Simulating the Stress-Strain Curves of Woollen Yarns

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Abstract A stochastic model is introduced for predicting the stress-strain curve of a woollen yarn under longitudinal extension. The important features of a woollen yarn that are included in this model are the distributions of fibre length, diameter and strength, the effect of fibre reversals, the variation in linear density and twist along the length of a yarn, the radial variation in fibre packing, and the non-uniform arrangement of fibres in any particular yarn cross-section. The key deformation mechanisms within the model include variation in the stress between and along individual fibres, fibre breakage, fibre slippage, non-uniform strain along the yarn, and twist flow within the extending yarn. It is shown that the model can predict with reasonable accuracy the tenacity of woollen yarns which are made from fibres with a wide range of properties, and which contain a wide range of twist levels.

1. INTRODUCTION

It is well known that staple fibre yarns, particularly those made from natural fibres, have highly variable structures. This variation is present at several levels. Along any staple yarn there are variations in local linear density and twist. This results in variation in the packing density of fibres along the yarn. On a smaller scale, there may be large variations in the length, cross-sectional area and strength of a yarn's constituent fibres. If a yarn is made from a blend of different fibre types, there may be large differences in physical properties between fibres. Even along a single fibre there may be large variations in cross-sectional area and local breaking stress.

This irregular structure results in complicated mechanical behaviour. There are several important mechanisms involved in the tensile behaviour of staple fibre yarns. As a yarn extends there is uneven strain along, and twist redistribution within the structure. There is also lateral movement of fibres towards the yarn axis. Within the yarn individual fibres may partially or completely slip relative to the rest of the structure; the degree of this slippage depends on conditions which change as the yarn extends. Furthermore individual fibres break as the yarn extends. Ultimate yarn rupture is due to a combination of fibre slippage and fibre breakage.

The theoretical mechanics of staple fibre yarns has been the subject of intense research activity. Previous workers have developed models to predict the tension [Hearle, 1965; Carnaby and Grosberg, 1976; van Luijk et al., 1985] and torque [Tandon et al., 1995] of a staple yarn undergoing longitudinal extension. Although there has been considerable progress towards understanding the mechanical behaviour of staple yarns, none of these models is capable of accurately predicting their tensile behaviour.

Despite the obvious importance of structural variation on mechanical behaviour, previous yarn models essentially neglected this variation when analysing yarn mechanical behaviour. In particular, they ignored the effect of longitudinal yarn irregularity. Only comparatively simple models [Zhurek, 1960; Mandle, 1981] of the effect of longitudinal irregularity on the mechanical properties exist; these models take no account of the complicated mechanisms involved in staple yarn deformation.

This paper presents a stochastic model of the tensile behaviour of an irregular staple yarn. This model includes the effect of structural variability, and considers the following important mechanisms within an extending staple fibre yarn: uneven strain along a yarn, twist flow within a yarn, non-uniform lateral fibre movement, fibre breakage and fibre slippage.

2. FIBRE MODEL

Fibres in the model yarn structure may be of different types, each having different physical properties. Let q denote the number of different fibre types in the structure. The discrete probability distribution function of the number of the fibres of type k in the yarn structure is given by T. That is, for $k = 1,\ldots,q$, T_k is the proportion by number of fibre type k in the yarn.

Each fibre is assumed to have a circular cross-section, and a constant diameter along its length. Any two fibres may vary in their length and diameter, even if they have the same fibre type. With some natural fibres, fibre length is correlated with fibre diameter. For this reason the fibre length (length biased) and diameter distributions of fibre type k are represented with a bivariate probability distribution function, L_k.

Each fibre is considered to consist of a number of fibre segments of equal length $\zeta \le \zeta_{max}$ arranged in series, where ζ_{max} is the maximum length of a fibre segment, and is a parameter of the yarn model. Given the length of a fibre, l, the length of a segment within this fibre is given by $\zeta = l / v$, where v is the smallest integer such that $l / v \le \zeta_{max}$. Fibre strength is modelled using the "weakest-link" hypothesis [Pierce, 1926], which states that the strength of a fibre can be represented as the minimum of the statistically independent strengths of its fibre segments. It is assumed that the breaking stress of a fibre segment is independent of its diameter. The breaking stress distribution of fibre segments of length ζ that are of type k is represented by $B_{k,\zeta}$.

