INTEGRATED HYDROGEOLOGICAL MODEL DEVELOPMENT FOR THE WAKOOL IRRIGATION DISTRICT

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Abstract This paper describes the development and application of an integrated hydrogeological model for the Wakool Irrigation District (WID) in New South Wales Australia. The WID has experienced high water table levels and increasing land salinisation since the mid 1950s. In 1980, 10% of the total area in the district had a shallow water table (less than two metres below ground surface). The affected area has since been reduced through operation of 59 pumps extracting approximately 12,500 ML/year of salty water, which is, directed to two evaporation basins. Nevertheless, due to large-scale irrigation, the water table is still rising at an average rate of 30-70 mm/year. The community concerns on the expanding high water table areas, land salinisation, the role of groundwater pumping, and the general sustainability of the area, has led to the development of a Land and Water Management Plan (LWMP) for the WID. One of the main objectives of the LWMP is to encourage landholders to adopt sustainable land and water management practices, decrease and control the water table rise, and monitor the progress of subsequent strategies. An important component in the development and execution of the LWMP is to establish a hydrogeological model, which will be able to analyse the complex hydrogeological regime in the WID, and predict the environmental impacts of various planned management options. This paper presents the Wakool Irrigation District Model (WIDM) based on the MIKE SHE integrated catchment modelling system. The WIDM provides a complete description of the complex hydrological regime in WID involving temporal and spatial variations in the exchange of water between the ground surface, drainage and supply systems, and the groundwater aquifers within the area. Management options have been analysed for a period between 1975 and 2020. Results from scenarios such as the implementation of on-farm recycling ponds, laser leveling, deep-rooted perennials, tree planting, installation of deep groundwater pumps and the effect of shallow groundwater pumping will be presented. The close interaction between evaporation and shallow groundwater tables will be demonstrated as well as the effect of rainfall distribution on the rise of water table.

1. INTRODUCTION

Rising groundwater tables and subsequent land salinisation has been observed during the last three decades in many irrigation districts throughout the south-western part of NSW. Crop yield is significantly influenced by shallow saline groundwater table lying within a critical depth (less than 2 m) below the land surface. Concerns about the increasing environmental problems have encouraged communities to develop a Land and Water Management Plan (LWMP) to ensure a future sustainable agricultural industry. The various management options proposed in the LWMPs depend on the individual characteristics of the districts, and include common features concerning surface drainage, infrastructure, flood plain management, sub-surface drainage schemes, and on-farm management.

Several hydrological and hydrogeological studies have been carried out in order to increase our understanding of the hydrological systems and assess future conditions in the districts (e.g. Bogoda et. al. [1994]). These include development of a comprehensive hydrological model, which will be a valuable tool not only in the planning phase, but also during implementation and operation of selected options. The existing model for the Wakool Irrigation District (WID) is a regional model covering the entire district (Storm and Punthakey [1996]). This model covers an area of 2760 km^2 and has a spatial resolution of 2×2 km. The regional model provides a good framework for analysis of a range of options, in particular those relating to groundwater management. This paper presents the Wakool Irrigation District Model (WIDM) based on the MIKE SHE integrated modelling system.

The WIDM provides a complete description of the complex hydrological regime in WID involving temporal and spatial variations in the exchange of water between the ground surface, drainage and supply systems, and the groundwater aquifers within the area. Different scenarios on water management have been analysed for a period between 1975 and 2020. Results from options such as the implementation of on-farm recycling ponds, laser leveling, deep rooted perennials, tree planting, installation of deep groundwater pumps, effect of shallow groundwater pumping will be presented. The close interaction between evaporation and the shallow groundwater table will be demonstrated as well as the effect of rainfall distribution on the rise of water table.

2. DESCRIPTION OF THE AREA

The Wakool Irrigation District is located in the central part of the Murray Darling River Basin west of the town Deniliquin and north-east of Swan Hill. The district is bounded by the Wakool River in the south and Edward River in the north (Figure 1). Irrigation commenced in WID in 1936 and slowly expanded substantially and today covers approximately 220,000 ha.

