Modelling The Population Dynamics Of
A Stock Of The Green Sea Turtle (*Chelonia mydas*)

C.S. Lee,[1] C.H. Diong & S.S.L. Lim,[2] and C.M.U. Leh[3]

Abstract. Data on turtle landings and average monthly egg collections of the green turtle from a group of three uninhabited islands off Sarawak, Malyasia over the last 50 years indicate a declining trend in the number of nesting females since the 1950s'. In this paper, we divide the stock of turtles into three life history stages, namely, a group consisting of eggs and hatchlings, a group consisting of juvenile turtles at different developmental stages, and a group of mature adult turtles. We attempt to model the population dynamics of the existing stock of green sea turtles using a system of three ordinary differential equations. Our simulation results reveal that in order to reverse the declining trend of the green turtle population, it is imperative to protect all stages of development of the stock of turtles.

1 Introduction.

Many species of sea turtles have been reported in recent years to be experiencing sharp decline in their population numbers, see for example Chan & Liew 1996; Chan 1987, 1991; Chua 1988; Brahim et al., 1987; Leh 1990 and references cited therein. The decline of turtle populations, as has been pointed out by Chan & Liew 1996; Frazer 1992; Canin 1989 and many others, is caused by many factors. The degraded environment, the increase in undesirable forms of fishing activities such as drift net fishing, the destruction of nesting habitats, and the consumptive and commercial exploitation of eggs, meat and shell are but just a few factors that threatened the survival of living cheloniids.

Conservation of sea turtles and other marine resources needs the cooperation of all parties involved. To effectively protect sea turtles, we believe that a better understanding of their population dynamics is important; and conservation measures need to be carried out at all stages of development of the sea turtles in order to improve their survival rates.

In this paper, we attempt to construct a mathematical model to describe the population dynamics of a stock of green sea turtles which nests on three uninhabited islands located 15 km off the coast of Sarawak. The turtle population is classified into three life-history stages, namely, the eggs and hatchlings, the juveniles, and the mature adults. Data on turtle landings and average monthly egg collections in this stock of turtles spanning from 1945 to 1995 are used to estimate some of the system parameters.

2 The Model.

Let $H(t)$, $J(t)$ and $M(t)$, respectively, denote the population numbers of a stock of the sea turtles in the egg and hatchling stage, the juvenile stage, and the mature adult stage at time t. Let $\dot{H}(t)$, $\dot{J}(t)$, and $\dot{M}(t)$ represent the time derivatives of $H(t)$, $J(t)$ and $M(t)$ respectively. Consider the following model

$$\begin{aligned} \dot{H}(t) &= \alpha M - k_1 H - k_2 H^2 \\ \dot{J}(t) &= k_3 H - k_4 J - k_5 J^2 \\ \dot{M}(t) &= k_6 J - k_7 M - k_8 M^2 \end{aligned} \tag{1}$$

where α and k_i, $i = 1, \cdots, 8$ are constant coefficients. We assume that the rate of increase of egg and hatchling population is directly proportional to the number of mature adults. The parameters k_1, k_4 and k_7 are coefficients associated with terms $k_1 H$, $k_4 J$ and $k_7 M$, hereinafter

[1]Division of Mathematics, School of Science, National Institute of Education, Nanyang Technological University, 469 Bukit Timah Road, Singapore 259756.
[2]Division of Biology, School of Science, National Institute of Education, Nanyang Technological University, 469 Bukit Timah Road, Singapore 259756.
[3]Sarawak Museum, Jalan Tun Abang Haji Openg, 93566 Kuching, Sarawak, Malaysia.
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referred to as the 'loss terms' which, respectively, represents the rate at which individuals from the egg and hatchling group, the juvenile group, and the mature adult group leave the respective groups for the next life history stage. Each of the 'loss terms' not only includes those members that have successfully completed their physical development of that stage and are proceeding to become new members of the next group, but also those that have died during the unit of time. The term k_7M is used specifically to denote those individuals that have died during the unit of time as well as those that are still surviving after the unit of time, but are not reproductively active anymore.