3. INITIAL YARN STRUCTURE

The yarn is modelled as a series of segments of equal length. Let Q be the length of the yarn whose tensile behaviour we are simulating, and let N be the number of yarn segments that will be used to model the behaviour of the yarn. Then the initial length of a single segment, λ_0, is given by $\lambda_0 = Q / N$.

Within each yarn segment, the linear density and twist are assumed to be constant, but between segments they may vary. In general, the linear density and twist in a short yarn segment are correlated, with twist tending to be higher at points of low linear density. The distribution of linear density and twist in short yarn segments also depends on the segment length, with both the variance of segment linear density and that of segment twist decreasing with an increase in segment length. For this reason, the linear density and twist of yarn segments of length λ are modelled by a bivariate probability distribution function, V_λ. The linear density and twist are generated independently for each yarn segment.

4. INITIAL YARN SEGMENT STRUCTURE

A yarn segment is modelled by considering all fibres that pass through its central cross-section. All fibres are assumed to follow helical paths which are centred on the yarn axis and have the same pitch. The pitch of the helical paths are taken to be equal to the length of a single turn of twist.

Fibres passing through a yarn cross-section are ordered in increasing distance between the yarn axis and the outermost edge of the fibre. Let f_i denote the i-th fibre in this ordering. Because fibres are assumed to be circular, and to follow helical paths, the distance between the yarn axis and the outermost edge of f_i is given by $s_i = r_i + \mu_i / 2$, where r_i is the distance between the axis of f_i and the yarn axis, and μ_i is the diameter of f_i.

An important concept in characterising the structure of a yarn is that of a radial packing ratio. Consider a cross-section of a yarn. The proportion of the boundary of the circle of radius r, centred on the yarn axis, that intersects fibre is called the *fibre packing ratio at r* of this cross-section. In this yarn model it is assumed that the initial fibre packing ratio varies between yarn segments, and that it is a function of both the initial linear density, W, and the initial twist, T, of a yarn segment. For a given fibre blend, the initial fibre packing ratio at r is given by $\phi_{W,T}(r)$.

The *local packing ratio* of the fibre f_i, ϕ_i, is the ratio of the area of f_i in the yarn cross-section to the area of the annulus formed between s_{i-1} and s_i. The area of the intersection between f_i and the yarn cross-section is approximately that of an ellipse with a minor axis of $\mu_i / 2$, and a major axis of $\mu_i / (2\cos\alpha_i)$, where α_i is the helix angle of the path of f_i. Hence we have

$$\phi_i = \frac{\mu_i^2}{4(s_i^2 - s_{i-1}^2)\cos\alpha_i}, \tag{1}$$

where $s_i = r_i + \mu_i / 2$, for $i = 1, \ldots, n$, and $s_0 = 0$.

Given the initial fibre packing ratio of the yarn segment, $\phi_{W,T}(r)$, the model determines the initial radial position of each fibre so that the resulting structure approximately achieves this packing ratio. In general, it is not possible to determine the radial positions of a series of fibres with random diameters so that the resulting structure will match exactly an imposed fibre packing ratio function. The model approximates $\phi_{W,T}(r)$ by ensuring that for each i, the area of intersection between the first i fibres and the yarn cross-section is equal to the expected area of fibre within a radius s_i from the yarn centre, as calculated from $\phi_{W,T}(r)$. This is achieved by an iterative process. The first fibre is placed so that the area of f_1 in the cross section is equal to the expected area of fibre in the circle of radius s_1. That is,

$$\pi s_1^2 \phi_1 = 2\pi \int_0^{s_1} \phi_{W,T}(r) r dr. \tag{2}$$

Each subsequent fibre is added so that the area of fibre f_i in the cross section equals the expected area of fibre that should be encountered in the annulus with inner radius s_{i-1} and outer radius s_i. Thus

$$\pi(s_i^2 - s_{i-1}^2)\phi_i = 2\pi \int_{s_{i-1}}^{s_i} \phi_{W,T}(r) r dr. \tag{3}$$