The climate in the district is semi-arid with an average rainfall of approximately 360 mm per year. High rainfalls occur mainly during the winter months June to August, and a large part of the district is occasionally prone to flooding. The topography is in general flat with a westerly direction of groundwater flow and natural surface drainage.

Three major hydrogeological formations are located within the area with the Shepparton Formation forming the upper aquifer system. The depth and extent of the water bearing layers in the Shepparton are limited with pronounced variations in transmissivity across the district. Prior streams and ancestral river systems, which are remnants from former river routes, play an important role for groundwater flow paths. The deep regional aquifer systems, the Calivil and Renmark Formations play an important role for the regional groundwater flow in the Murray Basin, and the piezometric heads in these aquifers control the exchange to and from the Shepparton.

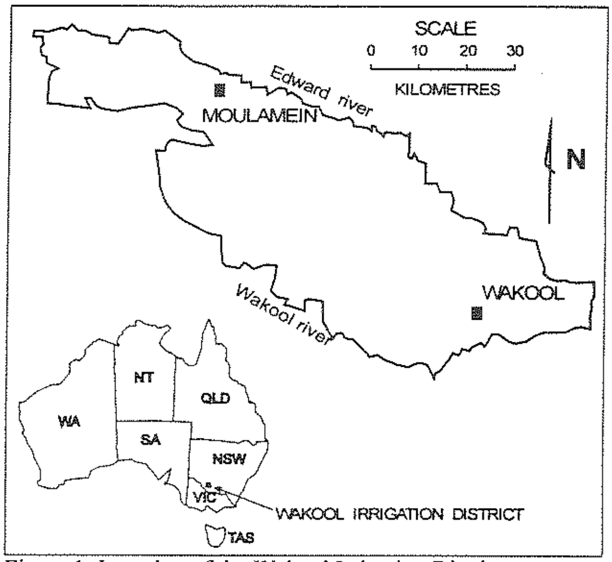

Figure 1: Location of the Wakool Irrigation District.

Within the WID, there seems to be a fine balance between heads in the Shepparton and the Calivil/Renmark aquifers, which determines the leakage between the aquifers. In contrast to more easterly located irrigation areas, the piezometric heads in the deeper aquifers have reached the same overall level as the groundwater table, and therefore the potential for deep recharge within the area is decreased.

In general, the Shepparton provides a relatively poor drainage capacity for the infiltrating water through the rootzone. Thus, natural rainfall combined with large-scale irrigation in the district have caused the area of shallow water table to increase from 7,200 ha to 47,500 ha during the period 1960 – 75. A large scale groundwater pumping scheme, the Wakool Tullakool Sub-Surface Drainage System (WTSSDS), which was established in the early 1980's has partially alleviated the problems in the south-eastern part of the district. However, currently an area of 26,100 ha is affected by high water table, which is continuing to expand under the present irrigation management. The current irrigation practices and possible further rise in the piezometric head in the deeper aquifers are expected to cause a further increase in groundwater table in many parts of the district.

3. METHODOLOGY

The development of the Wakool farm Model has involved the following steps:

- Identification of all relevant information;
- Collation of information and data processing;
- Setting up of the model;
- Calibration of the model;
- Perform model runs to predict the effect of the proposed management options on the regional groundwater environment.

A common belief is that only very sparse information is available to support a comprehensive model development for the irrigation districts in the Murray Region. This may be true considering direct usable information only. However, for hydrogeological information, at least for the Wakool Irrigation District, there is considerable indirect information, which after processing has been very useful for the model development. A broad list of the most important information utilized in the development of the model is presented in Table 1.

3.1 Model Conceptualisation

In order to develop a reliable modelling tool, the mathematical model should reflect all the important natural processes as well as provide the possibility to impose relevant human interventions in the natural system. Important features, which are common for irrigation areas include: natural and constructed drainage lines; supply channels with particular emphasis on representation of sites where significant seepage losses may occur; pumping schemes, and pumping of saline groundwater to evaporation basins.