The terms k_3H and k_6J, respectively, represents the proportions of egg and hatchling group and the juvenile group that have successfully completed physical development in the respective groups and are now ready to enter the next life-history stage in the group we have defined. The terms k_2H^2, k_5J^2 and k_8M^2 model the quadratic parts of the respective mortality rates of the three groups owing to environmental and resource factors, such as natural predation, direct and indirect human-related mortality and other causes.

Before we estimate the parameters, observe that our model has a nontrivial equilibrium point at which

$$\frac{M}{H} = \frac{k_1 + k_2H}{\alpha}, \quad \frac{H}{J} = \frac{k_4 + k_5J}{k_3}, \quad \frac{J}{M} = \frac{k_7 + k_8M}{k_6}, \tag{2}$$

hence

$$\alpha k_3 k_6 = (k_1 + k_2H)(k_4 + k_5J)(k_7 + k_8M) \tag{3}$$

3 Parameters Estimation.

According to Mortimer 1989 and Miller 1996 and references cited therein, turtles are known to take 20 to 50 years to reach sexual maturity, and 40 to 70 years or more can pass between the time a female first hatches out of her egg and when she stops laying eggs herself (Mortimer 1989; Miller 1996; Limpus & Walter 1980; Balazs 1980; Bjorndal & Bolten 1988; Limpus 1979). Thus, we assume that a mature female sea turtle has a reproductive lifespan of 20 years. If we accept a 1: 1 sex ratio in the green turtle and assume a mature female sea turtle to lay a total of 3000 eggs in her reproductive lifetime, then on the average, each adult turtle has a mean reproductive output of 75 eggs a year.

To estimate k_1 and k_2, we solve the following differential equation associated with the first equation in (1).

$$\dot{H} = -(k_1 + k_2H)H \tag{4}$$

Equation (4) is solved analytically, and the solution corresponding to $H(0) = H_0$ is:

$$H(t) = \frac{H_0 k_1 e^{-k_1 t}}{k_1 + k_2 H_0 (1 - e^{-k_1 t})}. \tag{5}$$

From our data for the Sarawak stock of green turtles, and also from Mortimer 1989, the average number of eggs collected annually in the period 1970-1995 is 228082, and a portion of the collected eggs was replanted each year. It is found that in the period 1970-1994, the average number of hatchlings released per year was 43913. Suppose that there are no limiting environment and resource related factors, $(k_2 = 0)$, and $\frac{1}{5}$ or 20% of the initial population still remains in the egg and hatchling group at the end of five years, then $k_1 = 3.2189 \times 10^{-1}$. To estimate k_2, we may assume that the term k_2H^2 will only start to play an important role in affecting the population growth when $H \geq 200,000$. Thus, using (5) and $k_1 = 3.2189 \times 10^{-1}$, and assuming that when both the linear and quadratic 'loss terms' in the egg and hatchling equation are effective, then at the end of five years, the egg and hatchling population will be reduced to $\frac{1}{10}$ or 10% its initial value, we find that $k_2 = 1.6094 \times 10^{-6}$ for $H_0 = 250,000$.

The term k_3H represents the proportion of individuals from the egg and hatchling group that joined the juvenile group at time t after having survived the egg and hatchling stage. We assume that only 40% of individuals that leave the egg and hatchling group at the end of five years survive to recruit into the juvenile stage. Thus $k_3 = 0.4k_1 = 1.2876 \times 10^{-1}$.

To estimate k_4 and k_5, we solve

$$\dot{J} = -(k_4 + k_5 J)J \tag{6}$$

analytically. The solution of (6) corresponding to $J(0) = J_0$ is:

$$J(t) = \frac{J_0 k_4 e^{-k_4 t}}{k_4 + k_5 J_0(1 - e^{-k_4 t})}. \tag{7}$$

If we assume that when there is no environmental resistance, $\frac{1}{10}$ of the initial population still remains in the group at the end of 15 years, then $k_4 = 1.5351 \times 10^{-1}$. To estimate k_5, the parameter associated with the term $k_5 J^2$ in the equation for the juvenile group that accounts for the mortality of juveniles owing to predation, resource and habitat factors, we may assume that the term $k_5 J^2$ only starts to play an important role in affecting growth when $J \geq 20,000$. Thus, using (7), $k_4 = 1.535 \times 10^{-1}$, $J_0 = 25,000$, and assuming that when both 'loss terms' are effective in the equation for the juvenile group, then at the end of 15 years, only $\frac{1}{25}$ of the initial population still remains in the juvenile group, we obtain $k_5 = 1.0234 \times 10^{-5}$.