Generating the fibres in the initial yarn structure is an iterative process. Set $s_0 = 0$, and suppose the properties of the first i-1 fibres have already been determined. In order to generate the properties of f_i the following steps are carried out:

- The fibre type of f_i, k_i, is randomly generated[1] from T. The fibre length, l_i, and fibre diameter, μ_i, are randomly generated from L_{k_i}.
- The breaking stress of each fibre segment in f_i is independently randomly generated from B_{k_i,ζ_i}, where ζ_i is the length of a segment of f_i.
- The radial position of f_i is found by solving (3) for r_i.

This procedure is repeated while the linear density of the generated yarn cross-section is less than the desired linear density, w. That is, the procedure stops after the n-th iteration if

$$\sum_{i=1}^{n-1} \frac{\pi\mu_i^2 \nu_{k_i}}{4\cos\alpha_i} < w \le \sum_{i=1}^{n} \frac{\pi\mu_i^2 \nu_{k_i}}{4\cos\alpha_i}, \tag{4}$$

where ν_k is the density of fibre type k.

Each fibre is assumed to intersect the yarn cross-section at a random point along its length. Each fibre is also assumed to be initially stress-free.

[1] All random variables are randomly generated from the appropriate distribution using the inverse transform method [Law and Kelton, 1982].

5. STAPLE YARN MODEL

Consider the yarn structure as modelled in Section 2. In order to determine the tension-extension and torque-extension behaviour of this structure, it is subjected to a series of extensions by a small length ι. Suppose that the properties of the yarn structure are known after the $(k-1)$-th incremental yarn extension, and we are interested in finding the properties of the yarn structure after the k-th incremental yarn extension.

Let $\lambda_{j,k}$ be the length of the j-th yarn segment after the k-th incremental yarn extension, and let $\Delta\lambda_{j,k}$ be the difference between the length of the j-th yarn segment after the k-th incremental yarn extension and the initial length of the j-th yarn segment. That is,

$$\Delta\lambda_{j,k} = \lambda_{j,k} - \lambda_0 . \tag{5}$$

As the total extension in the yarn structure after the k-th incremental extension must be $k\iota$, we have

$$\sum_{j=1}^{N} \Delta\lambda_{j,k} = k\iota . \tag{6}$$

Similarly, let $\theta_{j,k}$ be the number of radians of twist in the j-th segment after the k-th incremental yarn extension, and let $\Delta\theta_{j,k}$ be the difference between the number of radians of twist in the j-th segment after the k-th incremental yarn extension and the number of radians of twist in the undeformed j-th yarn segment, $\theta_{j,0}$. That is,

$$\Delta\theta_{j,k} = \theta_{j,k} - \theta_{j,0} . \tag{7}$$

We assume that the yarn ends are not free to rotate during yarn extension, which is the case during the practical testing of yarn strength. Hence there can be no change in the total turns of twist in the yarn structure during extension. That is

$$\sum_{j=1}^{N} \Delta\theta_{j,k} = 0 . \tag{8}$$

Let the tension and torque in the j-th segment after the k-th incremental yarn extension be denoted by $F_{j,k}$ and $M_{j,k}$, respectively. For equilibrium, both the tension and the torque must be constant along the length of the yarn at any yarn extension. Hence after the k-th incremental yarn extension we must have

$$F_{j-1,k} = F_{j,k}, \quad M_{j-1,k} = M_{j,k}, \tag{9}$$

for each $j = 2,\ldots,N$.

The tension and torque in the j-th segment after the k-th incremental yarn extension are determined from the positions and strains in the fibres in the segment at the end of the $(k-1)$-th incremental extension, and from $\lambda_{j,k}$ and $\theta_{j,k}$ using the model described in Cassidy et al. [1996]. In order to determine the length and radians of twist in each segment after the k-th incremental yarn extension, it is necessary to solve (7), (9) and (10) for all $\lambda_{j,k}$ and $\theta_{j,k}$, $j \in \{1,2,\ldots,N\}$. This is a system of $2N$ non-linear equations in $2N$ unknowns.