The model must further be able to simulate the infiltration and evapotranspiration for agricultural crops on specific soils. In this respect, the capillary rise mechanism in shallow water table areas is an important factor in the overall and local water balances.

The groundwater system in the WID area can be divided into two aquifer systems, Shepparton and Calivil/Renmark. The former is further divided into two layers, representing a sandy and relatively permeable layer in the upper 15 to 20 meters from the surface. We assume that the main groundwater movement in horizontal directions within the area occurs in this layer. Beneath it is the low permeable Lower Shepparton, which primarily is responsible for the vertical flow exchange between the Upper Shepparton and the Calivil / Renmark

2.4 Method of characteristics

Consider the first order ODE (also called characteristic equation)

$$\frac{dx}{dt} = u(x,t) \tag{5}$$

Solution to (5) is a family of characteristic pathways $(x, t(x))$ parameterized by $x \in [0, L]$. Along any such path the PDE (2) becomes the ODE

$$\frac{dc}{dx} + \frac{wp}{Q}c = \frac{s}{Q}$$

where it has to be understood that the argument for all variables is $(x, t(x))$, i.e. taken on a characteristic pathway. Upon integration along the specific characteristic pathway $(x, t(x))$ originating from the origin $(x,t) = (0,0)$ down to the stream's exit point $(L, t(L))$ yields the following relationship

$$\begin{aligned} c(L, t(L)) &= \frac{1}{\psi(L)}\Bigg(c(0, t(0)) \\ &+ \int_0^L \psi(\xi)\frac{s(\xi, t(\xi))}{Q(\xi, t(\xi))}d\xi\Bigg) \end{aligned} \tag{6}$$

where $\psi(x) = e^{wp\int_0^x d\xi/Q(\xi,t(\xi))}$.

Let $\tau(t)$ denote the time it takes for a water parcel to move at time t from the upstream location $x = 0$ to the downstream location $x = L$. Then clearly

$$\tau(t) = t(L) - t(0) \tag{7}$$

With this, equation (6) shows that at the downstream location, the sediment concentration at time t will be composed of the initial upstream concentration at time $t - \tau(t)$, reduced by the factor $1/\psi(L)$ because of settling p, plus a concentration due to resuspension, bank erosion, and rainfall along the stretch appearing in the form of the source/sink term s.

If detailed information on the stream system is available, the characteristic equation (5) can easily be solved numerically and so yield for each time t the characteristic pathway originating at the upstream location at time $t-\tau(t)$. With such characteristic paths available and detailed information on sediment sources, $\psi(L)$ and the integral on the right hand side of (6) are easily computed numerically.

An advantage of the characteristic pathway approach is that it allows sharp concentration fronts to be moved without spurious numerical effects. This is in contrast with other schemes such as finite difference of the form (4) which are notoriously difficult to fine tune in hyperbolic PDEs because of their tendency to add spurious numerical effects to the solution (such as numerical diffusion and oscillatory components near sharp concentration fronts).

However, as already mentioned above, in many field studies there is not enough information about the catchment to determine within the required time and space intervals all necessary modelling variables and parameters. This means that in general the detailed information required to solve the characteristic equation (5) and compute the integrals in (6) is not available. In such cases, use of the above described characteristic pathway approach requires simplifying assumptions.