If we assume that only 25% of individuals leaving the juvenile group survive to become mature adults, then $k_6 = 0.25k_4 = 3.8376 \times 10^{-2}$. For the estimation of parameters k_7 and k_8 in the 'loss terms' $k_7 M$ and $k_8 M^2$ in the equation for the mature adults, we solve the following associated differential equation:

$$\dot{M} = -(k_7 + k_8 M)M \tag{8}$$

The analytical solution of (8) that satisfies the initial condition $M(0) = M_0$ is

$$M(t) = \frac{M_0 k_7 e^{-k_7 t}}{k_7 + k_8 M_0(1 - e^{-k_7 t})} \tag{9}$$

where M_0 is a known constant. To estimate k_7, we assume that in general the reproductively active lifespan of a female is 20 years, and at the end of 20 years, $\frac{1}{20}$ or 5% of the initial mature adult population is still capable of reproducing. Thus, discounting the effects of environmental resistance, $k_7 = 1.498 \times 10^{-1}$. As in the estimation of k_2 and k_5, we have to make some assumptions in order to find k_8. Suppose that the internesting frequency in this population is four times on the average per reproductive season. Our analysis on the trends of turtle landings for the Sarawak green turtle population shows that the average number of turtle landings during the six-year period from 1950–1955 is 3996, while the average landings in the corresponding six-year period from 1990–1995 fell to only 550. Thus we may assume that the environmental resistance will only start to play an important role when the adult population exceeds 8000 individuals. Using $M_0 = 9000$, $k_7 = 0.1498$, and assuming that the number of mature adults at the end of 20 years are still reproductively active is reduced to $\frac{1}{40}$ or 2.5% of the initial population, then we obtain $k_8 = 1.7511 \times 10^{-5}$.

4 Simulation Results.

We have carried out some simulations of the model obtained in the previous section to check if it is suitable for describing the population dynamics of the stock of green turtles under consideration. For use as initial values in the simulations, the current population numbers in the three life-history stages, namely, the egg and hatchling group, the juvenile group, and the mature turtle group are estimated to be 150,000, 10,000, and 1100 respectively. Values of the system parameters are those estimated in the previous section, namely,

$$\alpha = 75, \quad k_1 = 3.2189 \times 10^{-1}, \quad k_2 = 1.6094 \times 10^{-6},$$

$$k_3 = 1.2876 \times 10^{-1}, \quad k_4 = 1.5351 \times 10^{-1}, \quad k_5 = 1.0234 \times 10^{-5},$$

$$k_6 = 3.8376 \times 10^{-2}, \quad k_7 = 1.498 \times 10^{-1}, \quad k_8 = 1.7511 \times 10^{-5}.$$

Figures 1, 2 and 3 display, respectively, the time histories of population numbers of the egg and hatchling group, the juvenile turtle group, and the mature turtle group for five simulation runs.

Curves with label A in the figures denote the time histories of population numbers of the three groups of turtles that correspond to the set of parameters mentioned above. Curves with label B show the time histories of population numbers of the three groups of turtles as a result of a reduction in the new mature turtle recruits from 25% to 10% of the linear 'loss term' k_4J in the equation for the juvenile turtle group. Curves with label C depict the time histories of population numbers of the turtles at three life-history stages as a result of a reduction in the new juvenile turtle recruits from 40% to 20% of the linear 'loss term' k_1H in the equation for the egg and hatchling group. Curves with label D display the time histories of population numbers of the three groups as a result of a reduction in the recruits of **both** the new juvenile turtle and the new mature turtle. In addition to a reduction in the recruitment of the number of juvenile turtles from 40% to only 20% of that in the linear 'loss term' k_1H in the equation for the egg and hatchling group, there is also a reduction of mature turtle recruitment number from 25% to only 10% of the linear 'loss term' k_4J. Consequently, the long term population numbers in all the three groups decline sharply in comparison with Curves A. Both the egg and hatchling group and the juvenile turtle group have their long term population numbers reduced by more than half of those shown in curves A, while that of the mature turtle group is reduced to less than one third of that depicted in Curve A.

