Development of a Simple, Catchment-Scale, Rainfall-Evapotranspiration-Runoff Model

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Abstract: Representation of the hydrological interaction between the land surface and the atmosphere requires considerable improvement, particularly for predicting evapotranspiration feedbacks as input to climate and earth system models. The predictive model developed here attempts to utilize a water balance approach that aims to extract information from the masses of catchment-scale time series data available on precipitation, energy-related variables and stream discharge. Thus it begins with a few simple assumptions in order to seek systematically some synthesis of the climate and landscape controls on evapotranspiration feedbacks and discharge yields. The model adopts the hydrograph identification approach in the linear module of the rainfall-runoff model IHACRES but replaces the previous statistically based non-linear evapotranspiration loss module by a catchment moisture deficit accounting scheme. One advantage of this more conceptual approach is that evapotranspiration can be output on the same time step at which precipitation and energy variables are available (such as from GCMs), and this time step can be shorter (e.g. half hourly) than the discharge time step (e.g. daily) used to calibrate the model parameters.

1. INTRODUCTION

Data requirements of existing rainfall-evapotranspiration-runoff models, whether physically based or of a more conceptual lumped parameter type, are usually too demanding for them to be applied outside experimental areas. Yet these models have been applied in areas where the data required to determine all their parameters are unavailable. This is often the case for Soil-Vegetation-Atmosphere Transfer (SVAT) schemes used in General Circulation Models (GCMs) [Sellers *et al.*, 1986; Dickinson *et al.*, 1986]. When applying models under these circumstances parameter identifiability problems become apparent.

The Project for Intercomparison of Land-Surface Parameterization Schemes (PILPS) is currently in the process of evaluating the performance of 23 SVATs [Timbal, 1997]. In phase 2 of the project they chose two experimental sites (Cabauw, The Netherlands and the HAPEX-Mobilhy site in the south of France) on which to evaluate the SVAT schemes performance. The use of these experimental sites reduces the parameter identifiability problem, allowing some evaluation of the process representations in the models. But just as has been argued for physically-based hydrological models [Beven, 1989] one cannot easily measure parameters which must effectively characterize a heterogeneous landscape. Unfortunately, these SVAT schemes are used to represent the land-surface processes for the entire land surface of the planet when incorporated as components of GCMs.

The most widely available data used in this sort of modeling are time series of precipitation, temperature and streamflow. With only these data sets the danger of over-parameterization has been recognized [Beven, 1989; Hornberger *et al.*, 1985; Jakeman and Hornberger, 1993; van Genuchten, 1991]. It is apparent that when confined to the above data sets models need to be kept simple in their process representation. Too many parameters quickly leads to ambiguously estimated parameter values and questionable physical characteristics being attributed to the catchment.

Estimation of areal evapotranspiration (ET) using techniques such as those of Penman [1948], or amendments of this technique by Monteith [1965] or Priestley and Taylor [1972], can require knowledge of variables such as wind speed, specific humidity, vapor pressure deficit, net radiation, soil heat flux and various vegetation related characteristics. Clearly the data required to explicitly use these techniques are not available extensively. Discussion of ET estimation methods can be found in Garratt [1992], Linacre [1992] and Brutsaert [1982]. Various more empirical techniques, which do not require masses of data, were developed by Thornthwaite [1948], Hamon [1961] and others. These techniques were generally designed to estimate ET at time scales of a month or longer.

Measuring ET is itself a difficult problem with the currently best accepted methods (eddy correlation, Bowen ratio energy balance) being both expensive and labor intensive. Because of this these data sets tend to have only small temporal extent. When attempting to verify ET models, one is therefore forced to use either the short term data given by the methods above (e.g. days of hourly data) or longer term data obtained from a catchment scale water balance approach (e.g. decades of yearly data). Which data are used for verification is usually determined by the time scale of interest, though it is worth noting that methods such as eddy correlation are point measurements, whereas the longer term water balance methods obtained using streamflow measurements can potentially better encapsulate the spatial variation of a catchment.