2.5 Model simplifications

The first assumption we shall invoke is that $Q(x,t)$ and $s(x,t)$ change only slowly over each travel time period $[t(0), t(L)]$ of length $\tau(t)$) associated with the given characteristic pathway $(x, t(x))$ solution to (5). A consequence of this assumption is that it is easy to derive a model for the water parcel travel time $\tau(t)$ as a function of stream discharge (Dietrich *et al*, [1989]). Furthermore, over such time intervals, $Q(x,t)$ and $s(x,t)$ can be replaced by their averages $\bar{Q}_t$ and $\bar{s}_t$ taken over the time interval $[t(0), t(L)]$ where the time subscript is used to indicate that these averages do depend on the time origin $t(0)$ and travel time $\tau(t)$. With this, upon integration, (6) becomes

$$\begin{aligned} c_L(t) &= c_0(t-\tau(t))e^{-\frac{wpL}{\bar{Q}_t}} \\ &+ \frac{\bar{s}_t}{wp}(1-e^{-\frac{wpL}{\bar{Q}_t}}) \end{aligned} \tag{8}$$

where, for convenience, the upstream and downstream concentrations at time t are denoted $c_0(t)$ and $c_L(t)$, respectively.

The average discharge $\bar{Q}_t$ can be estimated from available upstream and downstream discharge data. As for the source term $\bar{s}$ defined in (3), recall that its first component $wc_bf(u-u_*)$ accounts for resuspension. Although models for resuspension are available, they are of little use here because of scale differences and lack of data to fine tune resuspension parameters. For this reason, the bed concentration c_b has been assumed constant. Furthermore, the resuspension threshold velocity function $f(u-u_*)$ has been assumed to be proportional to a power of $\bar{Q}_t - Q_*$ when $\bar{Q}_t \geq Q_*$ and zero otherwise, i.e.

$$f(u-u_*) = \begin{cases} \nu(\bar{Q}_t - Q_*)^\mu & \text{if } \bar{Q}_t \geq Q_*; \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

As we shall see, both μ and Q_* have to be calibrated from the available data.

There remains to take into account the input of sediment mass per unit time and length ρ due to bank erosion and sediment entering the stream because of rainfall. The former can only be modelled very crudely. As for the latter, data for the Murray suggest that small bursts of rainfall may indeed cause short pulses of sediment. However, these pulses are small and essentially negligible when compared to the other processes. Here it will be assumed that ρ is constant. This is of course a very limiting assumption which we shall attempt to relax in the Namoi implementation of the model. With this and inclusion of (9), equation (8) can be rewritten in the final form

$$\begin{aligned} c_L(t) &= c_0(t-\tau(t))e^{-\frac{\alpha}{\bar{Q}_t}} \\ &+ (\frac{\beta\eta(\bar{Q}_t - Q_*)^\mu + \gamma}{\alpha})(1-e^{-\frac{\alpha}{\bar{Q}_t}}) \end{aligned} \tag{10}$$

where η equals 1 if $\bar{Q}_t \geq Q_*$ and 0 otherwise: and $\alpha =$

2. MODELLING APPROACH

2.1 Types of models

Models for sediment transport in streams can essentially be classified in three groups. These are: small scale models (a few meters) usually associated with flume experiments such as in Voogt *et al* [1991] and Bridge and Bennet [1992]; medium scale models (kilometres) based on coupled partial differential equations (PDEs) expressing conservation principles for water and sediments such as in van Niekerk *et al* [1992] and Celic and Rodi [1991]; and large scale empirical/stochastic models such as extension to the rating curve approach, or transfer function models identified and estimated from time series of flow, turbidity, and other available data such as in Lemke [1991], van Sickle and Beschta [1983] and Kelsey *at al* [1987].

2.2 The general approach

In this work our approach is to seek a model that has only modest data requirements, is relatively simple to implement, and that can be used to predict with sufficient accuracy daily downstream sediment concentration over stretches of up to 100 km. For this reason, we shall rely on sediment mass conservation principles, excluding the data and computer intensive momentum equation governing stream dynamics. Instead of the latter, a travel time model for water parcels is invoked. Furthermore, for simplicity, the stream channel is modelled as a rectangular box of constant width w extending along the x-axis with all stream properties constant over cross sections normal to it. Water flows in the x direction. This lay out is illustrated in Figure 1:

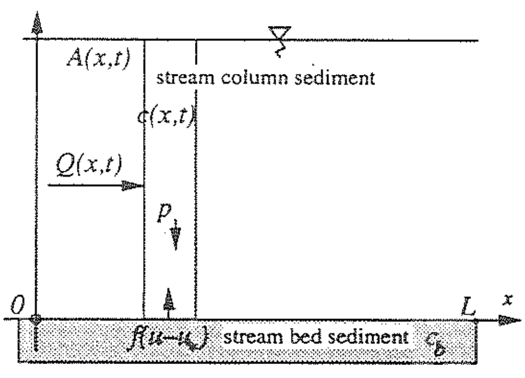

Figure 1: stream schematic representation

In Figure 1 and throughout the paper, variable definitions are

- A wetted stream cross-section;
- Q stream discharge;
- u stream water velocity Q/A;
- c stream column sediment concentration;
- c_b stream bed sediment concentration;
- p stream sediment settling rate;
- f resuspension rate; and
- t time.

Assuming negligible sources or sinks of water along the stretch of interest, water mass conservation yields the partial differential equation (PDE)

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

In addition, if sediments are carried by the stream with convection and resuspension assumed to be the dominant transport processes at play, equation (1) together with a sediment mass balance yields the sediment transport PDE

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + \frac{wp}{A}c = \frac{s}{A} \tag{2}$$

where $s(x,t)$ denotes the total sediment mass source per unit time and length, here set to

$$s = wc_b f(u - u_*) + \rho \tag{3}$$

where w is the stream channel width, $f(u-u_*)$ is a sediment resuspension velocity function with threshold u_*; and $\rho(x,t)$ is the sediment mass source per unit time and length from bank erosion and rainfall. In (2) no allowance has been made for diffusion as it is expected to be small when compared to convection and resuspension.

2.3 The ordinary differential equation approach

Upon a spatial discretization along the stream with nodes $\{x_i\}_{i=0}^n$ and vector $\mathrm{c}(t) = \{c(x_i,t)\}_{i=0}^n$ the PDE (2) can be approximated by the first order, linear system of ordinary differential equations (ODEs)

$$\frac{dc}{dt} = M\mathrm{c} + \mathrm{b} \tag{4}$$

where M is a square matrix and b is a vector, both independent of the sediment concentration c but generally functions of time t. Provided sufficient information is available about the stream system so that all entries in M and b are known *a priori*, a solution to (4) can easily be obtained using one of the many numerical schemes available to first order and linear systems of ODEs. If a few entries in M and b are missing, they can be calibrated via an inverse procedure, for example by minimizing over the space of unknown parameters a sum of squares between historical and computed sediment concentrations. However, in many practical situations the information content required to solve (2) via a discretization of the form (4) is not met; the parameters cannot be identified with reasonable accuracy (see Dietrich sl et al [1993] for a discussion of ill-posedness in environmental mass transport formulations). This is the case for example for the Murrumbidgee and Murray Rivers implementations presented in Sections 3 and 4 below. This is also the case for the Namoi catchment for which the modelling exercise presented in this paper was initiated. For this reason, we shall seek an approach that requires less information while retaining as much system dynamics as possible. To do so, we note that equation (2) is a first order hyperbolic PDE and so is best solved via the method of characteristics as follows.

3.3 Further simplifications

In the above, the ratio $\hat{\alpha}/\bar{Q}_t$ is very small indicating that the loss of sediment to the stream bed via settling is negligible compared to that gained from resuspension and other sources such as bank erosion and rainfall input. In such a case the term $e^{-\alpha/\bar{Q}_t}$ in (10) is not distinguishable from $1 - \alpha/\bar{Q}_t$. Thus, with the above parameter estimates, the downstream sediment concentration at time t as determined by model (10) is essentially equal to

$$c_L(t) = c_0(t-1) + \frac{\beta\eta(\bar{Q}_t - Q_*)^{\mu} + \gamma}{\bar{Q}_t} \qquad (12)$$

where $\bar{Q}_t$ is the discharge average at Wagga between the previous and present day. The first term on the right hand side of (12) is the upstream concentration on the previous day while the second term is the increase in sediment concentration due to resuspension and sediment sources along the stretch.