Finally, curves with label E may be used to compare with those labelled C as they correspond to the situation in which the number of new juvenile recruits is reduced drastically from 40% to only 10% of the linear 'loss term' k_1H in the equation for the egg and hatchling group. We observe that the long term population number of the juvenile turtle has decreased tremendously, while the long term population numbers in the other two groups are now only slightly more than those shown in Curves B.

5 Discussion.

Our simulation results show that a decreased recruitment of juveniles to the mature adult stage adversely affects the long term number of eggs and hatchlings (compare Curves A and B in Fig.1) as well as the long term number of juveniles (compare Curves A and B in Fig.2). This is to be expected as a decrease in mature adults translates to a decrease in the number of egg-laying females, and hence a lower reproductive output in terms of total number of eggs laid. The long term juvenile turtle population, however, is more affected by a decrease in recruitment of eggs and hatchlings to the juvenile stage than recruitment of juveniles to mature adults (compare Curves B and C in Fig.2). Once again, this trend is to be expected as the number of juveniles is directly derived (hence affected) from the number of eggs and hatchlings that survive to the juvenile stage.

Regardless of the recruitment success of eggs and hatchlings to juveniles, it is the number of juveniles which reach sexual maturity that will eventually affect the long term population numbers of mature adults (compare Curves A, C, and E with Curve B in Fig.3). It is evident that the long term population numbers in each life history stage is directly and adversely affected by the stage immediately preceeding it. Hence, it is imperative to protect all stages of development of the turtle in order to reverse the declining trend of the green turtle population.

6 Conclusion.

The population dynamics of a stock of green sea turtles which nests in a group of islands off Sarawak, Malaysia is described with a simple mathematical model using three state variables. Simulation results of the model showed that a decrease in either one or both of the number of recruits in the juvenile or mature turtle stage results in a decline in population numbers in all three life-history stages defined in this study. Thus, protection measures are needed at all stages of development of the species in order to reverse the declining trend in this green turtle population.

It is important to emphasize that while we have made an effort to estimate the system parameters, there is still a lot to be done to refine the model as well as its parameters. We are confident that as more data on the life-history of the green turtle become available, better estimates of the system parameters can be obtained. In the real world, there are many factors that affect population growth in sea turtles, and the intensity of these effects is difficult, if not impossible, to ascertain. For example, the impact of habitat deterioration, the destruction of nesting grounds, mortality causes due to increased fishing activities as well as climatic changes are but just a few examples of the deleterious effects on the growth of sea turtle population. It is a challenge indeed to develop a model that can describe the population dynamics of the green turtle in the light of all these uncertain human and environmental disturbances.

References

Balazs, G.C., *Synopsis of biological data on the green turtle in the Hawaiian islands*, NOAA-TM-NMFS SWFC Vol.7 (1980), pp.1–14.

Bjorndal, K.A. and Bolten, A.B., *Growth rates of immature green turtles, chelonia mydas, on feeding grounds in the southern Bahamas*, Copeia Vol.3 (1988), pp.555–564.

Brahim, S., E.H. Chan, and A.K. Rahman, *An update on the population status and conservation of the leatherback turtle of Terengganu.*, in "Towards Conserving Our Marine Heritage. Proc. Tenth Annual Seminar, Malaysian Society of Marine Science, Kuala Lumpur, Malaysia," A. Sasekumar, S.M. Phang, and E.L. Chong, editors, 1987, pp. pp.69–77.

Canin, J., *International trade in sea turtle products*, in "Proc. Ninth Annual Workshop on Sea Turtle Conservation and Biology, 7-11 Feb. 1989, Jekyll Island, Georgia, U.S.A.," 1989.

Chan, E.H. and H.C. Liew, *Decline of the leatherback population in Terengganu, Malaysia, 1956-1995*, Chelonian Conservation and Biology Vol.2, No.2 (1996), pp.196–203.

Chan, E.H., *Status of the leatherback turtle, Dermochelys coriacea*, "Workshop on Sea Turtle Conservation and Management in Malaysia, 14-17 December 1987, Tanjong Jara, Terengganu, Malaysia.," 1987.