2. DESCRIPTION OF THE MODEL

The rainfall-ET-runoff model used is based on the structure of the IHACRES metric/conceptual rainfall-runoff model. This model undertakes identification of hydrographs and component flows purely from rainfall, temperature and streamflow data [Jakeman and Hornberger, 1993; Jakeman *et al.*, 1990, 1994]. The IHACRES module structure consists of a non-linear loss module, which converts observed rainfall to effective rainfall or rainfall excess, and a linear streamflow routing module, which extends the concept from unit hydrograph theory that, the relationship between rainfall excess and total streamflow (not just quick flow) is conservative and linear.

The linear module allows any configuration of stores in parallel or series. From the application of IHACRES to many catchments it has been found that the best configuration is generally two stores in parallel, except in semi-arid regions or for ephemeral streams where often one store is sufficient [Ye *et al.*, 1997]. In the two-store configuration, at time step k, quickflow, $x_k^{(q)}$, and slowflow, $x_k^{(s)}$, combine additively to yield streamflow (discharge), q_k:

$$q_k = x_k^{(q)} + x_k^{(s)} \qquad (1)$$

with

$$x_k^{(q)} = -\alpha_q x_{k-1}^{(q)} + \beta_q U_k \qquad (2)$$

$$x_k^{(s)} = -\alpha_s x_{k-1}^{(s)} + \beta_s U_k \qquad (3)$$

where U_k is the effective rainfall.

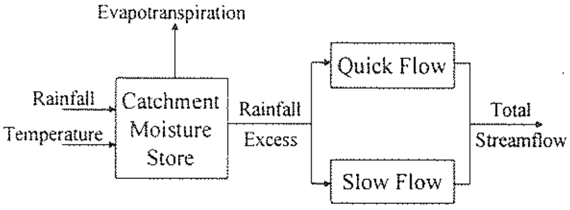

Figure 1: Structure of the rainfall-ET-runoff model.

The IHACRES loss module previously used a statistically based technique to account for antecedent soil moisture conditions and ET losses. Here this module is replaced by our more physically based catchment moisture store accounting scheme which uses rainfall and temperature as inputs and provides ET and rainfall excess as outputs, creating the overall model structure shown in Figure 1.

The catchment moisture store accounting scheme calculates Catchment Moisture Deficit at time step k, CMD_k, according to

$$CMD_k = CMD_{k-1} - P_k + E_k + D_k \qquad (4)$$

Such a scheme has been used previously by Littlewood 9in preparation). CMD is zero when the catchment is saturated and increases as the catchment becomes progressively drier. P is the precipitation, E is the ET loss and D is the drainage. Eventually, however, we wish to enhance the accuracy of the E_k term by utilizing other energy-related variables in its forcing, for example humidity.

Effective rainfall is calculated from

$$U_k = \begin{cases} D_k & CMD_k \le 0 \\ D_k - CMD_k & CMD_k > 0 \end{cases} \qquad (5)$$

Suitable parameterizations for both E_k and D_k were sort which minimized the number of parameters needed and for which the only data requirements are temperature, rainfall and streamflow. Several parameterizations were tried and some of the results are presented below.

For ET modeling, techniques vary considerably in their relationship between ET and temperature, T, with ET being proportional to T, T^2 or T^3 in most cases. The effects of vegetation on ET have been represented in various ways, most notably by the incorporation of a 'surface resistance' in the Penman-Monteith equation. This surface resistance has itself been estimated in many ways, commonly with a dependence on the available soil moisture such as that given by Stewart [1989] where ET is related to the soil moisture deficit, $\delta\theta$, using the expression $\exp[K(\delta\theta - \delta\theta_{\max})]$. Here K is a constant. Two of the parameterizations investigated are

$$E_k = c_1 T_k \exp(-c_2 CMD_k) \qquad (6)$$

$$E_k = \begin{cases} E_{\max}\left[1 + c_1 (T_k - 20)^3\right] & CMD_k \le c_2 \\ E_{\max}\left[1 + c_1 (T_k - 20)^3\right] & CMD_k > c_2 \\ \quad \cdot \exp(c_2 - CMD_k) & \end{cases} \qquad (7)$$