Having determined the properties of the yarn structure at the end of the k-th incremental yarn extension, this process is repeated until the tension in the yarn returns to zero. In this way the tension-extension and torque extension curves of a particular model yarn structure are determined.

To simulate the properties of the tension-extension and torque-extension curve distribution, statistics must be collected from the tensile curves generated from a number of randomly generated yarn structures.

6. EXPERIMENTAL WORK

In order to investigate the relationship between fibre properties and the stress-strain curves of woollen yarns, nine fibre blends were manufactured into woollen yarns. Six of these were wool blends selected for their mean fibre diameter and mean (barbe) length after carding. Blends with three levels of mean fibre diameter (20, 30 and 36 µm), each at two levels of mean fibre length after carding (50 and 70 mm) were selected. Two more blends were selected based on their core bulk; these were a high bulk down wool blend, and a low bulk lustre wool blend. Finally, to investigate the properties of yarns containing a mixture of fibre types, a typical 80% wool 20% nylon carpet blend was also manufactured into yarn. Each blend was spun into four yarns with twists of 150, 200, 250, and 300 tpm. Each yarn had a linear density of 200 tex.

The yarn tensile were determined on a Zwick 1510 yarn tensile testing machine (Zwick GmbH & Co., Ulm, Germany). This machine did not have the ability to record the entire tension-extension curve. In order to record the entire curve a CR10 datalogger (Campbell Scientific Inc., North Logan, USA) was connected to the Zwick. The CR10 measured the potential difference at a gauge on the Zwick which gave the percent of maximum load that the strain gauge was under. The voltage at this point varied linearly from zero volts when the strain gauge was unloaded, to one volt when it was at 100% of maximum load. The Zwick was operated in the 0-20 N load range. Thus the potential difference at the point measured by the CR10 was 1.0000 V when the tension between the sample jaws of the Zwick was 20.00 N. The CR10 recorded the voltage at this point in increments of 0.66 mV with a frequency of 16 Hz. The datalogger was connected to a notebook computer. After every series of twenty-five tensile tests the results were downloaded from the CR10 to the computer and saved to file.

All tests were carried out on conditioned specimens in a conditioned atmosphere of 65±2% relative humidity at 20°C. All specimens were conditioned for at least 24 hours prior to testing. The gauge length used for all tests was 0.5 m, and the strain rate used for all tests was 100% per minute. This gauge length and strain rate are those used in the commercial testing of yarn strength. For each yarn 25 tests were carried out on each of 2 cones of the yarn.

The maximum precision of the method used to record the stress-strain curves was ±0.03 N for the maximum tension, and ±0.002 for the breaking strain. The difference between the maximum tension recorded by the Zwick and that determined from the data recorded by the CR10 was in 99% of the yarn tests less than 0.03 N, and that for breaking strain was in 99% of the yarn tests less than 0.004. These results compare well with the precision limitations of the method used to record the tension-extension curves. These small differences were not of practical significance to this work.

7. NUMERICAL EVALUATION OF THE MODEL

The system of equations represented by (7), (9) and (10) are difficult to solve. As a result of fibre slippage and fibre breakage, these equations are not linear, continuous, or path independent. A variety of techniques were used in order to solve this system in an acceptable amount of time.

7.1 Newton's type methods

The methods most often used in this work were Newton's type methods for non-linear systems. Two different methods were used. These were Newton's method and Broyden's method [Press et al., 1993]. Both methods were modified by a line search algorithm [Press et al., 1993]. While these methods offer quadratic convergence for smooth continuous functions when the initial guess is close to the actual root, they can suffer from poor convergence when these conditions are not met.