Chan, E.H., *Sea turtles*, in "The State of Nature Conservation in Malaysia," R. Kiew, editor, Malayan Nature Society, Kuala Lumpur, Malaysia., 1991, pp. pp.120–135.

Chua, T.H., *On the road to local extinction: the leatherback turtle (Dermochelys Coriacea) in Terengganu, Malaysia.*, in "Thirty Years of Marine Science Research and Development. Proc. Eleventh Annual Seminar, Malaysian Society of Marine Sciences, Kuala Lumpur, Malaysia," A. Sasekumar, R. D'Cruz, and S.L.H. Lim, editors, 1988, pp. pp.153–158.

Frazer, N.B., *Sea turtle conservation and halfway technology*, Conservation Biology Vol.6 No.2 (1992), pp.179 –184.

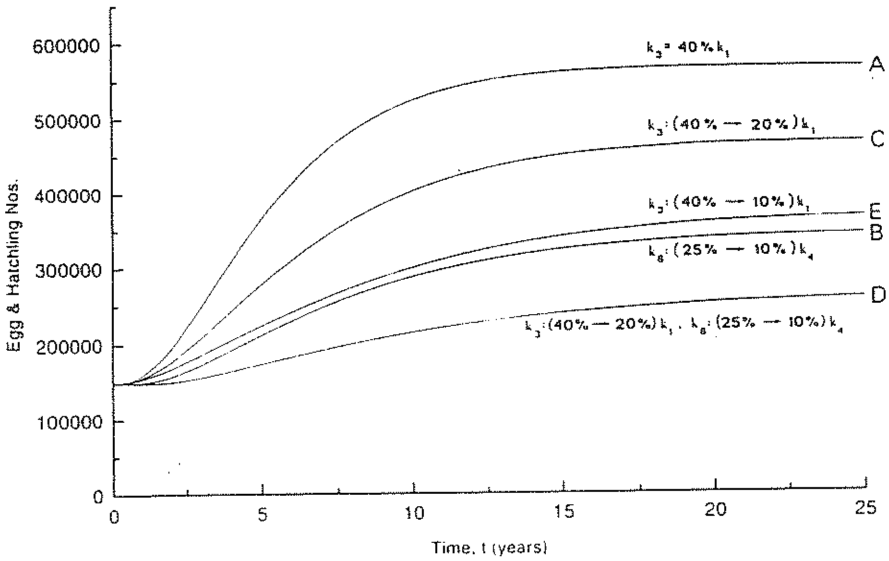
Leh, C.M.U., *The green turtle (Chelonia mydas (L.)) in Sarawak: Is there a future ?*, in "Research Priorities for Marine Sciences in the 90's. Proc. Twelfth Annual Seminar, Malaysian Society of Marine Science, Kuala Lumpur, Malaysia.," S.M. Phang, A. Sasekumar, and S. Vickineswary, editors, 1990, pp. pp.219–225.

Limpus, C.J., *Notes on growth rates of wild turtles*, IUCN/SSC Marine Turtle Newsletter Vol.10 (1979), pp.3–5.

Limpus, C.J. and D. Walter,, *The growth of immature green turtles (Chelonia mydas) under natural conditions*, Herpetologica Vol.36 (1980), pp.162–165.