Table 1: Hydrometeorological characteristics of catchments investigated

Catchment	Area, km^2	Precipitation, mm/yr	Average Daily Maximum Temperature, °C	Annual Yield, %
Watershed 34 (Coweeta, North Carolina)	0.33	2012	19.6	46
Watershed 36 (Coweeta, North Carolina)	0.49	2012	19.6	64
Peel River @ Chaffey Dam	407	782	17.5	11.7
Scott Creek @ Scotts Bottom	27	1090	14.8	20.5

Equation (6) has ET directly proportional to temperature and decreasing exponentially as CMD increases. Equation (7) uses a step function with ET remaining at some maximum value (perturbed by temperature) until CMD reaches some critical value c_2, above which ET decreases exponentially as CMD increases. This reflects the idea that vegetation will continue to transpire at a maximum rate until the soil moisture is too low for the root system to access without stress.

It was assumed that drainage is not temperature dependent. As with the modeling of ET several parameterizations of drainage were investigated. Two of those investigated are

$$D_k = \begin{cases} \dfrac{-c_3}{c_4} CMD_k + c_3 & CMD_k < c_4 \\ 0 & CMD_k \geq c_4 \end{cases} \tag{8}$$

$$D_k = \begin{cases} D_{\max} & CMD_k \leq c_3 \\ D_{\max} \exp(c_3 - CMD_k) & CMD_k > c_3 \end{cases} \tag{9}$$

where $D_{\max}$, c_3 and c_4 are non-negative constants. To maximize simplicity and minimize computing time required, $[CMD_{k-1} - P_k]$ was used in place of CMD_k in the ET and drainage equations, along with the extra requirement that if $[CMD_{k-1} - P_k] < 0$ then it is considered as equal to zero.

To measure the performance of the model estimate of streamflow, $\hat{q}_i$, two performance statistics are used: the bias (B) and the observed streamflow variance explained (R^2). These are defined as

$$B = \frac{1}{n} \sum_{i=1}^{N} \left(q_i - \hat{q}_i \right) \tag{10}$$

$$R^2 = 1 - \alpha_e^2 / \alpha_q^2 \tag{11}$$

where α_e^2 and α_q^2 are the variance of the model residuals $(q_i - \hat{q}_i)$ and of the observed streamflow respectively.

Unfortunately, adequate ET data for the catchments investigated were not available so no direct performance measure of the ET estimate could be made. It should be noted though that low bias in predicting streamflow indicates that losses from the catchment were overall accounted for well. The distribution of these losses from day-to-day remains in question.

3. EXAMPLE CATCHMENTS

To investigate the applicability of this non-linear module for ET loss and discharge prediction, it was applied on a daily time step to a selection of catchments covering a range of scales and climatic conditions. The hydrometeorological characteristics of the selected catchments are given in Table 1.

Two small catchments were selected from the Coweeta Hydrological Laboratory in the United States. Coweeta is located in the Nantahala Mountains of western North Carolina. Watershed 36 is a high-elevation, steeply sloping catchment with shallow soils, and a high annual yield and a large proportion of quick flow [Swift *et al.*, 1988]. Watershed 34 is a mid-elevation catchment with somewhat deeper soils and, consequently, substantially more delayed flow. Details of the physical characteristics of the Coweeta catchments are given by Swank and Crossley [1988].

The largest catchment investigated is the Peel River upstream of the Chaffey Dam. The Chaffey catchment is within the Namoi Basin and is located south of the city of Tamworth in northeastern New South Wales, Australia. The topography comprises precipitous slopes and rough terrain along its southern boundary and eastern region. A north-south trending fault escarpment bisects the catchment and separates this precipitous country from gently undulating terrain in the western section. The Peel River upstream of Chaffey Dam is a quickly responding ephemeral river with a low annual yield.