4. MURRAY RIVER IMPLEMENTATION

4.1 Calibration

A set of data for the Murray River containing discharge and turbidity measured upstream at Cobram and at Tocumwal about 20 km downstream was used to calibrate and validate model (10). The data period used was 1/1/91 to 31/12/93. Since only stage height data is available at Cobram and the distance between Cobram and Tocumwal is small, flow data at Cobram was set to that at Tocumwal.

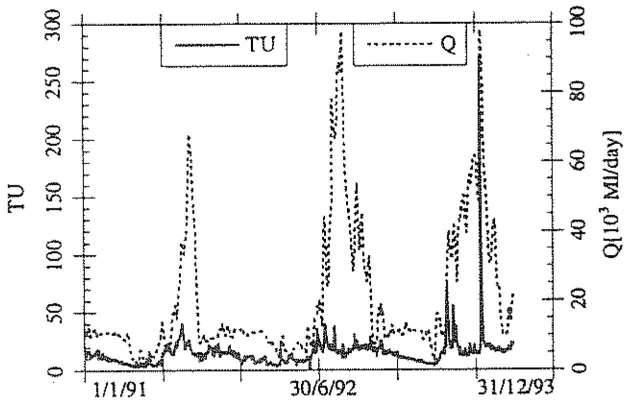

Figure 4: Discharge Q and turbidity TU values downstream at Tocumwal over the period 1/1/91 to 31/12/93.

As was the case for the Murrumbidgee, over the chosen test period there were a few missing flow and turbidity measurements which were linearily interpolated. With the stretch quite short so that travel times were significantly less than the daily sampling time intervals, the travel time $\tau(t)$ was set to zero. Plots of discharge and turbidity for this period are given in Figure 4.

If the whole data set is used for calibration, parameter estimates are

$$\hat{\alpha} = .004 \; ; \; \hat{\beta} = 16 \; ; \; \hat{\gamma} = 6,790 \; ; \; \hat{Q}_* = 4,500 \; ; \; \hat{\mu} = 1$$

4.2 Validation

Adopting again the testing procedure described in Section 3, we partitioned the available data in two portions, one extending over a year for calibration and the other extending over two years for validation.

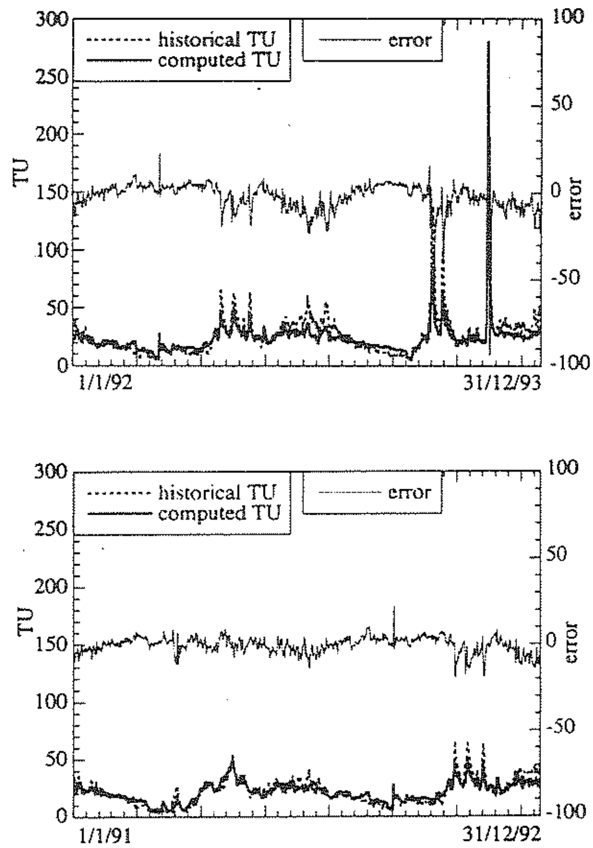

Figure 5: Computed vs historical turbidity TU values at Tocumwal over the 1991-1992 (top) and 1992-1993 (bottom) validation periods.