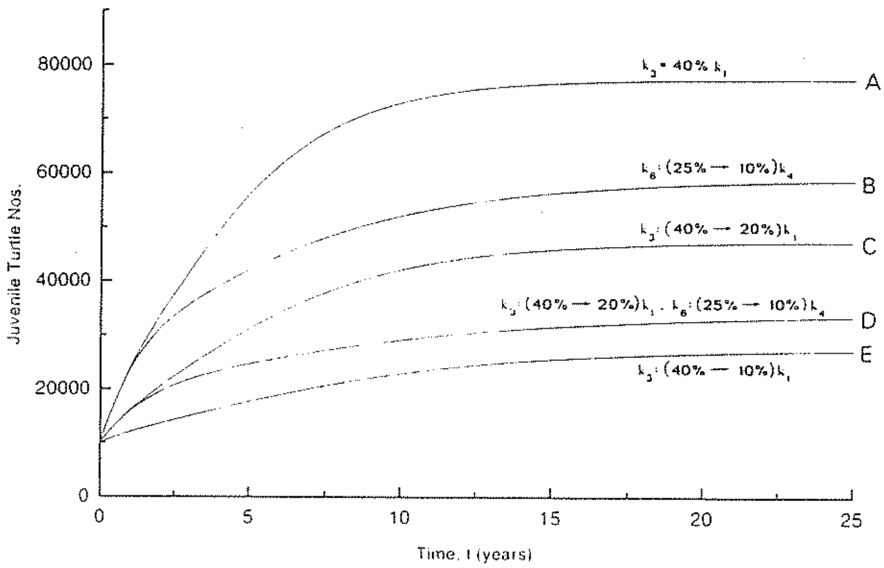
Miller, J.D., *Reproduction in sea turtles*, in "The Biology of Sea Turtles," Lutz, P.L. and Musick, J.A. (eds), CRC Press, New York, U.S.A., 1996, pp. pp.51–81.

Mortimer, J.A., *Recommendations for the management of the green turtle (Chelonia mydas) population nesting at the turtle islands of Sarawak*, Report submitted to WWF Malaysia. Produced under WWF Project 3868 WWF, Malaysia..



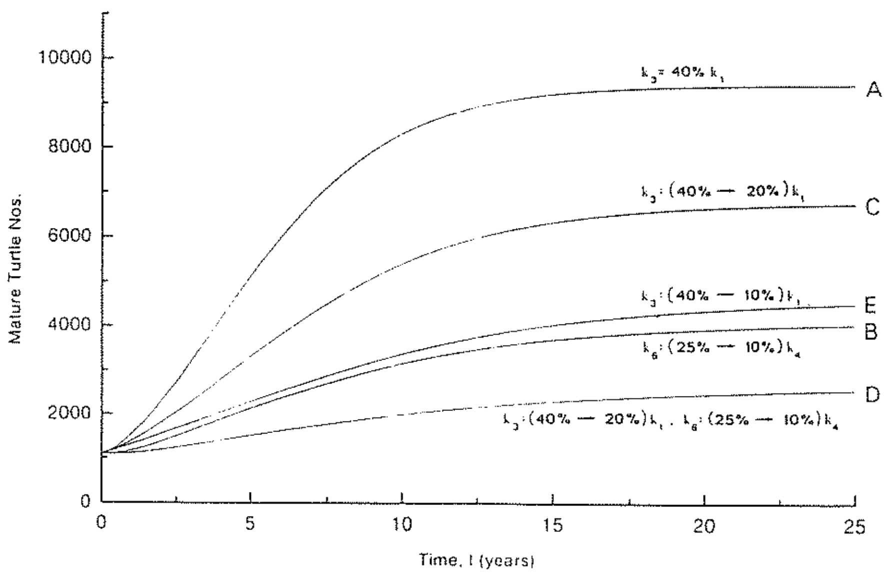
600000
500000
400000
300000
200000
100000
0
Egg & Hatchling Nos.
0
5
10
15
20
25
Time, t (years)
$k_3 = 40\% k_1$
A
k_3: (40% — 20%) k_1
C
k_3: (40% — 10%) k_1
E
B
k_6: (25% — 10%) k_4
D
k_3: (40% — 20%) k_1, k_6: (25% — 10%) k_4

Fig 1



80000
60000
40000
20000
0
Juvenile Turtle Nos.
0
5
10
15
20
25
Time, t (years)
$k_3 = 40\% k_1$
A
k_6: (25% — 10%) k_4
B
k_3: (40% — 20%) k_1
C
k_3: (40% — 20%) k_1, k_6: (25% — 10%) k_4
D
E
k_3: (40% — 10%) k_1

Fig 2



10000
8000
6000
4000
2000
0
Mature Turtle Nos.
0
5
10
15
20
25
Time, t (years)
$k_3 = 40\% k_1$
A
k_3: (40% — 20%) k_1
C
k_3: (40% — 10%) k_1
E
B
k_6: (25% — 10%) k_4
D
k_3: (40% — 20%) k_1, k_6: (25% — 10%) k_4