Table 2: Calibration results for the four catchments using (7) and (9) in the non-linear loss module.

Catchment	c_1	c_2	c_3	E_{max}	D_{max}	R^2	*B (mm/d)*
Watershed 34, Coweeta	0.0001	30	12	6	1	0.89	-0.06
Watershed 36, Coweeta	0.0000	32	11	2	5	0.77	0.03
Chaffey	0.0008	103	24	2	1	0.88	-0.01
Scott	0.0003	128	10	3	1	0.87	0.00

Table 3 : Calibration results for the four catchments using (6) and (8) in the non-linear loss module.

Catchment	c_1	c_2	c_3	c_4	R^2	*B mm/d*
Watershed 34, Coweeta	0.43	0.06	1	9	0.90	-0.06
Watershed 36, Coweeta	0.04	0.00	7	55	0.79	0.00
Chaffey	0.17	0.03	0	*	0.84	0.02
Scott	0.24	0.01	2	7	0.86	0.00

The final catchment used in this investigation is Scott Creek at Scotts Bottom. It is situated near the city of Adelaide in South Australia. The catchment is dominated by winter rainfall and can completely dry up over summer. Soils in the catchment are made up of a sandy loam top layer with a clay subsoil.

4. RESULTS

The model, using (7) and (9) in the non-linear loss module and two stores in parallel in the linear module, was calibrated on all four catchments and the results are given in Table 2.

The model, using (6) and (8) in the non-linear loss module, was also calibrated on all four catchments and the results are given in Table 3. Figure 2a-d shows the modeled streamflow against observed streamflow for this model. Figure 3 shows the modelled catchment moisture deficit changing over time in the Scott catchment. It clearly demonstrates the seasonal nature of streamflow in Scott creek. Each year CMD gradually builds over summer, a period of little rain, and then it is reduced quickly when the rains return in winter. Comparing Figure 3 and Figure 2d demonstrates the strong relationship between CMD and streamflow. Figure 4 shows the ET predicted for the Scott catchment.

5. DISCUSSION AND CONCLUSION

A comparison of Tables 2 and 3 show the model performs similarly using either (6) and (8) or (7) and (9) in the non-linear loss module. Using (6) and (8) may provide a slight advantage in keeping the bias closer to zero. Clearly the extra parameter in (7), E_{max}, provides no increase in model performance while increasing model complexity and parameter uncertainty. From this evidence we could conclude that the extra parameter is not warranted on these data sets, at least in this functional form, and so we focus our attention on the simpler model. However, selection of the most appropriate parameterization in the future will require more extensive testing on longer data sets and across more hydroclimatologies.

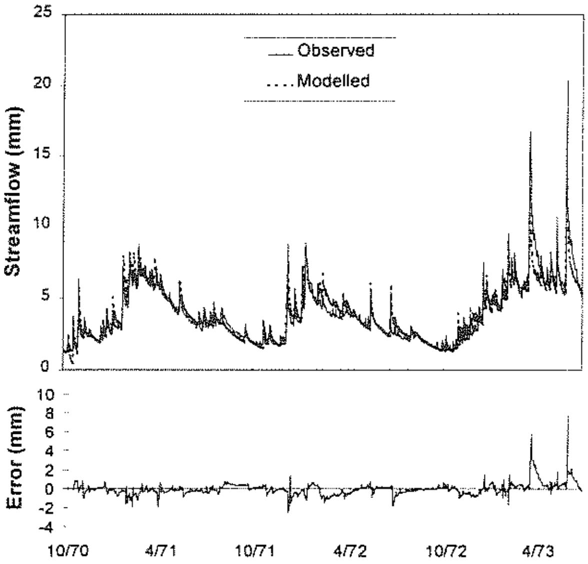

Figure 2a : Model fit for Coweeta watershed 34

Clearly from Table 3 and Figure 2 the model captures the streamflow characteristics of each catchment quite well, with R^2 ranging from 0.79 to 0.90. Notably, the bias remains low in all cases. Coweeta watershed 36 and Scott produce zero bias, Chaffey producing a bias of only 0.02 mm/day and Coweeta watershed 34 having a bias of -0.06 mm/day. These low biases indicate that the total ET losses have been estimated well. ET data, preferably on a daily basis, is required though if we are to have confidence in the estimated ET losses per day. Figure 4 shows an example of the daily ET losses predicted by the model.

