) into the mass balance model (Figure 1). By introducing typical depth distribution functions of  $^{137}\text{Cs}$  (Equations 18 to 20 and Figure 2) into the model, we get detailed equations for the model (Equations 21 to 23) and numerical simulation were carried out.

Our model proves that depth distribution patterns of  $^{137}\text{Cs}$  is a major factor for estimating the rate of soil loss (Figure 3). Soil erosion rate is directly related with the fraction of  $^{137}\text{Cs}$  content near the soil surface. Since this fact has not been considered in many theoretical models, the erosion rate might be overestimated or underestimated by these models, especially using proportional model. The amount of soil loss is also influenced by sampling year (Figure 4) and  $^{137}\text{Cs}$  input fraction (Figure 5). However, the extent of the influence of  $^{137}\text{Cs}$  input fraction is not large rather than method of how to get annual relative  $^{137}\text{Cs}$  loss or the erosion constant ( $\lambda$ ). If annual relative  $^{137}\text{Cs}$  loss was get by arithmetic mean (divided by time period) extremely

difference will be deduced even using depth distribution function.

In the vast majority of the published literature, soil erosion is occurring in cultivated agricultural fields. Although our model is deduce for uncultivated soil, it is worth to apply the model to cultivated soil due to  $^{137}\text{Cs}$  is not uniformly distributed in soil profile even in some cultivated fields as mentioned in the beginning of the paper. For the cultivated fields with well tillage mixing,  $^{137}\text{Cs}$  is uniformly distributed in soil profile, introduction of soil erosion constant ( $\lambda$ ) is still helpful.

## 5. ACKNOWLEDGMENTS

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