Broyden's and Newton's methods are used to solve the system of equations in the following way. Suppose we are trying to solve the system for the k-th incremental yarn extension. Initially, Broyden's method is used to try and solve the system of equations using the change in segment length and twist that occurred during the $(k-1)$-th deformation as the initial guess of the solution. This initial guess is called the *constant deformation guess*, and is given by

$$\begin{aligned} \lambda_{j,k} &= \lambda_{j,k-1} + (\lambda_{j,k-1} - \lambda_{j,k-2}), \\ \theta_{j,k} &= \theta_{j,k-1} + (\theta_{j,k-1} - \theta_{j,k-2}), \end{aligned} \tag{10}$$

for each $j = 1, \ldots, N$. For the evaluations of the model reported in this paper, the convergence criterion used to determine a solution was

$$\begin{aligned} \frac{\left|F_{j,k} - F_{j-1,k}\right|}{F_{j,k}} &< 10^{-4}, \\ \frac{\left|M_{j,k} - M_{j-1,k}\right|}{M_{j,k}} &< 10^{-4}, \end{aligned} \tag{11}$$

for all $j \in \{2,3,\ldots,N\}$, and the maximum number of iterations that were used to find a solution was $5+N$. If Broyden's method with this initial guess fails to converge to a solution after this number of iterations, then Broyden's method is used with a different initial guess, called the *uniform deformation guess*. The uniform deformation guess is given by

$$\lambda_{j,k} = \lambda_{j,k-1} + \frac{1}{N}, \quad \theta_{j,k} = \theta_{j,k-1}, \tag{12}$$

for each $j = 1, \ldots, N$. If after $5+N$ iterations Broyden's method with the uniform initial guess also fails to converge to a solution, then Newton's method with the constant deformation guess is used to find a solution. Finally, if after $5+N$ iterations Newton's method with the constant initial guess also fails to converge to a solution, then Newton's method with the uniform deformation guess is used to find a solution.

If after this procedure no solution has been found, then the attempted yarn extension is reduced by a factor of 1/2, and the four approaches of the previous paragraph are attempted again in turn. If no solution is again found, then the attempted yarn extension is reduced by a further factor of 1/2, and these four approaches tried again. If after this procedure no solution to the system of equations has been found, then a different solution technique is used: simulated annealing.

If at any stage a solution is found which results in an attempted deformation that is too large in terms of the change in segment extension or segment twist, then the incremental yarn strain is reduced by a factor of 1/2, and another solution is attempted using the method described above.

7.2 Simulated annealing

Simulated annealing is an optimisation technique that can be used on cost functions with arbitrary degrees of nonlinearities, discontinuities, and stochasticity, and where the cost function domain is restricted by arbitrary boundary conditions [Ingber, 1993]. This technique was used in this work to solve the system of equations when the continuous methods based on Newton's method failed to converge to a solution.

There are a wide range of simulated annealing type methods, see Ingber [1993] for a review. The method of Press et al. [1992] was used in the work described in this paper. This is based on a modification of the downhill simplex method of Nelder and Mead [1965].

For applying this simulated annealing method to the yarn model, the temperature T was defined as

$$T = \sum_{j=2}^{N}\left(\frac{F_{j,k} - F_{j-1,k}}{F_{j,k-1}}\right)^2 + \sum_{j=2}^{N}\left(\frac{M_{j,k} - M_{j-1,k}}{M_{j,k-1}}\right)^2. \tag{13}$$

The $2N-1$ points in $\Re^{2N}$ which formed the initial simplex were taken to be the constant deformation guess, and the $2N$ points taken by increasing the attempted deformation in each component of this guess in turn by 50%. A solution was taken to be any point with temperature $T < (2N-2)^{-1} \times 10^{-4}$. This convergence criterion is in general more restrictive than that used for the Newton's methods.

One of the problems with simulated annealing type methods is the difficulty of specifying a successful *annealing schedule* [Press et al., 1993]. The annealing schedule refers to the number of moves that are made at each temperature, and the rate at which the temperature is reduced. For the evaluations of the model whose results are presented in this paper, the initial temperature was taken to be 10 times the highest function value of any point on the initial simplex. One hundred moves were taken at each temperature, after which the temperature was reduced by a factor of 0.9. In addition, after every 700 moves the point on the simplex with the highest function value was replaced by the point with the lowest function value ever found (if this point was not already a member of the simplex). While this annealing schedule was probably somewhat conservative, it proved successful in solving the system of equations in an acceptable time in the majority of cases.