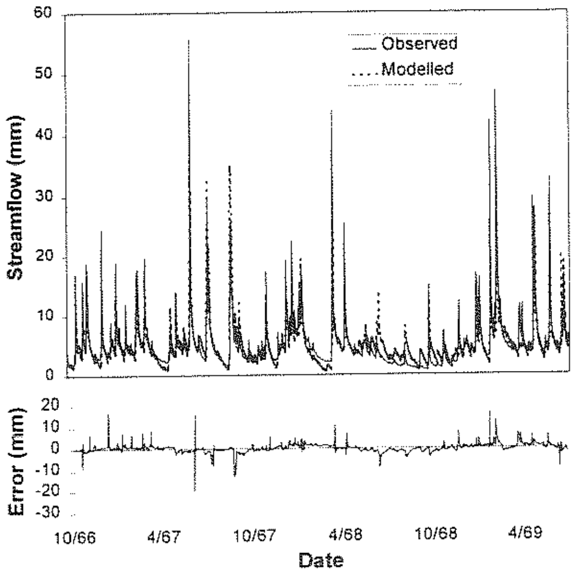

Figure 2b : Model fit for Coweeta watershed 36

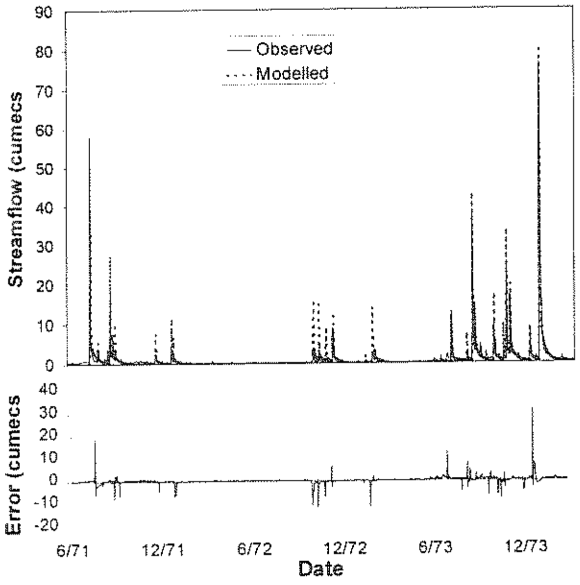

Figure 2c : Model fit for Chaffey catchment

From Table 3 we see that the best calibration results for the Chaffey catchment occur when c_3 is zero. So in the Chaffey catchment the drainage is always zero and we only get effective rainfall when CMD is forced below zero by the rainfall. This is an ephemeral catchment that often has very little or no streamflow. The non-linear module is acting as a threshold mechanism adjusted by the antecedent moisture conditions. Here the model has collapsed down to five parameters and the non-linear module is acting similarly to the bucket model of Manabe *et al.* [1965].