If the year 1991 is used for calibration, parameter estimates are

$$\hat{\alpha} = .004 \; ; \; \hat{\beta} = 80 \; ; \; \hat{\gamma} = 5,688 \; ; \; \hat{Q}_* = 4,000 \; ; \; \hat{\mu} = 0.8$$

Based on the above estimates, a plot of computed and historical turbidity values for the validation period 1992-1993 is at the top of Figure 5.

If model (10) is calibrated over 1993, we obtain the following least squares estimates

$$\hat{\alpha} = .004 \; ; \; \hat{\beta} = 19 \; ; \; \hat{\gamma} = 9,677 \; ; \; \hat{Q}_* = 4,500 \; ; \; \hat{\mu} = 1$$

Based on the above estimates, a plot of computed and historical turbidity values for the validation period 1991-1992 is at the bottom of Figure 5.

4.3 Further simplifications

Similar to the Murrumbidgee implementation of Section 3, the ratio $\alpha/\bar{Q}_t$ in the two above model implementations is very small so that model (10) is in this case of the very simple form

$$c_L(t) = c_0(t) + \frac{\beta\eta(\bar{Q}_t - Q_*)^\mu + \gamma}{\bar{Q}_t} \tag{13}$$

5. COMMENTS

The validation results of Sections 3 and 4 indicate that the model does not perform very well for the Murrumbidgee River. Indeed, Figure 3 shows that for low TU values, relative errors between computed and historical TU values are often larger than 100%. Furthermore, Figure 3 indicates that the peak computed TU values often underestimate significantly the historical peaks. These peaks are often generated by high stream discharges that followed periods of low to medium discharges, indicating that resuspension is probably the mechanism at play. We can only conclude that the resuspension power function $f(u - u_*)$ as defined in (9) is not adequate as it ought to put more weight on high discharges. This would be the case if instead of a power law, $f(u - u_*)$ had an exponential structure. Tests on this issue remain to be performed.

In contrast to the Murrumbidgee River, the tests performed on the Murray River indicate that TU concentrations are predicted with good accuracy. Therefore, over small stretches of a few tens of kilometres, it appears that our model has the ability to perform reasonably well. This also suggests that in order to predict sediment concentration over such long stretches with sufficient accuracy, data may need to be sampled over a network with node spacing no larger than a few tens of kilometres. Of course, the sampling interval will depend upon the particular flow and sediment response dynamics of the reach in question.

We also note that our approach could be invoked to generate estimates of weekly or monthly average sediment loads.

6. MODEL EXTENSIONS

In addition to a more adequate structure for the resuspension function $f(u - u_*)$, one may seek model improvements by invoking a more refined mechanism for the exchange of sediment between the bed and the stream. For example, (2) does not take into account the fact that as the bed mass increases during low flows due to settling and the absence of resuspension, more sediments will be available for resuspension for the next flood event. Such a mechanism could be included by considering the bed as a reservoir having a bulk sediment concentration which is constant in space but varying in time. In this case the bed concentration c_b in (2) becomes a function of time $c_b(t)$ satisfying the exchange mass balance equation

$$\frac{dc_b}{dt} = \theta\bar{c}p - \kappa c_b f(u - u_*) \tag{14}$$

where $\bar{c}$ is a stream spatial aggregate sediment (lumped) concentration while θ and κ are constants to be determined indirectly through the model calibration exercise. With this, (2) and (14) are now coupled. In this case, the calibration of the unknown parameters and the model implementation are not any more a straightforward exercise. In this light, it is not clear at present whether or not the added complexity associated with using (14) is warranted for systems characterized by a data base similar to that which was available for the Murrumbidgee and Murray modelling exercise presented in Sections 3 and 4.

Acknowledgments

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