If this simulated annealing method also fails to find a solution of the equations, the step with the lowest temperature found using this method was used. This provided a mechanism for taking the best possible small step over a point where a solution that met the convergence criteria could not be found. Such steps were rare; less than 10% of all model evaluations resulted in such a step being taken. If three such bad steps were taken in a row, then the programme terminated with an appropriate error message. In every case that this was a result of a rapid increase in strain in one segment and a reduction in strain in the other segments. Combined with a rapid reduction in yarn tension, this was effectively due to the onset of yarn rupture. This model is in general not capable of predicting the tension and torque of a yarn beyond the onset of rupture - this process occurs over very short times and goes beyond the assumptions of statics into the realm of dynamics.

8. MODEL VALIDATION

8.1 Single segment model

When the yarn model was evaluated with only one yarn segment, it was called the single segment model. To compare the results of the single segment model with the experimental results, 10 stress-strain curves were predicted using the properties of each yarn manufactured in the practical work. Each curve was predicted up to a strain of 0.2, which was greater than the breaking strain of any of the yarns tested in the practical work.

In general, the predicted yarn tenacity increased with increasing twist. This was in agreement with the experimental observations. As with the measured stress-strain curves, the rate of increase of tenacity with an increase in twist was greater for coarse fibre yarns.

The effect of bulk on predicted yarn tensile properties was the same as observed in the experimental work. The yarn modulus decreased with an increase in bulk, as did the yarn tenacity.

The relationship between the predicted and measured tenacity of the 36 yarns is shown in Figure 1. The predicted tenacity was in general very close to the measured tenacity over the entire tenacity range. The predictions were slightly higher than the measured values. This was not surprising considering the expected effect of yarn irregularity, which was largely neglected in the single segment model.

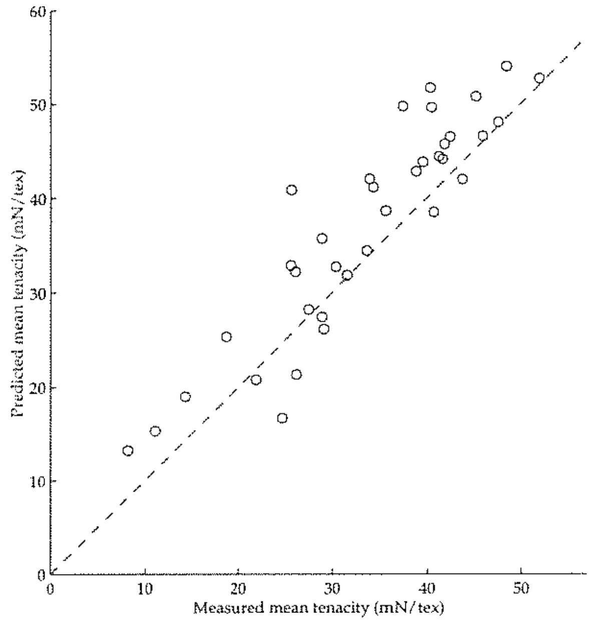

Figure 1. Predicted versus measured mean tenacity.

8.2 Multiple segment model

When the yarn model was evaluated with more than one yarn segment it was called the *multiple segment model*. With the exception of the number of segments, the evaluation of the multiple segment model used the same parameters as those used in the evaluation of the single segment model.

Unfortunately, it was beyond the scope of this work to perform a full validation of the multi-segment model based on the results of the recorded stress-strain curves of all 36 of the yarns whose stress-strain curves were measured. This was primarily a consequence of the large computational requirements of the model. An evaluation of the single segment model typically required less than 1 minute of CPU time on dual 200 MHz Intel PentiumPro processor machine with 128 megabytes of RAM. In contrast, evaluation of a 10 segment model on the same machine could require over 300 hours of CPU time, depending on the number of fibres passing through each cross-section and on their length.