Table 3 also shows that the best calibration results for

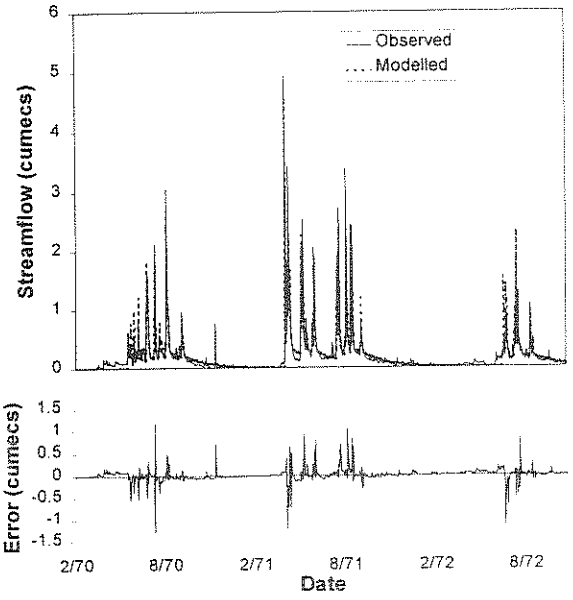

Figure 2d : Model fit for Scott catchment

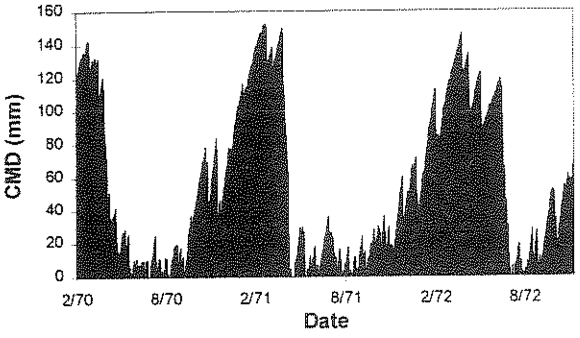

Figure 3 : Catchment moisture deficit in the Scott catchment

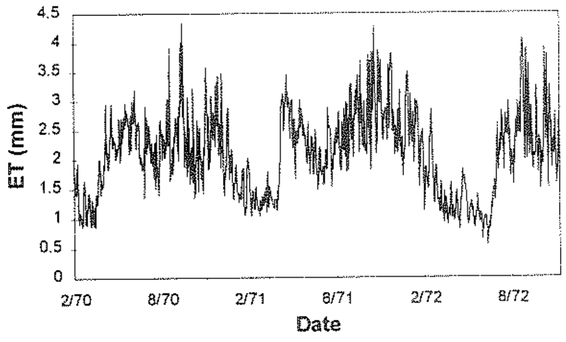

Figure 4 : Evapotranspiration in the Scott catchment

Coweeta watershed 36 occur when c_2 is zero. This means that the ET has no dependence on CMD. For the period of calibration this is not entirely surprising as Coweeta has relatively high rainfall and so the CMD may rarely get high enough to affect the transpiration of the vegetation and therefore ET.

By using this semi-physical approach as described in the accounting scheme in (4) the physical interpretation of the parameters will be made easier than with the previously used statistical approach. It is hoped that, by studying catchments with gauged discharge data and possibly measured ET data, relationships can be constructed between the model parameters and landscape attributes. These relationships would permit the simulation of streamflow and ET for changes in land use and for ungauged catchments, at least within a similar region. The construction of these relationships is only made possible by the minimal parameterization of catchment hydrologic response, with the model, using (6) and (8), having at most 7 parameters. Three of these are the routing parameters in (2) and (3).

Only temperature and catchment moisture are investigated as forcing variables for ET here. However it is envisaged that other important variables will be considered as the methodology is developed further.

When refined, this approach to modeling rainfall-ET-runoff could be used to provide the land surface feedbacks for a climate model. It will be necessary to spatially disaggregate climate forcing variables from the grid scale down to the relevant catchment scale, and to temporally disaggregate daily ET and energy feedbacks from the land surface to the atmosphere down to a sub-daily time step of the order of an hour. These scale issues are recognized problems [e.g. Kalma and Sivapalan, 1995] and the focus of continuing research. The accounting scheme in (4) potentially permits this disaggregation. Provided the ET expression, such as (6), is applicable for the sub-daily time step, the model can be calibrated by fitting daily or higher – average discharge data.

In this approach, parameterization of all gauged catchments must be undertaken off-line, but once inserted in a GCM the overall computational complexity is little greater than that of a bucket model. These parameterizations are valid at least for historically tested climate conditions, vegetation and land use status. With the construction of relationships between the model parameters and the landscape attributes the model parameterizations would gain a much wider applicability.

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