For this section of the work it was decided to restrict attention to predicting the stress-strain properties of a single yarn, called Yarn 23. This yarn was selected for two reasons. Firstly, it was made from fibres that were coarse and short; this means that there were less fibres passing through each yarn cross-section and that these fibres on average contained fewer fibre segments. Secondly, it was of intermediate twist, so that both fibre slippage and fibre breakage would be expected to play an important role in its tensile deformation.

Ten stress-strain curves were predicted using a 10 segment model. The use of 10 segments was somewhat arbitrary. With larger numbers of segments the time required for computation of the stress-strain curve became impractical on the computer equipment available. Further research is required on the effect of the number of segments on the predictions of this model.

Qualitative comparison of the predicted and measured stress-strain curves shown in Figure 2 shows that the predicted curves were very similar to the measured curves at all strain levels. The most obvious differences were the slightly higher modulus of the predicted curves when the strain level was in the range 0.02 to 0.04, and that one of the predicted curves had a breaking strain 0.014 greater than any of the measured curves. The differences in modulus possibly reflect differences between the predicted and actual initial fibre packing ratio.

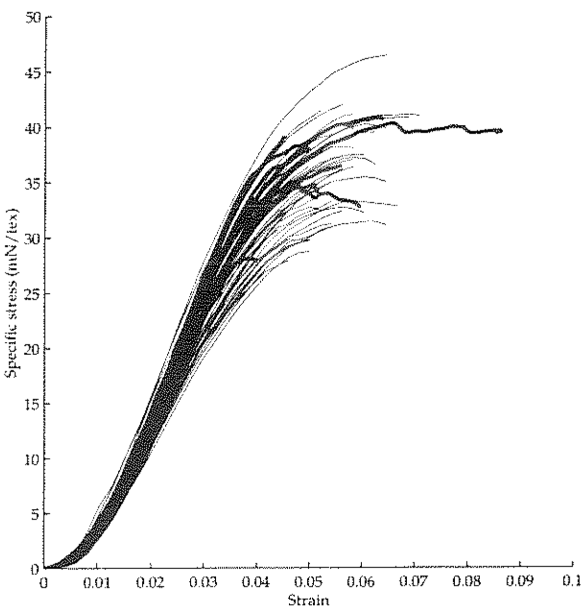

Figure 2. The predicted (bold line) and measured (light lines) stress-strain curves of Yarn 23.

The modulus, tenacity, and breaking strain distributions of the predicted curves were compared statistically with the distributions of these properties obtained from the measured stress-strain curves of Yarn 23. These distributions were compared with the Kolmogorov-Smirnov two-sample test [Press et al., 1993], using the KOLMOG2 procedure of Genstat 5 Release 4.1 (Rathamsted Experimental Station, Harpenden, U.K.). The attained significance levels of this test when comparing the modulus, tenacity and breaking strain distributions were $p < 0.005$, $p = 0.32$, and $p = 0.38$ respectively.

9. CONCLUSIONS

In this paper a model is developed which is capable of predicting the stress-strain curve of a general staple fibre yarn. Within this model, several of the most important yarn structural features are described by arbitrary distribution functions. These include the mass and twist variation along a yarn, the fibre length, the fibre diameter, and the fibre strength. The initial packing of fibres over any cross-section is considered to be a function of both the local linear density and twist level, and the core bulk of the fibre material.

The predictions of the model are compared with the experimentally measured stress-strain curves of 36 different woollen yarns. These yarns varied in twist, and the fibres they were made from varied in diameter, length, core bulk, and fibre type. Limited evaluation of the full model results in predicted tenacity and breaking strain distributions that can not be differentiated from the measured distributions by the Kolmogorov-Smirnov two-sample test. A comprehensive validation of a simplified model shows that the model can predict with reasonable accuracy the tenacity of woollen yarns which are made from fibres with a wide range of properties and which contain a wide range of twist levels.

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